Introduction to Bayesian Networks

Alice Gao

Lecture 10

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

Examples of Bayesian Networks

Semantics of Bayes Net
- Representing the joint distribution
- Encoding the conditional independence assumptions

Constructing Bayes Nets

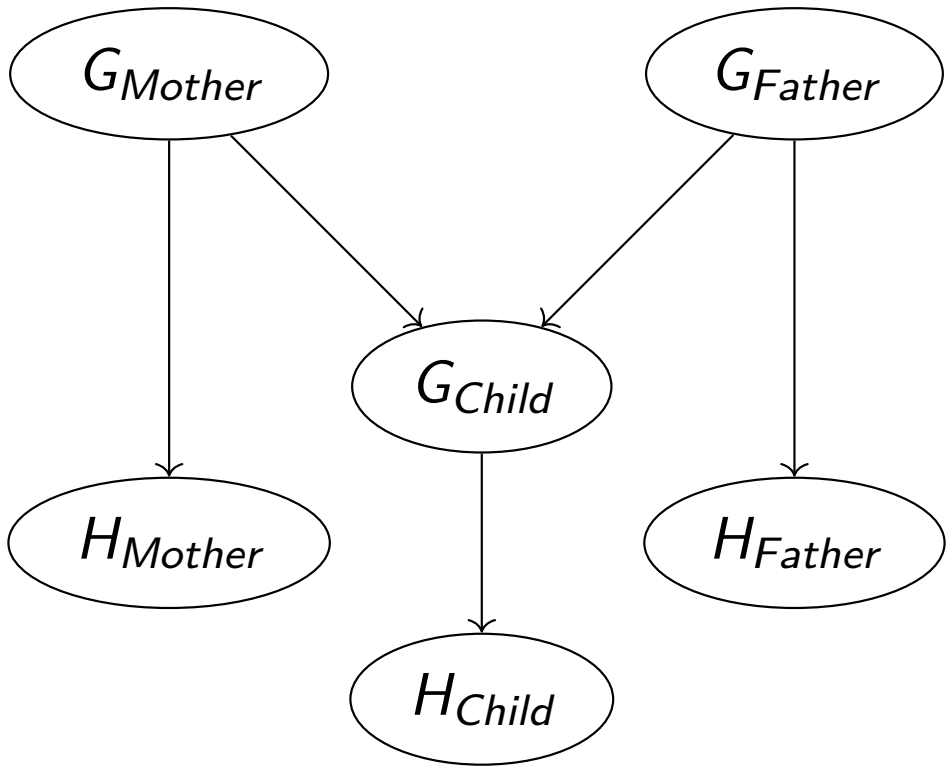
Revisiting the Learning goals

Learning Goals

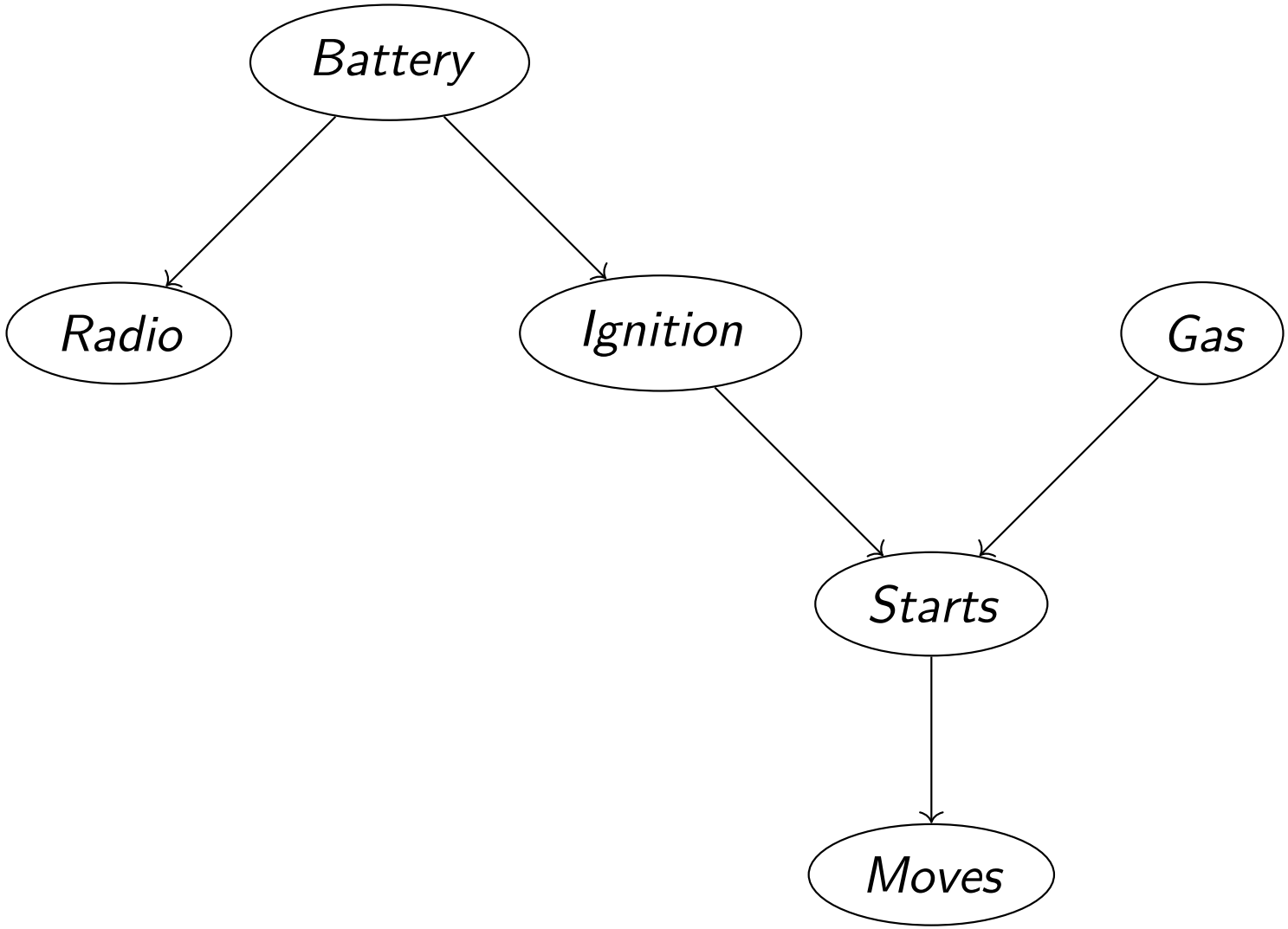
By the end of the lecture, you should be able to

- Compute a joint probability given a Bayesian network.
- Identify the conditional independence assumptions of a Bayesian network.
- Given a Bayesian network, determine if two variables are independent or conditionally independent given a third variable.

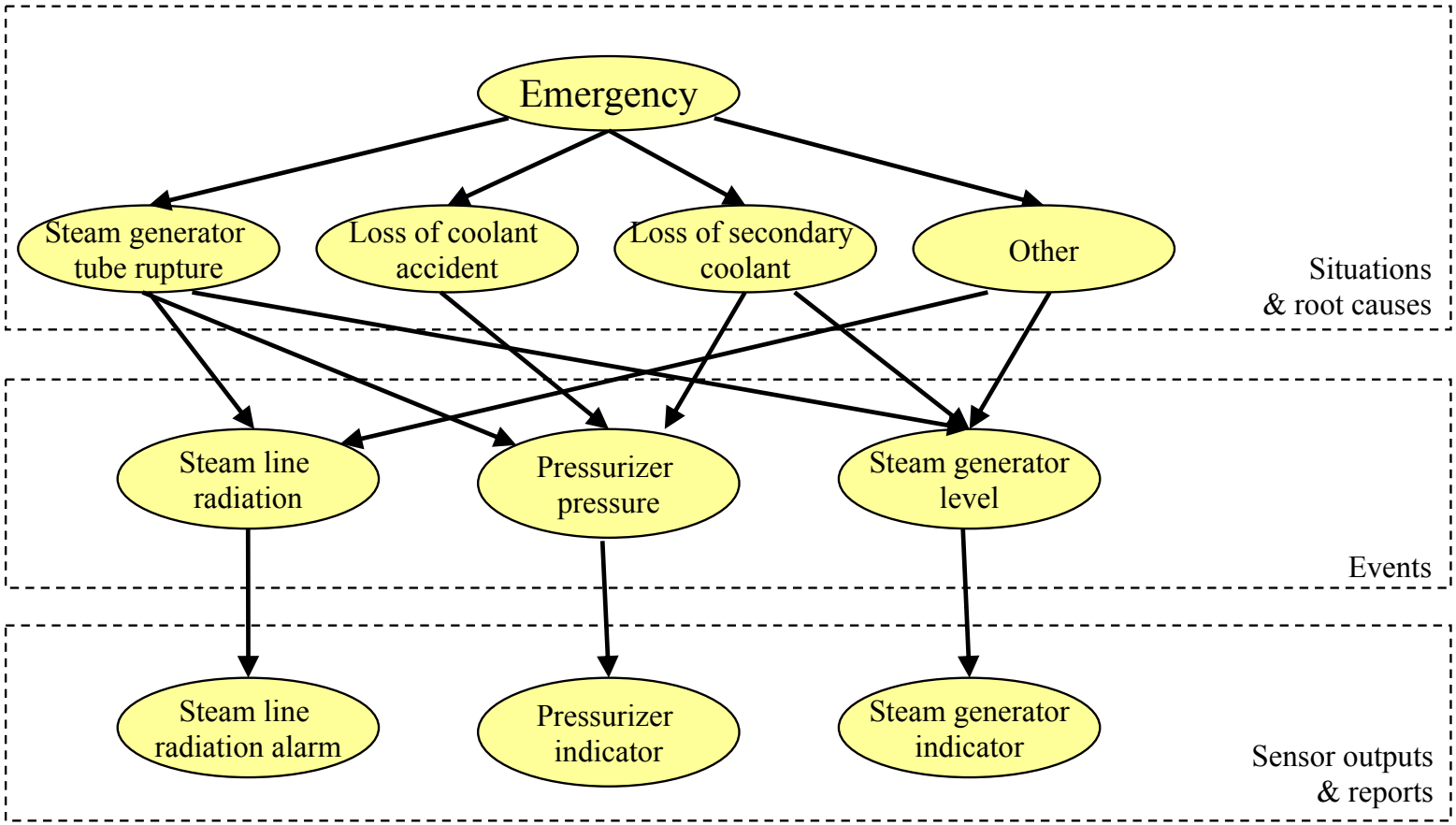
Inheritance of Handedness



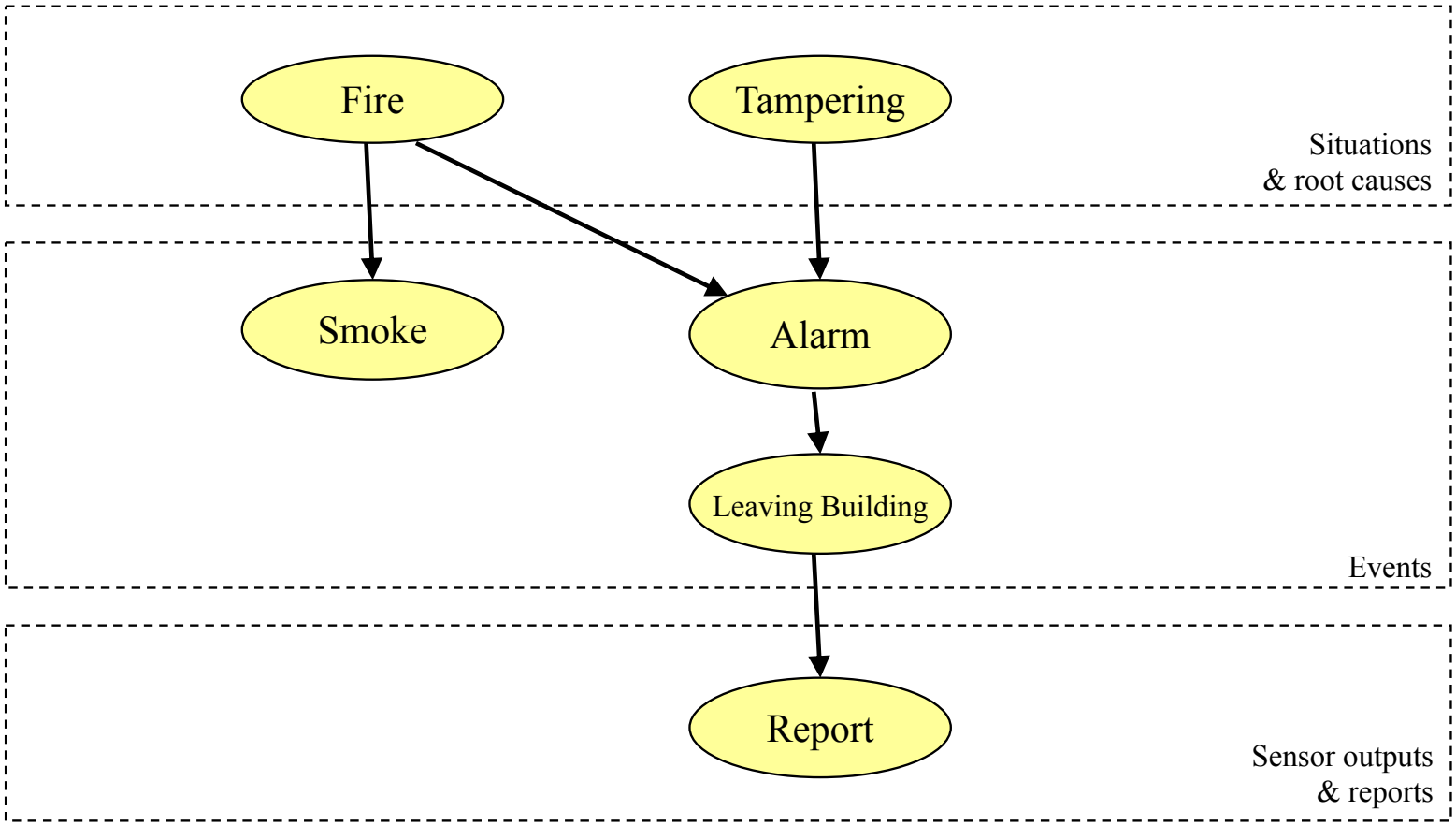
Car Diagnostic Network



Example: Nuclear power plant operations

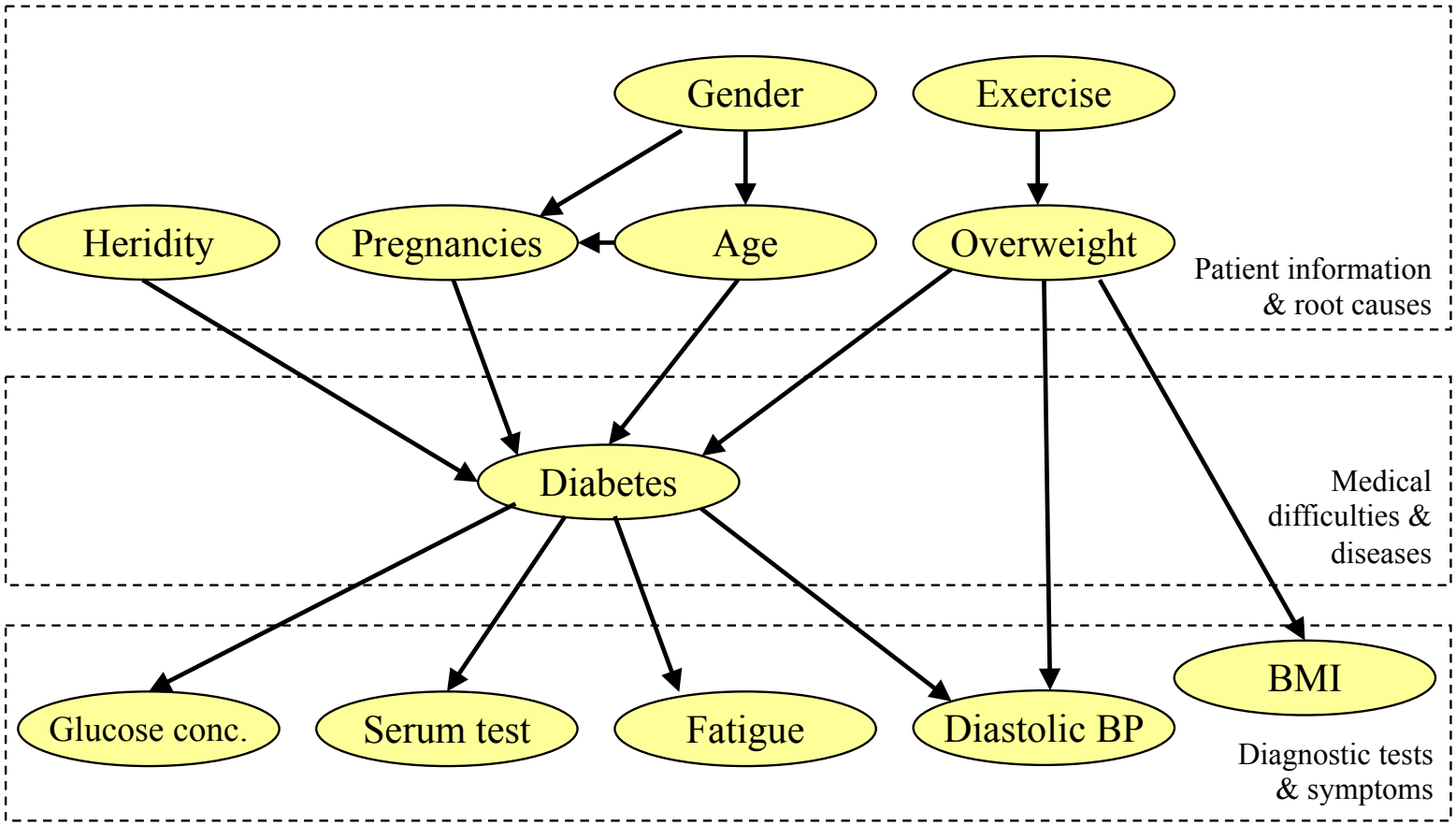


Example: Fire alarms



Report: “report of people leaving building because a fire alarm went off”

Example: Medical diagnosis of diabetes



Why Bayesian Networks?

A probabilistic model of the Holmes scenario:

- The random variables: Earthquake, Radio, Burglary, Alarm, Watson, and Gibbon.
- # of probabilities in the joint distribution: $2^6 = 64$.
- For example,
  $P(E \wedge R \wedge B \wedge A \wedge W \wedge G) = ?$
  $P(E \wedge R \wedge B \wedge A \wedge W \wedge \neg G) = ?$
  ... etc ...

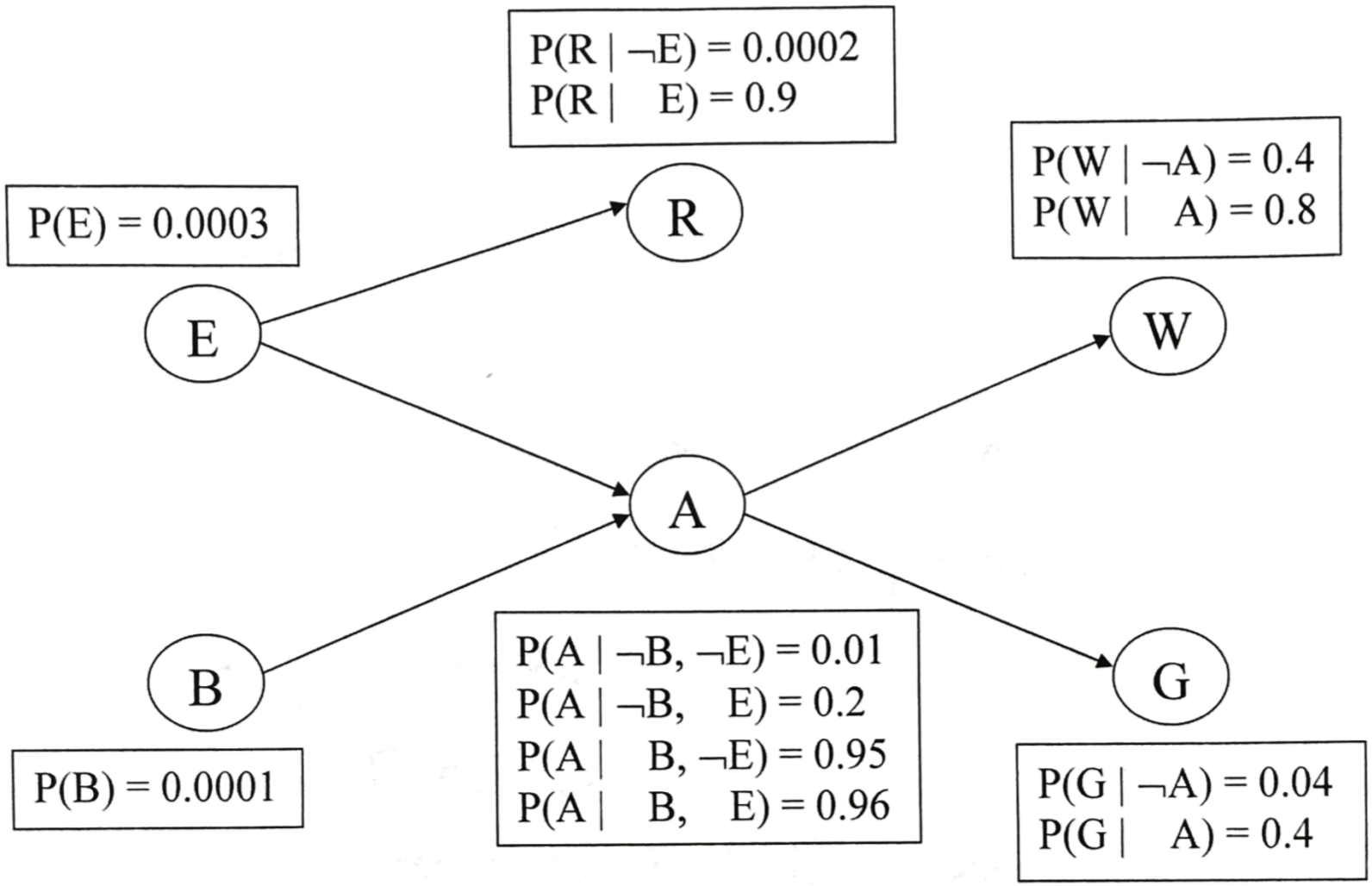
We can answer any question about the domain using the joint distribution, but

- Quickly become intractable as the number of variables grows.
- Unnatural and tedious to specify all the probabilities.

Why Bayesian Networks?

A Bayesian Network is a compact version of the joint distribution and it takes advantage of the unconditional and conditional independence among the variables.

A Bayesian Network for the Holmes Scenario



Bayesian Network

A Bayesian Network is a directed acyclic graph.

- Each node corresponds to a random variable.
- X is a parent of Y if there is an arrow from node X to node Y. The graph has no directed cycles.
- Each node X_i has a conditional probability distribution $P(X_i|Parents(X_i))$ that quantifies the effect of the parents on the node.

Learning Goals

Examples of Bayesian Networks

Semantics of Bayes Net
- Representing the joint distribution
- Encoding the conditional independence assumptions

Constructing Bayes Nets

Revisiting the Learning goals

The Semantics of Bayesian Networks

Two ways to understand Bayesian Networks:

- A representation of the joint probability distribution
- An encoding of the conditional independence assumptions

Representing the joint distribution

We can calculate every entry in the joint distribution using the Bayesian Network. How do we do this?

1. Choose an order of the variables that is consistent with the partial ordering of the nodes in the Bayesian Network.
2. Compute the joint probability using the following formula.

$$P(X_n, \ldots, X_1) = \prod_{i=1}^{n} P(X_i | Parents(X_i))$$

Representing the joint distribution

Example: What is the probability that

- The alarm has sounded,
- Neither a burglary nor an earthquake has occurred,
- Both Watson and Gibbon call and say they hear the alarm, and
- There is no radio report of an earthquake?

CQ: Calculating the joint probability

CQ: What is the probability that

- NEITHER a burglary NOR an earthquake has occurred,
- The alarm has NOT sounded,
- NEITHER of Watson and Gibbon is calling, and
- There is NO radio report of an earthquake?

(A) $0 \leq p \leq 0.2$

(B) $0.2 < p \leq 0.4$

(C) $0.4 < p \leq 0.6$

(D) $0.6 < p \leq 0.8$

(E) $0.8 < p \leq 1$

Encoding the Conditional Independent Assumptions

By modeling a domain using a Bayesian network,
we are making the following key assumption.

> *For a given ordering of the nodes,*
> *each node is conditionally independent of its predecessors given its parents.*

$$P(X_i|Parents(X_i)) = P(X_i|X_{i-1}, \ldots, X_1), \forall i = 1, \ldots, n$$

Identifying the Conditional Independence Assumptions

Given a Bayesian Network,

- Consider all orderings of the variables that are consistent with the partial ordering in the Bayesian network.
- Based on the Bayesian Network,

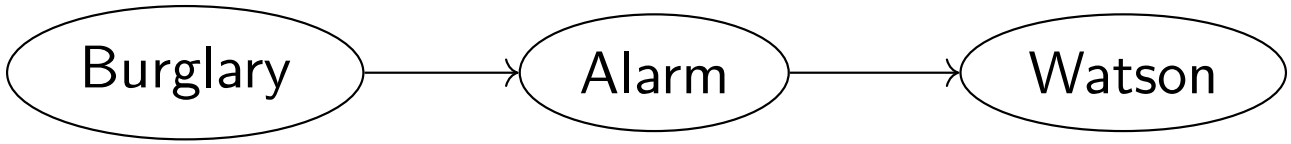
$$P(X_n, \ldots, X_1) = \prod_{i=1}^{n} P(X_i | Parents(X_i))$$

Based on the chain rule,

$$P(X_n, \ldots, X_1) = \prod_{i=1}^{n} P(X_i | X_{i-1}, \ldots, X_1)$$

- The difference between the RHS of the equations give the conditional independence assumptions.

CQ: Independence and Conditional Independence



CQ: True or False.

1. Watson is independent of Burglary.
2. Watson is conditionally independent of Burglary given Alarm.

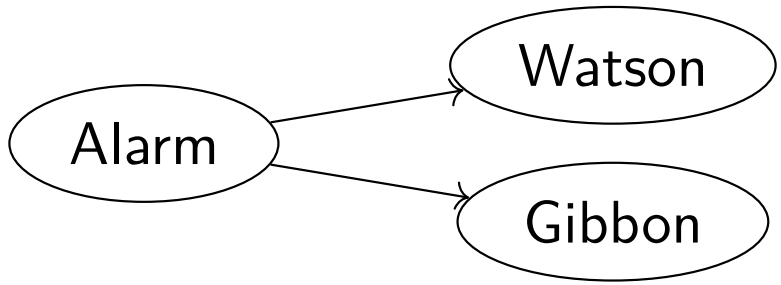
(A) Both True

(B) Both False

(C) (1) is True and (2) is False.

(D) (1) is False and (2) is True.

CQ: Independence and Conditional Independence



CQ: True or False.

1. Watson is independent of Gibbon.
2. Watson is conditionally independent of Gibbon given Alarm.

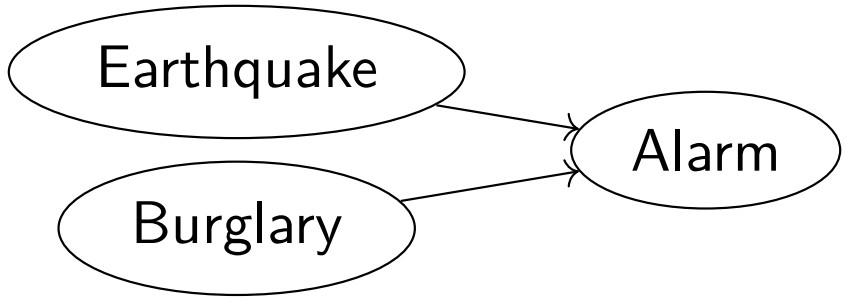
(A) Both True

(B) Both False

(C) (1) is True and (2) is False.

(D) (1) is False and (2) is True.

CQ: Independence and Conditional Independence



CQ: True or False.

1. Burglary is independent of Earthquake.
2. Burglary is conditionally independent of Earthquake given Alarm.

(A) Both True

(B) Both False

(C) (1) is True and (2) is False.

(D) (1) is False and (2) is True.

CQ: Conditional Independence

CQ: Is Radio conditionally independent of Gibbon given Earthquake?

(A) Yes

(B) No

(C) I don't know.

CQ: Conditional Independence

CQ: Is Radio conditionally independent of Burglary given Alarm?

(A) Yes

(B) No

(C) I don't know.

Constructing Bayes Nets

Two questions to consider

- Given a Bayesian network, is it a correct and good representation of the domain?
- How do we construct a Bayesian network that is a correct and good representation of the domain?

Correct and Good Bayes Networks

A Bayes network is a correct representation of the domain iff

- it makes the correct independence assumptions.

Among all the correct Bayes network representations,
a Bayes network is a good representation of the domain iff

- the number of required probabilities is relatively small, and
- the probabilities required are natural to specify.

Constructing a Correct Bayesian Network

1. Determine the set of variables that are required to model the domain.
2. Order the variables, $\{X_1, ..., X_n\}$.
3. For $i = 1$ to n, do the following
   - 3.1 Choose a minimum set of parents from X_1, ..., X_{i-1} such that $P(X_i|Parents(X_i)) = P(X_i|X_{i-1}, \ldots, X_1)$ is satisfied.
   - 3.2 Create a link from each parent of X_i to X_i.
   - 3.3 Write down the conditional probability table $P(X_i|Parents(X_i))$.

Example: Construct a Bayes Net

Construct a correct Bayesian network using the following ordering. (Let's drop Radio.)

$$B, E, A, W, G$$

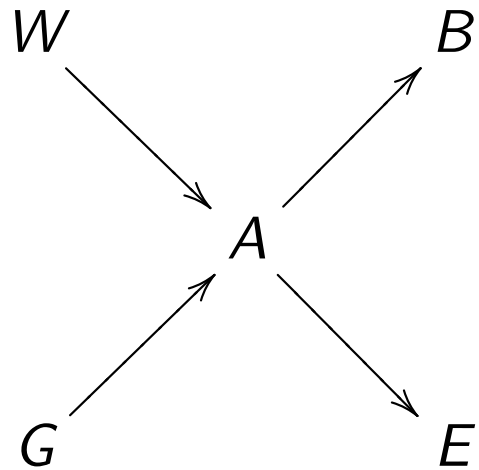
Example: Construct a Bayes Net

Construct a correct Bayesian network using the following ordering. (Let's drop Radio.)

$$W, G, A, B, E$$

CQ Is this Bayes Net correct?

CQ: Consider the node ordering: W, G, A, B, E. Is the following Bayesian network a correct representation of the domain?



Hint: In our domain, Watson and Gibbon are not independent of each other. What about in this network?

(A) Yes

(B) No

(C) I don't know.

Exercise: Construct a Bayes Net

Construct a correct Bayesian network using the following ordering.

$$W, G, E, B, A$$

Revisiting the Learning Goals

By the end of the lecture, you should be able to

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