Propositional Logic: Natural Deduction

Alice Gao

Lecture 7

Based on work by J. Buss, L. Kari, A. Lubiw, B. Bonakdarpour, D. Maftuleac, C. Roberts, R. Trefler, and P. Van Beek

Outline

Learning goals

By the end of this lecture, you should be able to

(Natural deduction)

- Describe rules of inference for natural deduction.
- Prove that a conclusion follows from a set of premises using natural deduction inference rules.

The Natural Deduction Proof System

We will consider a proof system called Natural Deduction.

- It closely follows how people (mathematicians, at least) normally make formal arguments.
- It extends easily to more-powerful forms of logic.

Why would you want to study natural deduction proofs?

- Write proofs with symbols and nested boxes.
- Prove or disprove that Superman exists.
- Prove or disprove that the onnagata are correct to insist that males should play female characters in Japanese kabuki theatres.
- Practise writing proofs and problem solving.
- Develop problem solving strategies that can be used in many other situations.

A proof is syntactic

First, we consider proofs in a purely syntactic way.
A proof

- starts with a set of premises,
- transforms the premises based on a set of inference rules,
- and ends with a conclusion.

We write $\Sigma \vdash_{ND} \varphi$ or simple $\Sigma \vdash \varphi$. if we can find such a proof that starts with a set of premises Σ and ends with the conclusion φ.

Goal is to show semantic entailment

Next, we think about relate proofs and semantic entailment.
We will answer two questions:

- (Soundness) If there is a proof from Σ to φ,
  does Σ entail φ?
- (Completeness) If Σ entails φ, is there a proof from Σ to φ?