Constraint Satisfaction Problems: Backtracking Search

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Lecture 6

Based on work by K. Leyton-Brown, K. Larson, and P. van Beek

Outline

Learning Goals

Backtracking Search Algorithm

Revisiting the Learning goals

Learning Goals

By the end of the lecture, you should be able to

- Contrast naive depth-first search and backtracking search on a CSP.
- Describe/trace/implement the backtracking search algorithm.
- Describe/trace/implement the backtracking search algorithm with forward checking and/or arc consistency.
- Describe/trace/implement the backtracking search algorithm with forward checking and/or arc consistency and with heuristics for choosing variables and values.

Depth-first search on a CSP

Consider the incremental formulation of the four-queens problem. Let's run Depth-First Search on the CSP.

- How many successor states are there for any state?
- Each leaf node corresponds to a four-queen board. How many leaf nodes are there in the search tree?
- How many unique four-queen boards are there?

Why is the number of leaf nodes in the search tree much larger than the number of unique four-queen boards?

CQ: Naive Depth-First Search on a CSP

CQ: How does the number of leaf nodes in the search tree compare with the number of unique four-queen boards?

(A) Larger

(B) Smaller

A CSP is commutative

- A CSP is commutative. Assigning values to variables in different orders will arrive at the same state.
- In each node, we should only consider one variable when generating successor states.

Backtracking Search

Algorithm 1 BACKTRACK(assignment, csp)

```
 1: if assignment is complete then return true
 2: var ← SELECT-UNASSIGNED-VARIABLE(csp)
 3: for all value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
 4:     if adding {var = value} satisfies every constraint then
 5:         add {var = value} to assignment
 6:         result ← BACKTRACK(assignment, csp)
 7:         if result is true then return result
 8:     end if
 9:     remove {var = value} from assignment
10: end for
11: return false
```

CQ: Conditions for trying another value

CQ: In how many conditions do we try another value for a variable?

(A) 1

(B) 2

(C) 3

(D) 4

(E) More than 4

Questions to consider

1. Which variable should we choose next? Which value of the variable should we try next?
2. What inferences should we perform at each step of the search?

Interleaving search and inferences

What inferences should be performed at each step in the search?

- Execute arc consistency algorithm before search.

- **forward-checking**: Whenever a variable X is assigned, for each unassigned variable Y connected to X by a constraint, make Y arc-consistent with respect to X. (delete from Y's domain any value that is inconsistent with the assigned value for X.)

- **Maintaining Arc Consistency (MAC)**:
  - Forward checking and
  - Recursively propagate constraints when changes are made to the domains of variables.

Backtracking with Inferences

Algorithm 2 BACKTRACK-INFERENCES(assignment, csp)

```
1: if assignment is complete then return true
2: var ← SELECT-UNASSIGNED-VARIABLE(csp)
3: for all value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
4:     if adding {var = value} satisfies every constraint then
5:         add {var = value} to assignment
6:         inf-result ← INFERENCES(assignment, csp)
7:         if inf-result is true then
8:             add the inference results to assignment
9:             result ← BACKTRACK(assignment, csp)
10:            if result is true then return result
11:        end if
12:    end if
13:    remove {var = value} and the inference results from assignment
14: end for
15: return false
```

CQ: Interleaving search and inferences

CQ: Consider the 4-queens problem with an empty assignment. Choose $x_0 = 1$. After this assignment, which of the following is the result of performing forward checking?

(A) $x_0 = 1, x_1 \in \{0, 1, 2, 3\}, x_2 \in \{0, 1, 2, 3\}, x_3 \in \{0, 1, 2, 3\}$

(B) $x_0 = 1, x_1 \in \{0, 2, 3\}, x_2 \in \{0, 2, 3\}, x_3 \in \{0, 2, 3\}$

(C) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0, 2\}, x_3 \in \{0, 2, 3\}$

(D) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0, 2\}, x_3 \in \{0, 2\}$

CQ: Interleaving search and inferences

CQ: Consider the 4-queens problem with an empty assignment. Choose $x_0 = 1$. After this assignment, which of the following is the result of maintaining arc consistency?

(A) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0, 2\}, x_3 \in \{0, 2, 3\}$

(B) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0, 2\}, x_3 \in \{0, 2\}$

(C) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0\}, x_3 \in \{0, 2\}$

(D) $x_0 = 1, x_1 \in \{3\}, x_2 \in \{0\}, x_3 \in \{2\}$

Which variable and value should we choose next?

Heuristics for selecting a variable

- **minimum-remaining-values** (MRV) heuristic: Choose the variable with the fewest values left in its domain.
- **degree** heuristic: Choose the variable that is involved in the largest number of constraints on other unassigned variables.
- When choosing a variable, apply the MRV heuristic first. Whenever there is a tie, use the degree heuristic to break ties.

Heuristics for selecting a value for a variable

- **least-constraining-value** heuristic: Select the value that **rules out the fewest values** (or **leaves the largest number of values**) for the neighbouring unassigned variables.

Backtracking with Inferences and Heuristics

Algorithm 3 BACKTRACK-INF-HEUR(assignment, csp)

```
1: if assignment is complete then return true
2: choose var based on MRV and DEGREE HEURISTICS
3: for all value in dom(var) chosen based on LCV HEURISTIC do
4:     if adding {var = value} satisfies every constraint then
5:         add {var = value} to assignment
6:         inf-result ← INFERENCES(assignment, csp)
7:         if inf-result is true then
8:             add the inference results to assignment
9:         end if
10:        result ← BACKTRACK(assignment, csp)
11:        if result is true then return result
12:    end if
13:    remove {var = value} and the inference results from assignment
14: end for
15: return false
```

CQ: Applying the least-constraining-value heuristic

CQ: Consider the following partial assignment for the 4-queens problem. (x_i denotes the row position of the queen in column i.)

$$x_0 = 0, x_1 \in \{2, 3\}, x_2 \in \{1, 3\}, x_3 \in \{1, 2\}$$

Based on the least-constraining-value heuristic,
which value of x_1 should we choose?

(A) $x_1 = 2$

(B) $x_1 = 3$

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