Theorem: \( \{ \land, \lor \} \) is not an adequate set of connectives.

Proof: To prove this theorem, we will prove the following lemma first.

Lemma: For any formula which uses only \( \land \) and \( \lor \) as connectives, if every variable in the formula is true, then the formula is true.

See the next page for a proof of this lemma using structural induction. Now we use this lemma below.

It's sufficient to show that we cannot write \((-x)\) using only \( \land \) and \( \lor \).

Assume \( x \) is true. Then \((-x)\) is false. By the lemma, for any formula using \( x, \land, \lor \), the formula must be true when \( x \) is true. Therefore, we cannot use \( x, \land \) and \( \lor \) to write a formula that is false when \( x \) is true.

So we cannot write \((-x)\) using only \( \land \) and \( \lor \).

\( \{ \land, \lor \} \) is not an adequate set of connectives.

QED.
Lemma: For any formula which uses only $\land$ and $\lor$ as connectives, if every variable in the formula is true, then the formula is true.

Let $P(Y)$ denote $Y$ is true when every variable in $Y$ is true where $Y$ uses only $\land$ and $\lor$ as connectives.

Proof by structural induction on $Y$.

Base case: $Y$ is a propositional variable $p$.
If $p$ is true, then $Y$ is true. $P(Y)$ holds.

Induction step:

Case 1: $Y = (x \land y)$ where $x$ and $y$ use only $\land$ and $\lor$ as connectives.

Induction Hypothesis: if every variable in $x$ (and $y$) is true, then $x$ (and $y$) is true.
Assume every variable in $Y$ is true. Then every variable in $x$ and $y$ is true. By induction hypothesis, $x$ and $y$ are true. So $Y \equiv (T \land T) \equiv T$.

Case 2: $Y = (x \lor y)$ where $x$ and $y$ use only $\land$ and $\lor$ as connectives.

Induction Hypothesis: if every variable in $x$ (and $y$) is true, then $x$ (and $y$) is true.
Assume every variable in $Y$ is true. Then every variable in $x$ and $y$ is true. By induction hypothesis, $x$ and $y$ are true. So $Y \equiv (T \lor T) \equiv T$.

By the principle of structural induction, $P(Y)$ holds for every $Y$ that uses only $\land$ and $\lor$ as connectives.

QED