Decisions with Multiple Agents: Game Theory

Alice Gao

Lecture 23

Readings: RN 17.5. PM 11.1 - 11.3.

Outline

Learning Goals

By the end of the lecture, you should be able to

- Determine Pareto optimal outcomes
  of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium
  of a 2-player normal form game.

Learning Goals

Dancing or Running? — Pareto Optimality

Prisoner's dilemma

Matching Quarters — Mixed-Strategy Nash Equilibrium

Dancing or concert?

Revisiting the Learning goals

Pareto dominance and Pareto optimality

Pareto dominance:
An outcome o Pareto dominates another outcome o' if and only if
every player is weakly better off in o and
at least one player is strictly better off in o.

A Pareto optimal outcome:
An outcome o is Pareto optimal if and only if
no other outcome o' Pareto dominates o.

What is the difference between the statements below?

- An outcome o Pareto dominates all other outcomes.
- An outcome o is NOT Pareto dominated by any other outcome.

CQ: Dancing or running - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

		Bob: $dancing$	Bob: $running$
Alice	$dancing$	$(2, 2)$	$(0, 0)$
	$running$	$(0, 0)$	$(1, 1)$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Prisoner's dilemma

		Bob	
		cooperate	defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

Alice and Bob have been caught by the police. Each has been offered a deal to testify against the other. They had originally agreed not to testify against each other. However, since this agreement cannot be enforced, each must choose whether to honour it. If both refuse to testify, both will be convicted of a minor charge due to lack of evidence and serve 1 year in prison. If only one testifies, the defector will go free and the other one will be convicted of a serious charge and serve 3 years in prison. If both testify, both will be convicted of a major charge and serve 2 years in prison.

CQ: Prisoner's dilemma - DSE

CQ: Which outcome, if any, is a **dominant strategy equilibrium**?

(A) (cooperate, cooperate)

(B) (cooperate, defect)

(C) (defect, cooperate)

(D) (defect, defect)

(E) There is no dominant strategy equilibrium.

		Bob: cooperate	Bob: defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Bob	
		cooperate	defect
Alice	cooperate	$(-1, -1)$	$(-3, 0)$
	defect	$(0, -3)$	$(-2, -2)$

CQ: Prisoner's dilemma - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Alice \ Bob	cooperate	defect
cooperate	$(-1, -1)$	$(-3, 0)$
defect	$(0, -3)$	$(-2, -2)$

Matching quarters

		Bob heads	Bob tails
Alice	heads	$(1, 0)$	$(0, 1)$
Alice	tails	$(0, 1)$	$(1, 0)$

Alice and Bob are playing the game of matching quarters. They each show one side of a quarter. Alice wants the sides of the two quarters to match, whereas Bob wants the sides of the two quarters to NOT match.

CQ: Matching quarters - Pure-Strategy NE

CQ: How many of the four outcomes are **pure-strategy Nash equilibria**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Bob

		heads	tails
Alice	heads	$(1,0)$	$(0,1)$
	tails	$(0,1)$	$(1,0)$

Matching quarters - Mixed-Strategy NE

- How do we calculate a mixed-strategy Nash equilibrium?

- What does it mean if a player is mixing between two actions?

Dancing or concert?

		Bob	
		dancing	*concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Alice and Bob want to sign up for an activity together. They both prefer to sign up for the same activity. However, Alice prefers dancing over going to a concert whereas Bob prefers going to a concert over dancing.

CQ: Why is a player willing to mix between two actions?

Consider a 2-player normal form game and fix Bob's strategy. Alice is willing to play heads 60% of the time and tails 40% of the time. Which of the following statements is true?

(A) Alice's expected utility of playing heads is greater than her expected utility of playing tails.

(B) Alice's expected utility of playing heads is less than her expected utility of playing tails.

(C) Alice's expected utility of playing heads is same as her expected utility of playing tails.

CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Alice go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

		Bob	
		dancing	*concert*
Alice	*dancing*	$(2,1)$	$(0,0)$
	concert	$(0,0)$	$(1,2)$

CQ: Dancing or concert - mixed-strategy NE

CQ: At the mixed strategy Nash equilibrium, with what probability does **Bob go dancing**?

(A) 0 (B) 1/3 (C) 2/3 (D) 1

		Bob: *dancing*	Bob: *concert*
Alice	*dancing*	$(2, 1)$	$(0, 0)$
	concert	$(0, 0)$	$(1, 2)$

Revisiting the Learning Goals

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