Theorem: The halting-no-input problem is undecidable.

The halting-no-input problem: Given a program $P$ that takes no input, does $P$ terminate?

"Does $P$ terminate with input $I$?"

Algorithm for the halting problem

1. $P$ → Construct program $P'$ which runs $P$ with input $I$.
2. $I$ → $P'$ → Algorithm for halting-no-input problem.
3. "Does $P'$ terminate?"
4. yes → yes
5. no → no
Theorem: The both-halt problem is undecidable.

The both-halt problem: Given two programs $P_1$ and $P_2$, do both programs terminate?

Reduction #1: "Does $P$ terminate with input $I$?"
   - Algorithm for the halting problem
     - Let $P_1$ run $P$ with $I$
     - Let $P_2$ run $P$ with $I$

Reduction #2: Does reduction #2 lead to a valid proof?
   - "Does $P$ terminate with input $I$?"
   - Algorithm for the halting problem
     - Let $P_1$ run $P$ with $I$
     - $P_2$ does nothing and terminates

Reduction #3: Does reduction #3 lead to a valid proof?
   - "Does $P$ terminate with input $I$?"
   - Algorithm for the halting problem
     - Let $P_1$ run $P$ with $I$
     - Let $P_2$ run forever