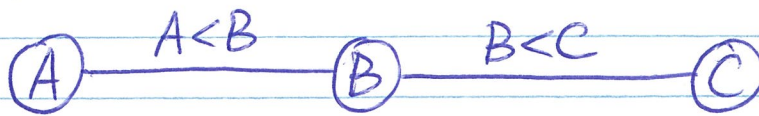


Arc Consistency Example Lecture 6

Alice Gao.



$$\text{dom}(A) = \{1, 2, \cancel{3}, \cancel{4}\}$$

$$\text{dom}(B) = \{\cancel{1}, 2, 3, \cancel{4}\}$$

$$\text{dom}(C) = \{\cancel{1}, \cancel{2}, 3, 4\}$$

queue: $\overset{\textcircled{1}}{(A, A < B)}$, $\overset{\textcircled{3}}{(B, A < B)}$, $\overset{\textcircled{2}}{(B, B < C)}$
 $\overset{\textcircled{5}}{(C, B < C)}$, $\overset{\textcircled{4}}{(A, A < B)}$

① remove $(A, A < B)$.

remove 4 from $\text{dom}(A)$.

add nothing to queue. (why not add back $(B, A < B)$?)

② remove $(B, B < C)$

remove 4 from $\text{dom}(B)$

add $(A, A < B)$ back into queue.

③ remove $(B, A < B)$.

remove 1 from $\text{dom}(B)$

add $(C, B < C)$ into queue (already there).

④ remove $(A, A < B)$.

remove 3 from $\text{dom}(A)$.

add nothing to queue.

⑤ remove $(C, B < C)$.

remove 1 and 2 from $\text{dom}(C)$.

add nothing to queue.

Arc Consistency Example Lecture 6.

Alice Gao

When we remove $(A, A < B)$ and reduce A 's domain, should we add $(B, A < B)$ back into the queue?

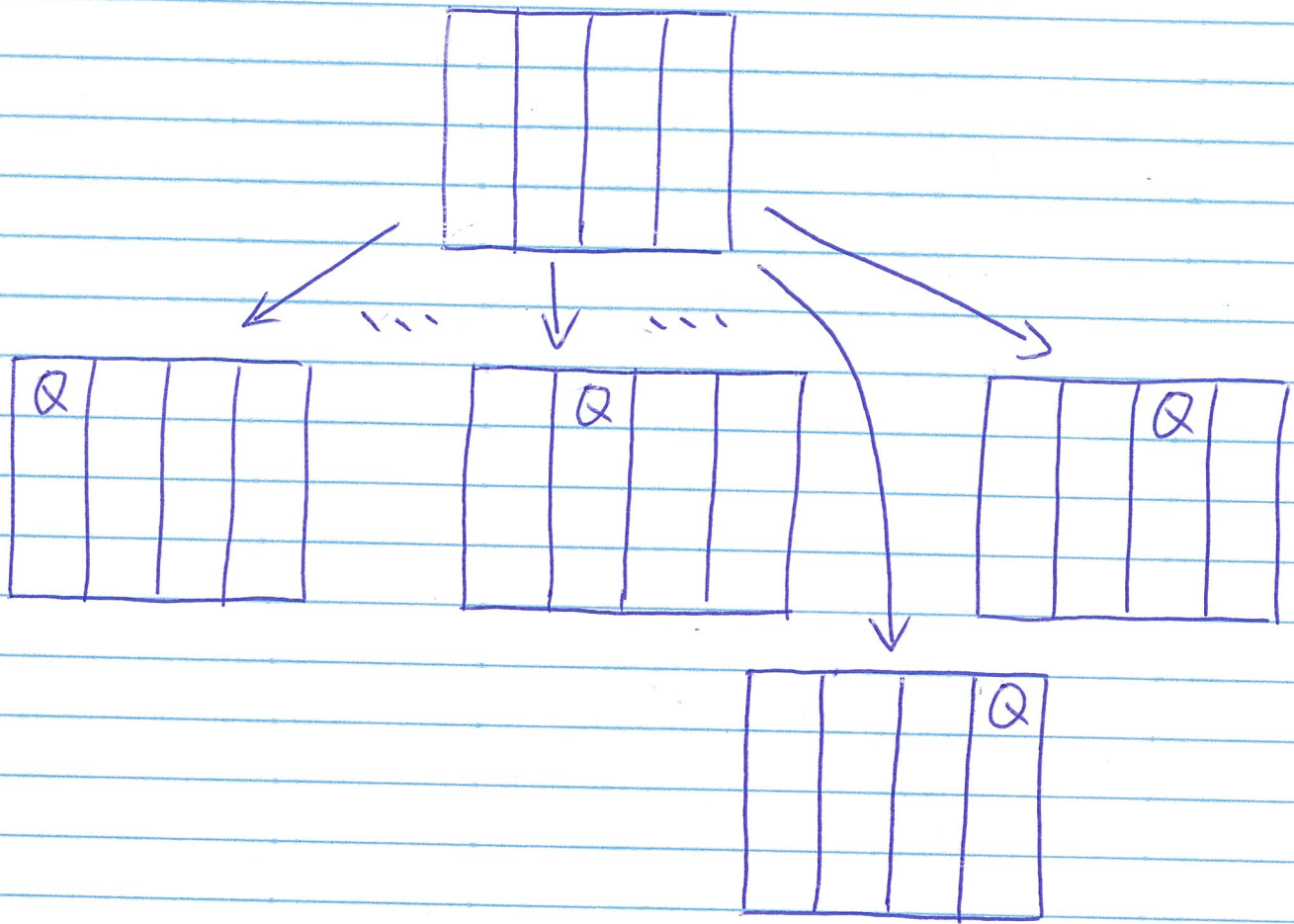
We don't add back arcs involving the same constraint and a different variable.

- If $(B, A < B)$ was not consistent before, it was in the queue. No need to add it again.
- If $(B, A < B)$ was consistent before, we will show that $(B, A < B)$ is still consistent after.

Proof by Contradiction:

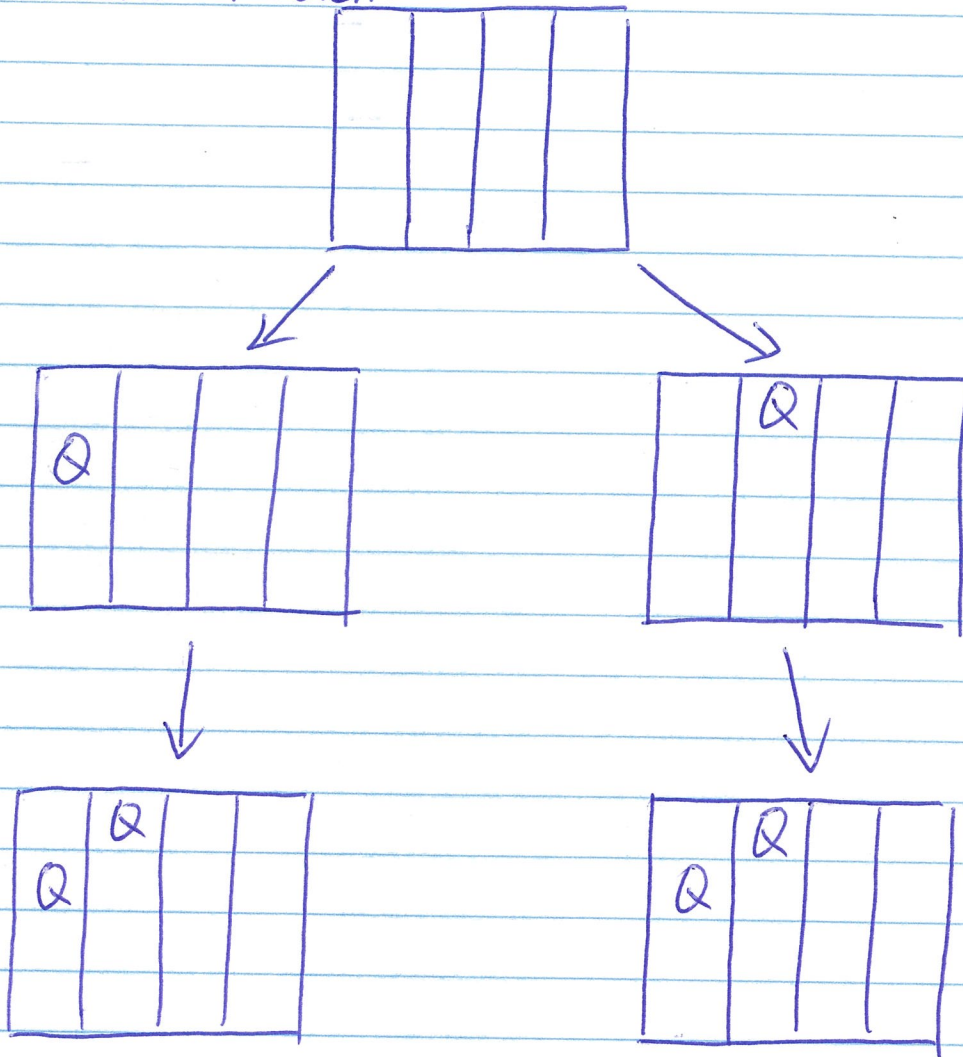
- Suppose $(B, A < B)$ is not consistent after.
- A value of B could only satisfy the constraint if A takes the dropped value.
- But we dropped the value of A because there were no value of B that ~~allow~~ satisfy the constraint when A takes the dropped value.
 - contradiction.

Four-Queens Problem.



- Count the # of successors.
 - Count the # of leaf nodes in the search tree. A
4-queen board.
 - Count the # of unique four-queen boards. B
- Why is A much larger than B?

Four-Queens Problem



- CSP is commutative.

Assigning values to variables in different orders will arrive at the same state.

- Consider one variable when generating successors.

Example 1: Backtracking Search Lecture 6.

Alice Gao

	0	1	2	3
0	Q			
1	X			
2	X			
3	X			

	0	1	2	3
0	Q	X	X	X
1	X	X		
2	X		X	
3	X			X

	0	1	2	3
0	Q	X	X	X
1	X	X		X
2	X	X	X	X
3	X		X	X

forward checking

no solution.
maintaining arc consistency.

Example 2:

	0	1	2	3
0	X			
1	Q			
2	X			
3	X			

	0	1	2	3
0	X	X		
1	Q	X	X	X
2	X	X		
3	X		X	

	0	1	2	3
0	X	X		X
1	Q	X	X	X
2	X	X	X	
3	X		X	X

forward checking.

maintaining arc consistency.