Bayesian Networks

Alice Gao

Lecture 12

Readings: RN 14.2. PM 8.3.

Outline

Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

Learning Goals

By the end of the lecture, you should be able to

- Determine whether an independence relationship holds by applying d-separation.
- Given a Bayesian network and an order of the variables, construct a Bayesian network that correctly represents the independence relationships among the variables.

Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

D-Separation

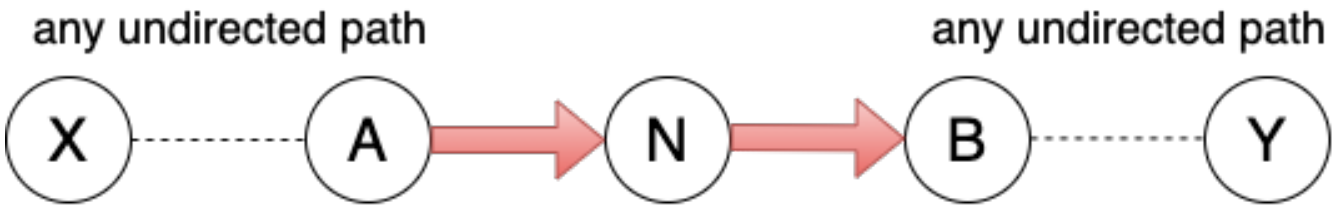
Are two variables X and Y independent
given the set of observed variables E?

Definition (D-Separation)

E d-separates X and Y
iff E blocks every un-directed path between X and Y.

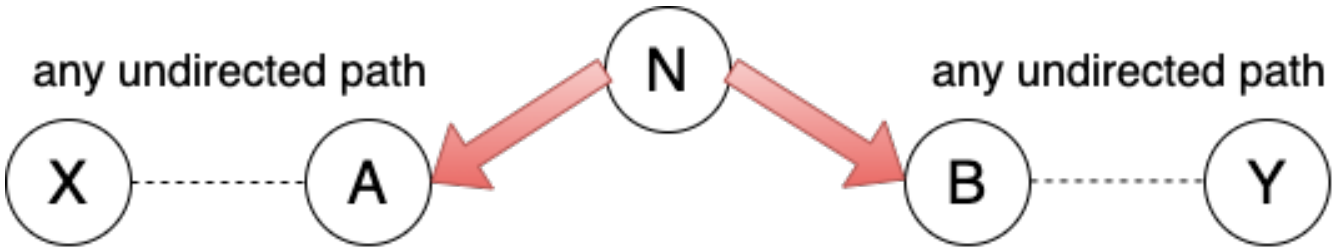
If E d-separates X and Y,
then X and Y are independent given E.

Blocked Path - Scenario 1/3



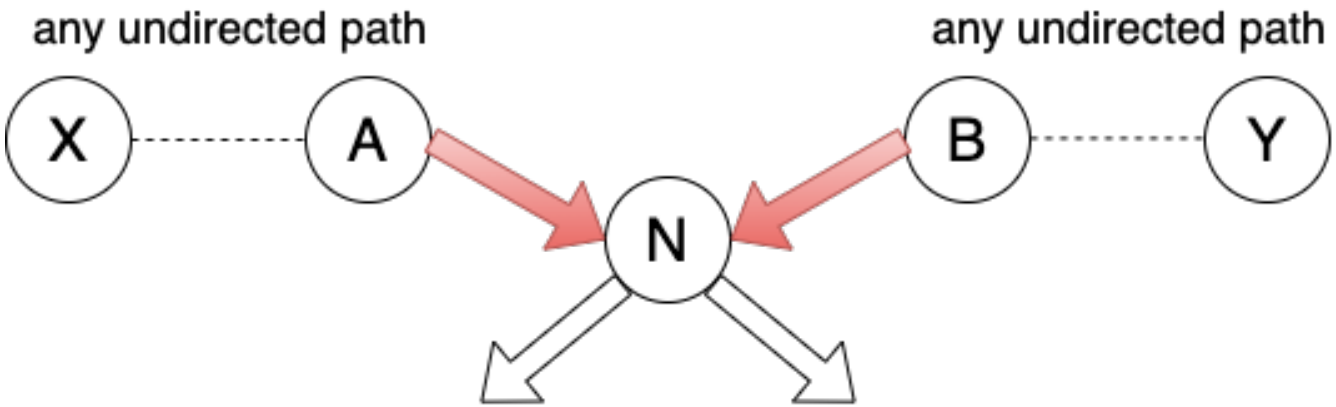
If N is observed,
then it blocks the path between X and Y.

Blocked Path - Scenario 2/3



If N is observed,
then it blocks the path between X and Y.

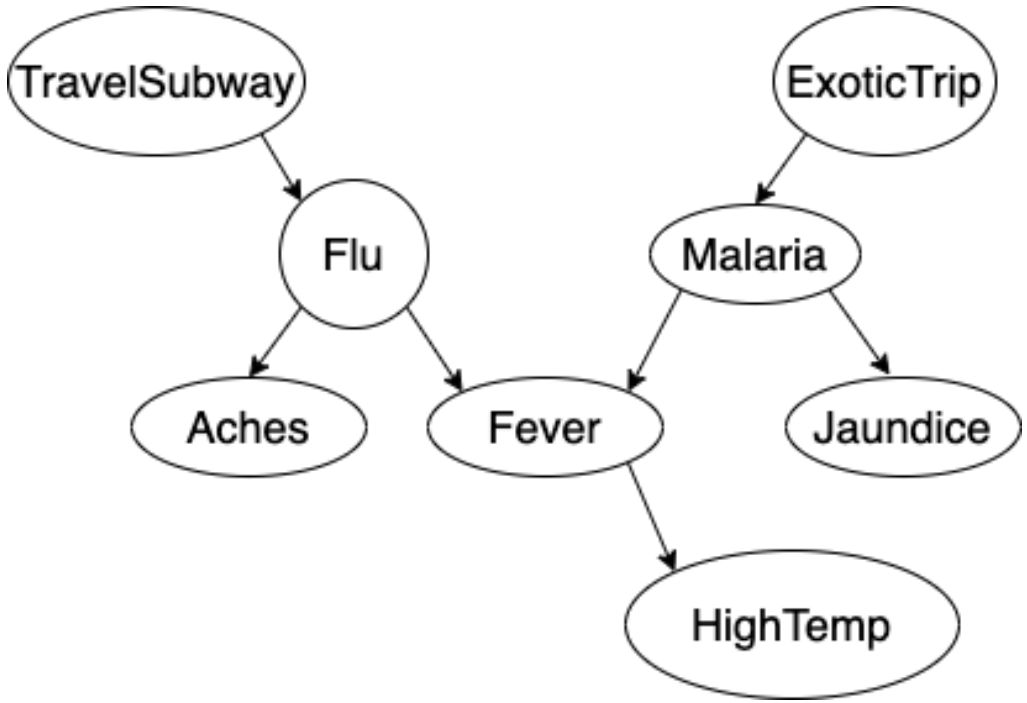
Blocked Path - Scenario 3/3



If N and N's descendants are NOT observed, then they block the path between X and Y.

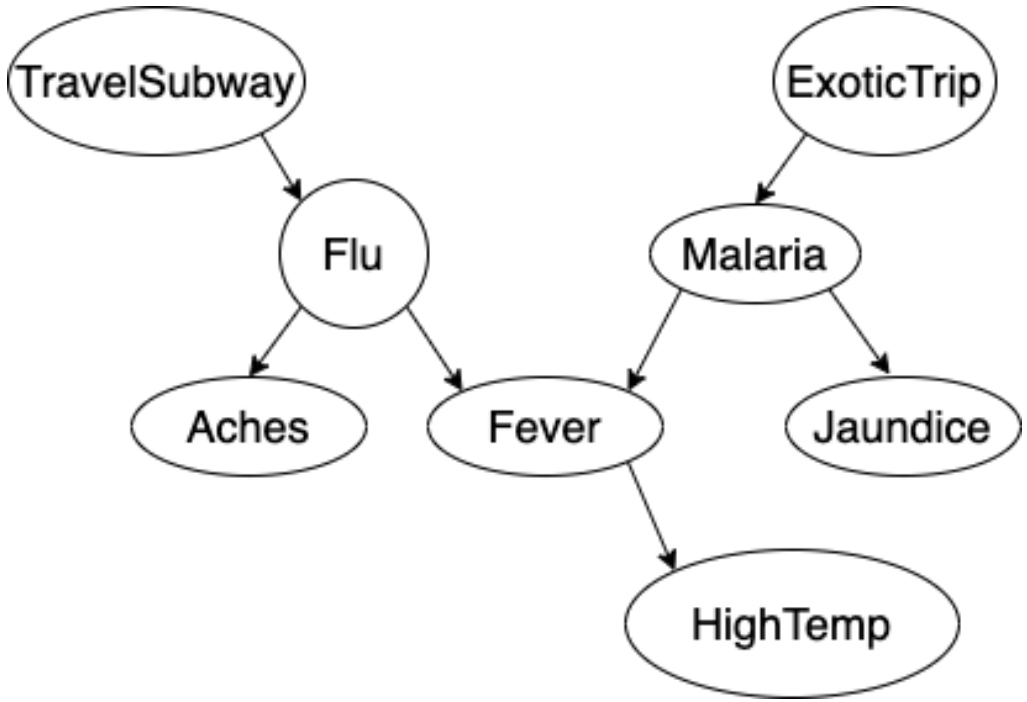
CQ: Applying D-separation

CQ 1a: Are **TravelSubway** and **HighTemp** independent?



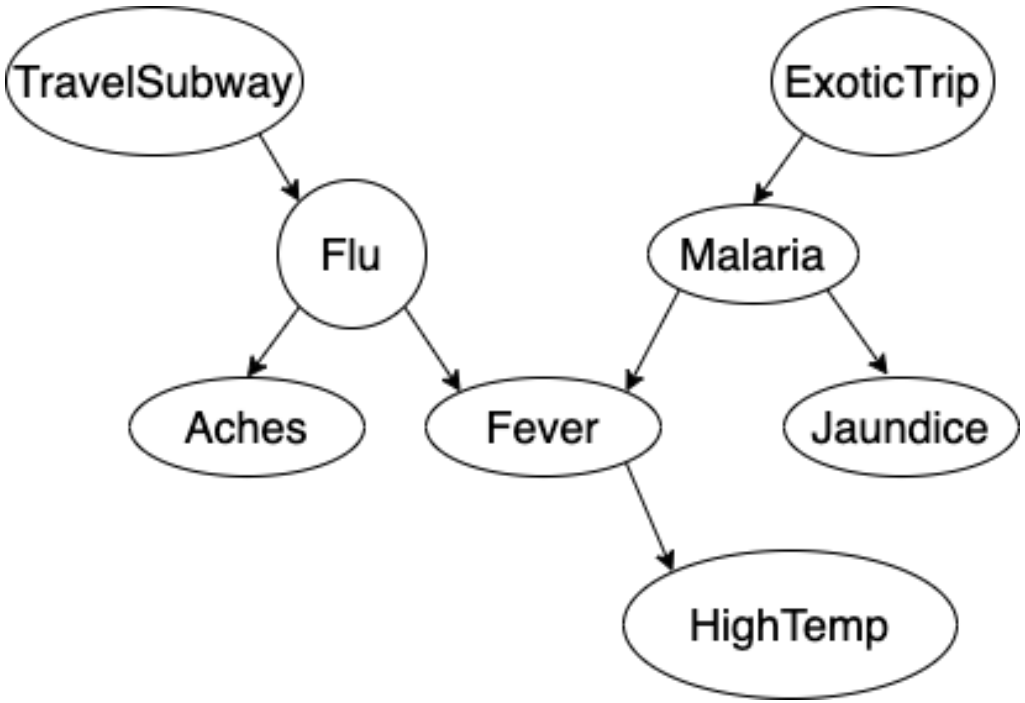
CQ: Applying D-separation

CQ 1b: Are **TravelSubway** and **HighTemp** independent given **Flu**?



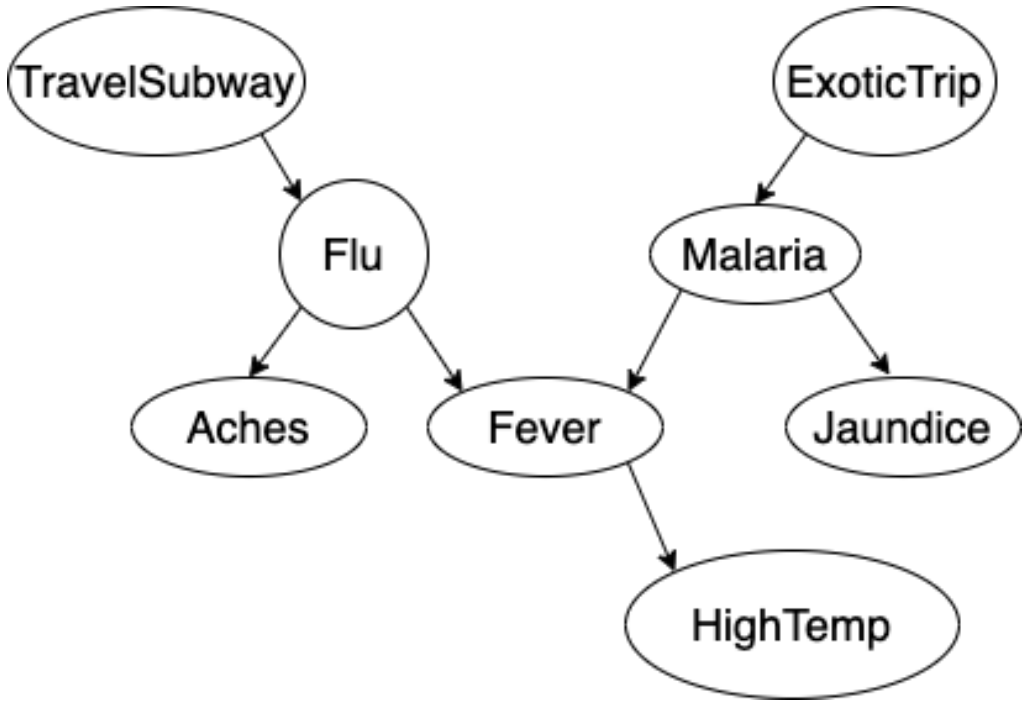
CQ: Applying D-separation

CQ 2a: Are **Aches** and **HighTemp** independent?



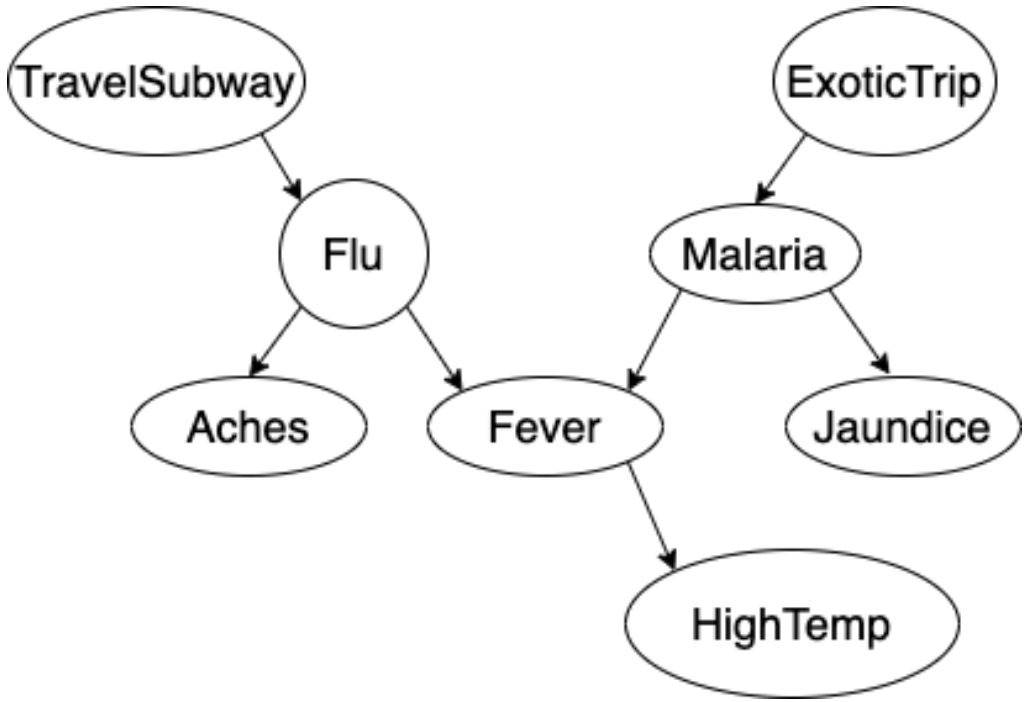
CQ: Applying D-separation

CQ 2b: Are **Aches** and **HighTemp** independent given **Flu**?



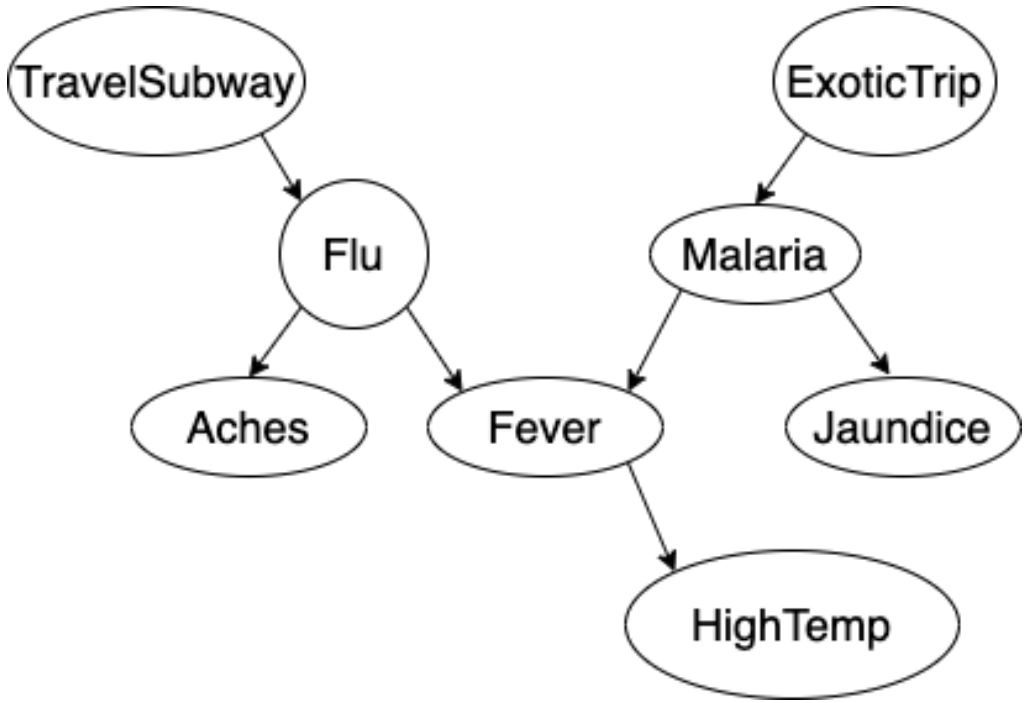
CQ: Applying D-separation

CQ 3a: Are **Flu** and **ExoticTrip** independent?



CQ: Applying D-separation

CQ 3b: Are **Flu** and **ExoticTrip** independent given **HighTemp**?



Learning Goals

D-Separation

Constructing Bayesian Networks

Revisiting the Learning goals

Constructing Bayesian Networks

- For a joint probability distribution, there are many correct Bayesian networks.

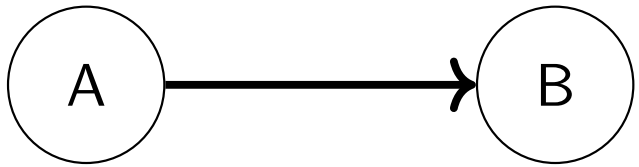
- Given a Bayesian network A, a Bayesian network B is correct if and only if

  If Bayesian network B requires two variables to satisfy an independence relationship, Bayesian network A must also require the two variables to satisfy the same independence relationship.

- Prefer a Bayesian network that requires fewer probabilities.

Requiring an Independence Relationship

- Having an edge between two variables DOES NOT mean that the two variables are DEPENDENT.



- The absence of an edge between two variables MEANS that the two variables satisfy an INDEPENDENCE relationship.

Constructing a Correct Bayesian Network

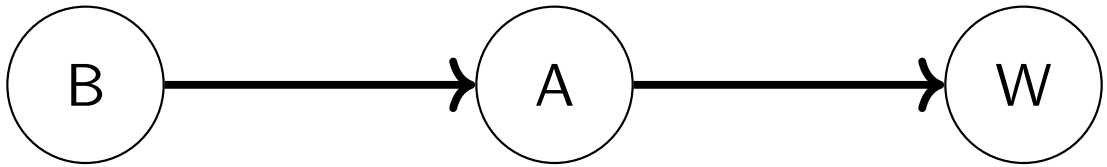
1. Order the variables $\{X_1, \ldots, X_n\}$.
2. For each variable X_i in the ordering,
   - 2.1 **Choose the node's parents:**
     Choose the smallest set of parents from $\{X_1, \ldots, X_{i-1}\}$ such that given $Parents(X_i)$ X_i is independent of all the nodes in $\{X_1, \ldots, X_{i-1}\} - Parents(X_i)$. Formally,

     $$P(X_i|Parents(X_i)) = P(X_i|X_{i-1} \wedge \cdots \wedge X_1).$$

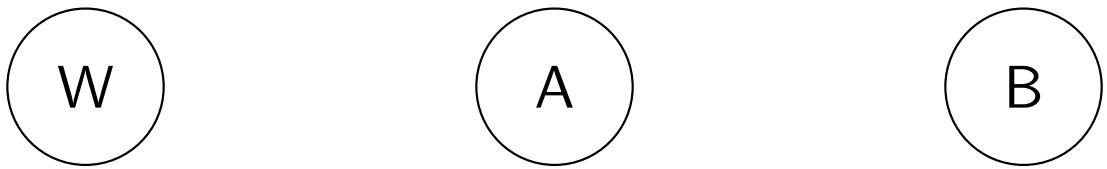
   - 2.2 Create a link from each parent of X_i to the node X_i.
   - 2.3 Write down the conditional probability table $P(X_i|Parents(X_i))$.

Example 1: Construct a Bayes Net

Consider the Bayesian network.



Construct a correct Bayesian network by adding the variables in the order: W, A, and B.



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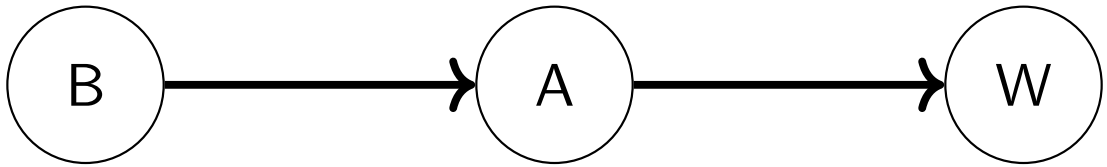

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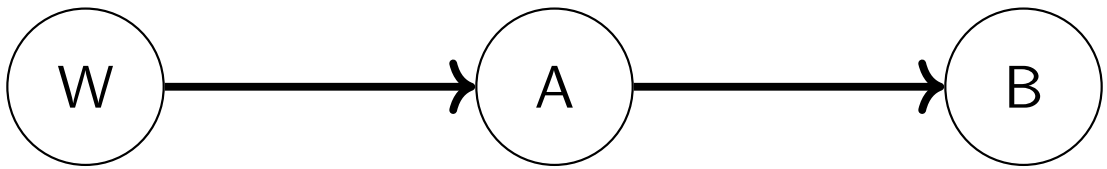

Example 2: Construct a Bayes Net

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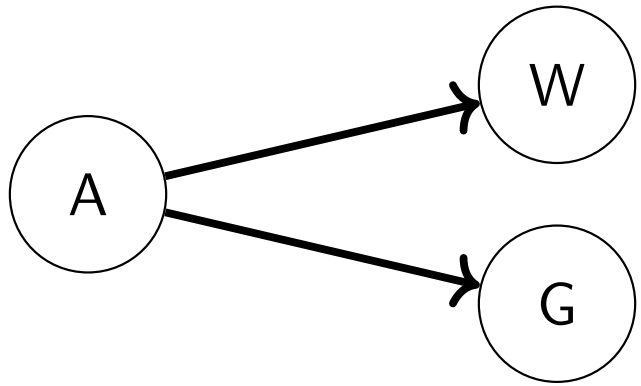
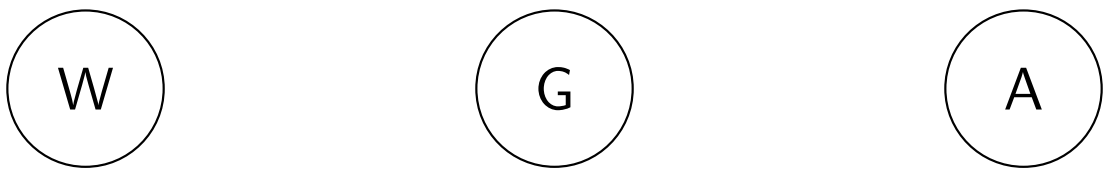


Construct a correct Bayesian network by adding the variables in the order: W, G, and A.



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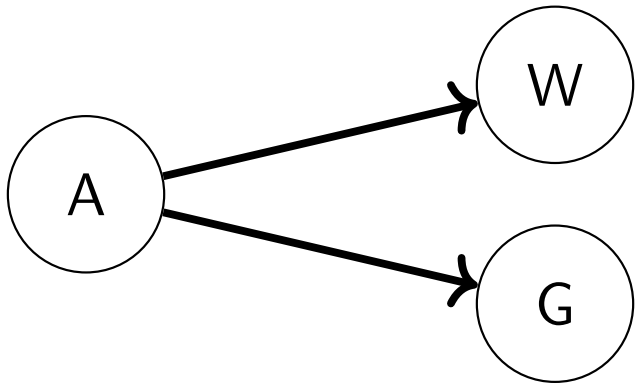

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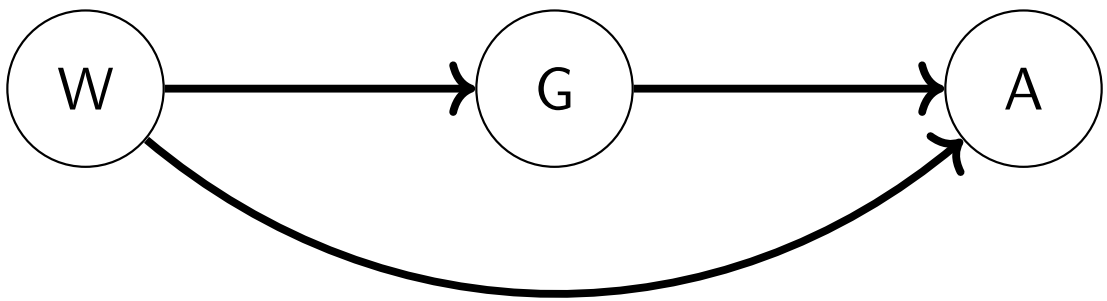

Example 3: Construct a Bayes Net

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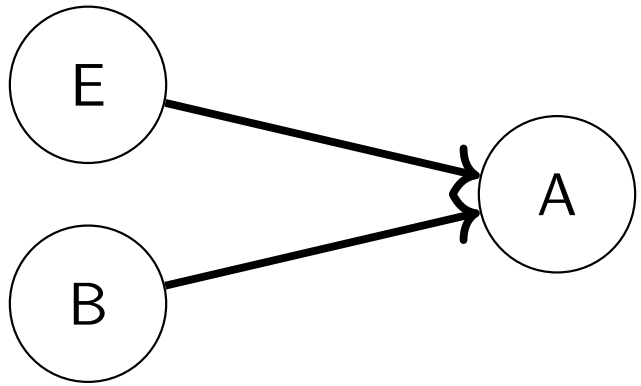
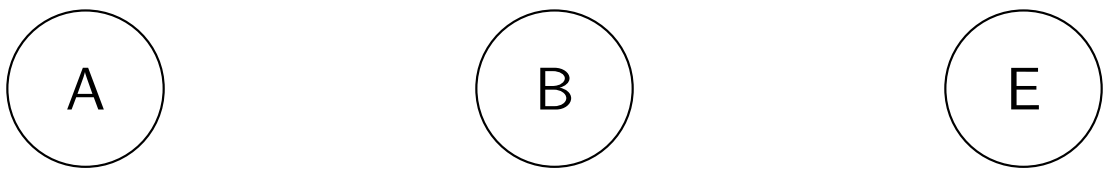


Construct a correct Bayesian network by adding the variables in the order: A, B, and E.



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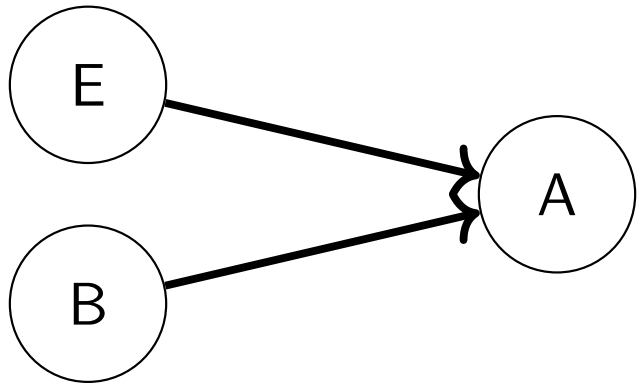

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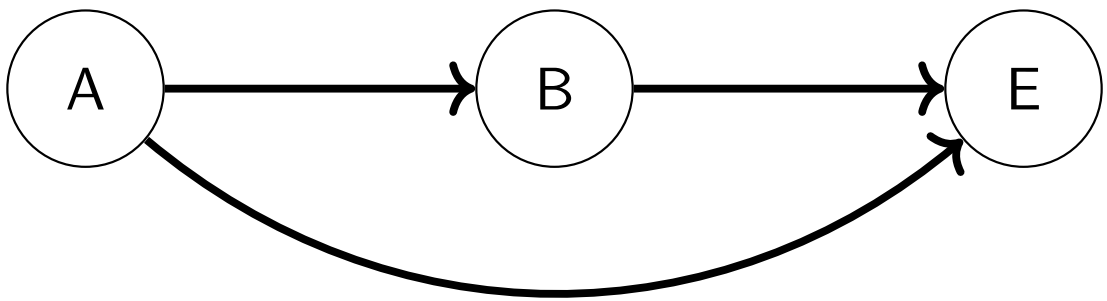

Constructing a Compact Bayesian Network

- What does an edge mean?
  Does an edge always represent a causal relationship?

- How can we construct a Bayesian network with the smallest number of edges?

Revisiting the Learning Goals

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