

Propositional Logic: Natural Deduction

Alice Gao
Lecture 7

Based on work by J. Buss, L. Kari, A. Lubiw, B. Bonakdarpour, D. Maftuleac, C. Roberts, R. Trefler, and P. Van Beek

Outline

Propositional Logic: Natural Deduction

Learning goals

An overview of natural deduction

Learning goals

By the end of this lecture, you should be able to

(Natural deduction)

- ▶ Describe rules of inference for natural deduction.
- ▶ Prove that a conclusion follows from a set of premises using natural deduction inference rules.

The Natural Deduction Proof System

We will consider a proof system called Natural Deduction.

- ▶ It closely follows how people (mathematicians, at least) normally make formal arguments.
- ▶ It extends easily to more-powerful forms of logic.

Why would you want to study natural deduction proofs?

- ▶ Write proofs with symbols and nested boxes.
- ▶ Prove or disprove that Superman exists.
- ▶ Prove or disprove that the onnagata are correct to insist that males should play female characters in Japanese kabuki theatres.
- ▶ Practise writing proofs and problem solving.
- ▶ Develop problem solving strategies that can be used in many other situations.

A proof is syntactic

First, we consider proofs in a purely syntactic way.

A proof

- ▶ starts with a set of premises,
- ▶ transforms the premises based on a set of inference rules,
- ▶ and ends with a conclusion.

We write $\Sigma \vdash_{ND} \varphi$ or simple $\Sigma \vdash \varphi$. if we can find such a proof that starts with a set of premises Σ and ends with the conclusion φ .

Goal is to show semantic entailment

Next, we think about relate proofs and semantic entailment.

We will answer two questions:

- ▶ (Soundness) If there is a proof from Σ to φ , does Σ entail φ ?
- ▶ (Completeness) If Σ entails φ , is there a proof from Σ to φ ?