

Predicate Logic: Semantic Entailment

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Lecture 14

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Outline

Predicate Logic: Semantic Entailment

- The Learning Goals

- Definition of Entailment

- Proving/Disproving an entailment

- Revisiting the Learning Goals

Learning goals

By the end of this lecture, you should be able to:

- ▶ Define semantic entailment for predicate logic.
- ▶ Prove that a semantic entailment holds.
- ▶ Prove that a semantic entailment does not hold.

Definition of Semantic Entailment

For propositional logic:

Let Σ be the set of propositional formulas. Let φ be a propositional formula.

$\Sigma \models \varphi$ iff for every truth valuation t , if $\Sigma^t = \mathsf{T}$ (t satisfies Σ), then $\varphi^t = \mathsf{T}$.

For predicate logic:

Let Σ be the set of predicate formulas. Let φ be a predicate formula.

$\Sigma \models \varphi$ iff for every interpretation I and environment E , if $I \models_E \Sigma$, then $I \models_E \varphi$.

Prove an entailment

Consider the entailment $\Sigma \vDash \varphi$.

To prove that the entailment holds, we need to consider

- (A) Every (I, E) such that $I \vDash_E \Sigma$.
- (B) Every (I, E) such that $I \not\vDash_E \Sigma$.
- (C) One (I, E) such that $I \vDash_E \Sigma$.
- (D) One (I, E) such that $I \not\vDash_E \Sigma$.

Disprove an entailment

Consider the entailment $\Sigma \vDash \varphi$.

To prove that the entailment does NOT hold, we need to consider

- (A) Every (I, E) such that $I \vDash_E \Sigma$ and $I \vDash_E \varphi$.
- (B) Every (I, E) such that $I \vDash_E \Sigma$ and $I \not\vDash_E \varphi$.
- (C) One (I, E) such that $I \vDash_E \Sigma$ and $I \vDash_E \varphi$.
- (D) One (I, E) such that $I \vDash_E \Sigma$ and $I \not\vDash_E \varphi$.

Disproving propositional entailment

A student is trying to prove that $\{(\alpha \rightarrow \beta)\} \models (\beta \rightarrow \alpha)$ where α and β are well-formed propositional formulas. The student starts the proof by writing down the following sentence.

There exists a truth valuation t such that $\beta^t = T$ and $\alpha^t = F$.

Is the above sentence true (a valid claim)?

- (A) Yes, it is true.
- (B) No, it is false.
- (C) There is not enough information to tell.

Revisiting the learning goals

By the end of this lecture, you should be able to:

- ▶ Define semantic entailment for predicate logic.
- ▶ Prove that a semantic entailment holds.
- ▶ Prove that a semantic entailment does not hold.