Decisions with Multiple Agents: Game Theory

Alice Gao

Lecture 16

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

Revisiting the Learning goals

Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine whether one outcome Pareto dominates another outcome of a game. Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.

CQ: Prior knowledge w/ GT and MD

CQ: Have you learned Game Theory and/or Mechanism Design in another course?

(A) Yes

(B) No

Decision making with multiple agents

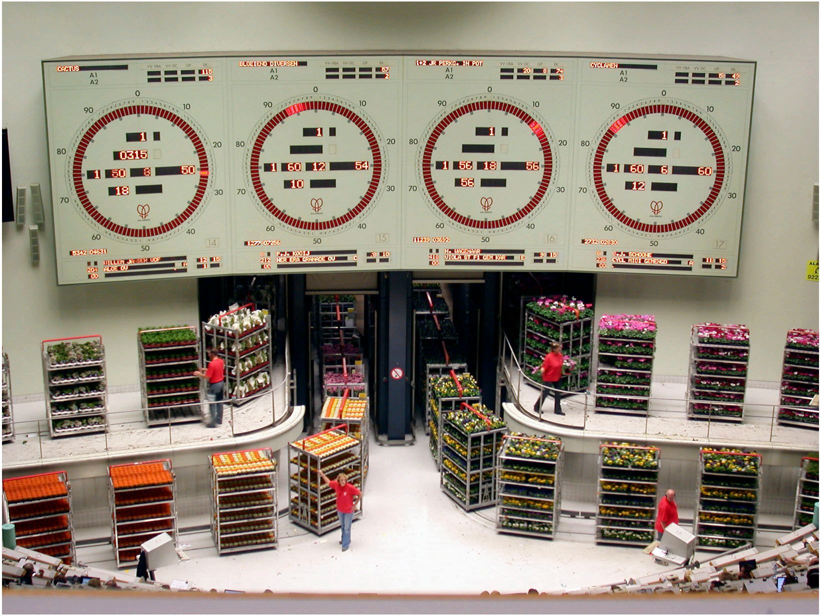
- Decision making in a multi-agent environment.
- When making a decision, each agent needs to take into account of the other agents' behaviour.

What is a game?

Game Theory

A game is a mathematical model of a strategic scenario.

Dutch flower auction



Matching problems

Examples: medical residency matching, school choice, and organ transplant, etc.

Crowdsourcing

Examples: 99 Designs, Topcoder, Duolingo, uwflow.com

Game Theory vs Mechanism Design

- Game theory: Given a game, how would agents play it?
- Mechanism design: How should we design the rules of the game so that the agents will behave the way we want them to?

The multi-agent framework

- Each agent decides what to do based on
  - their information about the world
  - their information about other agents
  - their utility function
- The outcome depends on the actions of all agents.

Relationship between utility functions

A game can be

- cooperative where agents have a common goal.
- competitive where agents have conflicting goals.
- or somewhere in between.

CQ: Friends who enjoy each other's company

CQ: Which of the following statements is correct?

(A) For Alice, staying home strictly dominates going dancing.

(B) For Alice, going dancing strictly dominates staying home.

(C) For Alice, neither action strictly dominates the other action.

		Anna	
		home	*dancing*
Alice	*home*	$(0, 0)$	$(0, 1)$
	dancing	$(1, 0)$	$(2, 2)$

CQ: Signing up for the same activity - strategy dominance

CQ: Which of the following statements is correct?

(A) For Alice, dancing strictly dominates running.

(B) For Alice, running strictly dominates dancing.

(C) For Alice, neither action strictly dominates the other action.

		Anna	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

CQ: Signing up for the same activity - dominant strategy equilibrium

CQ: Does this game have a dominant strategy equilibrium?
If so, which outcome is such an equilibrium?

(A) (dancing, dancing)
(B) (dancing, running)
(C) (running, dancing)
(D) (running, running)
(E) This game does NOT have a dominant strategy equilibrium.

		Anna	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

Nash equilibrium

- Won Nobel prize in Economics.
- One-page paper on Nash equilibrium and 26-page PhD thesis.
- Every finite game has at least one Nash equilibrium. (It may not be a pure strategy equilibrium though.)

CQ: Signing up for the same activity - Best response

CQ: Consider the strategy profile (*dancing*, *running*).
Which of the following statements is true?

(A) Alice's action is a best response to Anna's action.

(B) Anna's action is a best response to Alice's action.

(C) Both (A) and (B) are true.

(D) Neither (A) nor (B) is true.

		Anna	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

CQ: Signing up for the same activity - Nash equilibria

CQ: Which of the following is correct about the game below? Consider only pure-strategy Nash equilibria.

(A) (dancing, dancing) is the only Nash equilibrium.

(B) (running, running) is the only Nash equilibrium.

(C) (dancing, dancing) and (running, running) are both Nash equilibria.

(D) This game has more than 2 Nash equilibria.

		Anna	
		dancing	*running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

CQ: Signing up for the same activity - Pareto optimality

CQ: How many of the four outcomes are **Pareto optimal**?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

		Anna: *dancing*	Anna: *running*
Alice	*dancing*	$(2, 2)$	$(0, 0)$
	running	$(0, 0)$	$(1, 1)$

Revisiting the Learning Goals

By the end of the lecture, you should be able to

- Determine dominant-strategy equilibria of a 2-player normal form game.
- Determine pure-strategy Nash equilibria of a 2-player normal form game.
- Determine whether one outcome Pareto dominates another outcome of a game. Determine Pareto optimal outcomes of a 2-player normal form game.
- Calculate a mixed strategy Nash equilibrium of a 2-player normal form game.