

Predicate Logic: Soundness and Completeness of Natural Deduction

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Lecture 17

Outline

Soundness and Completeness of Natural Deduction

- The Learning Goals

- The soundness of an inference rule

- Satisfiable set of formulas

- Revisiting the Learning Goals

Learning goals

By the end of this lecture, you should be able to:

- ▶ Define soundness and completeness.
- ▶ Prove that an inference rule is sound or not sound.
- ▶ Prove that a semantic entailment holds using the soundness and completeness theorems.
- ▶ Show that no natural deduction proof exists for a semantic entailment using the soundness and completeness theorems.

CQ Choosing concrete formulas

True/False: Let α be a Predicate formula. There exists an interpretation and environment under which α is true.

- (A) True
- (B) False
- (C) Not enough information

CQ A set of formulas is unsatisfiable

Suppose that a set of formulas Σ is unsatisfiable.

Which of the following is correct?

- (A) For every pair (I, E) , at least one formula in Σ is false.
- (B) For one pair (I, E) , at least one formula in Σ is false.
- (C) For every pair (I, E) , at least one formula in Σ is a contradiction.
- (D) For one pair (I, E) , at least one formula in Σ is a contradiction.
- (E) None of the above

CQ Proving unsatisfiability

We want to prove that a set of formulas Σ is unsatisfiable. See the beginning of our proof below:

Consider any interpretation and environment (I, E) . Consider two cases.

1. At least one formula in Σ is false under (I, E) .
2. ...

What is the other case?

- (A) Every formula in Σ is true under (I, E) .
- (B) Every formula in Σ is false under (I, E) .
- (C) At least one formula in Σ is true under (I, E) .
- (D) At least one formula in Σ is false under (I, E) .
- (E) None of the above

Revisiting the learning goals

By the end of this lecture, you should be able to:

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- ▶ Prove that an inference rule is sound or not sound.
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- ▶ Show that no natural deduction proof exists for a semantic entailment using the soundness and completeness theorems.