1 The Mail Delivery Robot

The robot must choose its route to pickup the mail. There is a short route and a long route. The long route is slower, but on the short route the robot might slip and fall. The robot can put on pads. This won't change the probability of an accident, but it will make it less severe if it happens. Unfortunately, the pads add weight and slow the robot down. The robot would like to pick up the mail quickly with little/no damage.

What should the robot do?

2 Constructing a decision network for the mail delivery robot

What are the random variables?

- A: whether an accident occurs or not.

What are the decision variables (actions)?

- P: whether the robot puts on pad.
- S: whether the robot chooses the short route.

How do the random variables and the decision variables relate to one another?

- S affects A. If the robot chooses the short route, an accident may occur. If the robot chooses the long route, an accident won't occur.
- A is only affected by S and not affected by P.

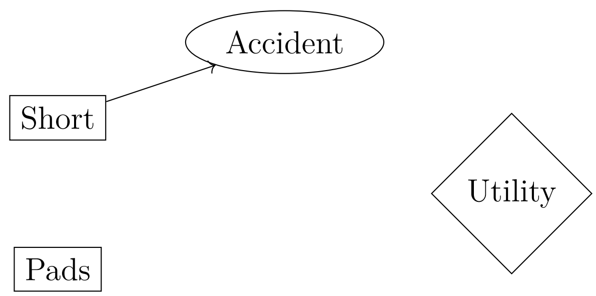
Based on the problem description, let's assume that

- If the robot goes on the long route, no accident occurs.
- If the robot goes on the short route, an accident occurs with a fixed probability q.

The conditional distribution of A given S is
$$\begin{aligned} P(A|\neg S) = &\quad 0 \\ P(A|S) = &\quad q \end{aligned}$$

The decision network so far

$$\boxed{\begin{aligned} P(A|\neg S) = &\quad 0 \\ P(A|S) = &\quad q \end{aligned}}$$

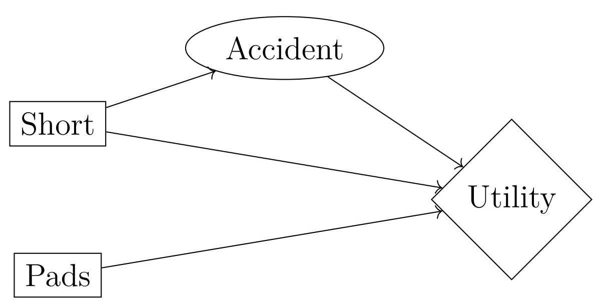


Which variables directly influence the robot's happiness?

Answer: All of PS, S and A directly influence the robot's happiness.

Let's add arcs to the utility node.

$$\boxed{\begin{aligned} P(A|\neg S) = &\quad 0 \\ P(A|S) = &\quad q \end{aligned}}$$



Here is the definition of the robot's utility function. Let's make sense of it.

	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	6
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	10
$\neg P, S, A$	w_3 severe damage	0
$P, \neg S, \neg A$	w_4 slow, extra weight	4
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	8
P, S, A	w_7 moderate damage	2

How does the robot's utility/happiness depend on the random variables and the decision variables?

- When an accident does not happen (w_0, w_2, w_4, w_6), the robot prefers not wearing pads than wearing pads because it's faster.

$$U(\neg P \wedge \neg S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
$$U(\neg P \wedge S \wedge \neg A) > U(P \wedge S \wedge \neg A)$$

- When an accident does not happen, the robot prefers the short route over the long one.

$$U(P \wedge S \wedge \neg A) > U(P \wedge \neg S \wedge \neg A)$$
$$U(\neg P \wedge S \wedge \neg A) > U(\neg P \wedge \neg S \wedge \neg A)$$

- When an accident occurs, the robot must have taken the short route. Thus, there is no utility for

  $\neg P \wedge \neg S \wedge A$ and $P \wedge \neg S \wedge A$.

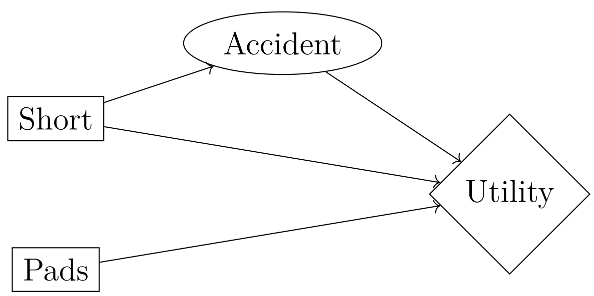
- When the robot took the short route and an accident occurrs, the robot prefers wearing pads than not wearing pads because pads reduce the severity of damage.

$$U(P \wedge S \wedge A) > U(\neg P \wedge S \wedge A)$$

Our final decision network:

	State	$U(w_i)$
$\neg P, \neg S, \neg A$	w_0 slow, no weight	6
$\neg P, \neg S, A$	w_1 impossible	
$\neg P, S, \neg A$	w_2 quick, no weight	10
$\neg P, S, A$	w_3 severe damage	0
$P, \neg S, \neg A$	w_4 slow, extra weight	4
$P, \neg S, A$	w_5 impossible	
$P, S, \neg A$	w_6 quick, extra weight	8
P, S, A	w_7 moderate damage	2

$$P(A|\neg S) = 0$$
$$P(A|S) = q$$



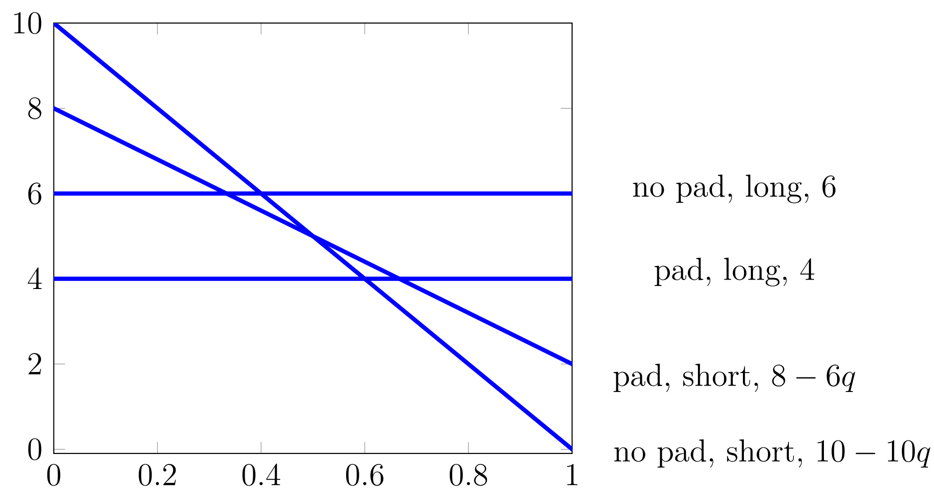
What should the robot do?

$$
\begin{aligned}
EU(\neg P, \neg S) =& P(w_0|\neg P \land \neg S) * U(w_0) \\
&+ P(w_1|\neg P \land \neg S) * U(w_1) \\
=& P(\neg P \land \neg S \land \neg A|\neg P \land \neg S) * U(w_0) \\
&+ P(\neg P \land \neg S \land A|\neg P \land \neg S) * U(w_1) \\
=& P(\neg A|\neg P \land \neg S) * U(w_0) \\
&+ P(A|\neg P \land \neg S) * U(w_1) \\
=& P(\neg A|\neg S) * U(w_0) \\
&+ P(A|\neg S) * U(w_1) \\
=& (1)(6) + (0)(-) \\
=& 6
\end{aligned}
$$

$$
\begin{aligned}
EU(\neg P, S) =& P(w_2|\neg P \land S) * U(w_2) \\
&+ P(w_3|\neg P \land S) * U(w_3) \\
=& P(\neg P \land S \land \neg A|\neg P \land S) * U(w_2) \\
&+ P(\neg P \land S \land A|\neg P \land S) * U(w_3) \\
=& P(\neg A|\neg P \land S) * U(w_2) \\
&+ P(A|\neg P \land S) * U(w_3) \\
=& P(\neg A|S) * U(w_2) \\
&+ P(A|S) * U(w_3) \\
=& (1-q)(10) + (q)(0) \\
=& 10 - 10q
\end{aligned}
$$

$$
\begin{aligned}
EU(P, \neg S) =& P(w_4 | P \wedge \neg S) * U(w_4) \\
& + P(w_5 | P \wedge \neg S) * U(w_5) \\
=& P(P \wedge \neg S \wedge \neg A | P \wedge \neg S) * U(w_4) \\
& + P(P \wedge \neg S \wedge A | P \wedge \neg S) * U(w_5) \\
=& P(\neg A | P \wedge \neg S) * U(w_4) \\
& + P(A | P \wedge \neg S) * U(w_5) \\
=& P(\neg A | \neg S) * U(w_4) \\
& + P(A | \neg S) * U(w_5) \\
=& (1)(4) + (0)(-) \\
=& 4
\end{aligned}
$$

$$
\begin{aligned}
EU(P, S) =& P(w_6 | P \wedge S) * U(w_6) \\
& + P(w_7 | P \wedge S) * U(w_7) \\
=& P(P \wedge S \wedge \neg A | P \wedge S) * U(w_6) \\
& + P(P \wedge S \wedge A | P \wedge S) * U(w_7) \\
=& P(\neg A | P \wedge S) * U(w_6) \\
& + P(A | P \wedge S) * U(w_7) \\
=& P(\neg A | S) * U(w_6) \\
& + P(A | S) * U(w_7) \\
=& (1 - q)(8) + (q)(2) \\
=& 8 - 6q
\end{aligned}
$$



What should the robot do?

- If $q \leq 2/5$, then wear no pad and go the short route.
- If $q > 2/5$, then wear no pad and go the long route.