Predicate Logic: Semantic Entailment

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Lecture 14

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Outline

Predicate Logic: Semantic Entailment
- The Learning Goals
- Definition of Entailment
- Proving/Disproving an entailment
- Revisiting the Learning Goals

Learning goals

By the end of this lecture, you should be able to:

- Define semantic entailment for predicate logic.
- Prove that a semantic entailment holds.
- Prove that a semantic entailment does not hold.

Definition of Semantic Entailment

For propositional logic:
Let Σ be the set of propositional formulas. Let φ be a propositional formula.
$\Sigma \models \varphi$ iff for every truth valuation t, if $\Sigma^{\mathrm{t}} = \mathsf{T}$ (t satisfies Σ), then $\varphi^{\mathrm{t}} = \mathsf{T}$.

For predicate logic:
Let Σ be the set of predicate formulas. Let φ be a predicate formula.
$\Sigma \models \varphi$ iff for every interpretation I and environment E, if $\mathrm{I} \models_{\mathrm{E}} \Sigma$, then $\mathrm{I} \models_{\mathrm{E}} \varphi$.

Prove an entailment

Consider the entailment $\Sigma \models \varphi$.
To prove that the entailment holds, we need to consider

(A) Every (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$.
(B) Every (I, E) such that $\mathrm{I} \not\models_{\mathbf{E}} \Sigma$.
(C) One (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$.
(D) One (I, E) such that $\mathrm{I} \not\models_{\mathbf{E}} \Sigma$.

Disprove an entailment

Consider the entailment $\Sigma \models \varphi$.
To prove that the entailment does NOT hold, we need to consider

(A) Every (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$ and $\mathrm{I} \models_{\mathbf{E}} \varphi$.
(B) Every (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$ and $\mathrm{I} \not\models_{\mathbf{E}} \varphi$.
(C) One (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$ and $\mathrm{I} \models_{\mathbf{E}} \varphi$.
(D) One (I, E) such that $\mathrm{I} \models_{\mathbf{E}} \Sigma$ and $\mathrm{I} \not\models_{\mathbf{E}} \varphi$.

Disproving propositional entailment

A student is trying to prove that $\{(\alpha \to \beta)\} \not\models (\beta \to \alpha)$ where α and β are well-formed propositional formulas. The student starts the proof by writing down the following sentence.

> *There exists a truth valuation* t *such that* $\beta^{\mathrm{t}} = T$ *and* $\alpha^{\mathrm{t}} = F$.

Is the above sentence true (a valid claim)?

(A) Yes, it is true.

(B) No, it is false.

(C) There is not enough information to tell.

Revisiting the learning goals

By the end of this lecture, you should be able to:

- Define semantic entailment for predicate logic.
- Prove that a semantic entailment holds.
- Prove that a semantic entailment does not hold.