Program Verification
Reversing an array

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Lecture 22

Based on work by J. Buss, L. Kari, A. Lubiw, B. Bonakdarpour, D. Maftuleac, C. Roberts, R. Trefler, and P. Van Beek
Outline

Program Verification: Reversing an array
Learning Goals
Introducing the array assignment rule
Revisiting the Learning Goals
Learning Goals

By the end of this lecture, you should be able to:
Partial correctness for array assignments
  ▶ Prove that a Hoare triple is satisfied under partial correctness
    for a program containing array assignment statements.
Let $A$ be an array of $n$ integers.

First, write down the sequence of changes. Resolve all of the changes when we prove the implied’s.

\[ Q[A\{e_1 \leftarrow e_2\}/A] \]
\[ A[e_1] = e_2; \]
\[ Q \] {array assignment}

- $A$ is the original array.
- $A\{e_1 \leftarrow e_2\}$ is the new array, which is identical to array $A$ except that the $e_1^{th}$ element is $e_2$. 
The array re-assignment notation

The array reassignment notation:

\[ A\{e_1 \leftarrow e_2\}[i] = \begin{cases} e_2, & \text{if } i = e_1 \\ A[i], & \text{if } i \neq e_1 \end{cases} \]

Note that \(e_1\) is an index whereas \(e_2\) is an array element.

We apply assignments from left to right.

**Examples:**

- \(A\{1 \leftarrow 3\}[1] = 3\)
- \(A\{1 \leftarrow 3\}\{1 \leftarrow 4\}[1] = 4\)
Consider an array $R$ of $n$ integers, $R[1], R[2], ..., R[n]$. We want to reverse the order of its elements.

Our algorithm:

For each $1 \leq j \leq \lfloor n/2 \rfloor$, we will swap $R[j]$ with $R[n + 1 - j]$. 
Reversing an array

$R$ is an array of $n$ integers, $R[1], R[2], \ldots, R[n]$. Prove that the following triple is satisfied under partial correctness.

\[
(\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_x)))
\]

\[j = 1;
\]

\[\textbf{while } (2 \times j \leq n) \{
\]

\[
t = R[j];
\]

\[
R[j] = R[n+1-j];
\]

\[
R[n+1-j] = t;
\]

\[
j = j + 1;
\]

\[
\}
\]

\[
(\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_{n+1-x})))
\]
Reversing an array

$R$ is an array of $n$ integers, $R[1], R[2], \ldots, R[n]$. Prove that the following triple is satisfied under partial correctness.

Let $Inv(j)$ denote our invariant.

$$\begin{align*} &\{(\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_x)))\} \\
j &= 1; \\
\textbf{while} \ (2 \times j \leq n) \ {\{ \\
&\quad t = R[j]; \\
&\quad R[j] = R[n+1-j]; \\
&\quad R[n+1-j] = t; \\
&\quad j = j + 1; \\
&\}\} \\
&\{(\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_{n+1-x})))\}\end{align*}$$
CQ 1: Consider the premise of implied (A).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 2: Consider the conclusion of implied (A).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.

(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.

(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.

(D) All swaps have been completed.

(E) None of the above
CQ 3: Consider the premise of implied (C). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 4: Consider the conclusion of implied (C). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in [1, j – 1] have been swapped, and elements in [j, (n + 1)/2] have NOT been swapped.
(C) Elements in [1, j] have been swapped, and elements in [j + 1, (n + 1)/2] have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 5: Consider the premise of implied (B). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 6: Consider the conclusion of implied (B). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in [1, \(j - 1\)] have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in [1, \(j\)] have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
Revisiting the learning goals

By the end of this lecture, you should be able to:
Partial correctness for array assignments

- Prove that a Hoare triple is satisfied under partial correctness for a program containing array assignment statements.