Consider the following argument about the existence of Superman.

If Superman were able and willing to prevent evil, he would do so.
If Superman were unable to prevent evil he would be impotent.
If Superman were unwilling to prevent evil, he would be malevolent.
Superman does not prevent evil.
If Superman exists, he is neither impotent nor malevolent.

Therefore, Superman does not exist.

Questions:

1. Translate the premises and the conclusion into propositional logic formulas. Define your own propositions.

Proposition definitions:

a: Superman is able to prevent evil.
w: Superman is willing to prevent evil.
p: Superman prevents evil.
i: Superman is impotent.
m: Superman is malevolent.
e: Superman exists.

Premises:

1. $((\boldsymbol{a} \wedge \boldsymbol{w}) \rightarrow \boldsymbol{p})$
2. $\big((\neg \boldsymbol{a}) \rightarrow \boldsymbol{i}\big)$
3. $\big((\neg \boldsymbol{w}) \rightarrow \boldsymbol{m}\big)$
4. $(\neg \boldsymbol{p})$
5. $(\boldsymbol{e} \rightarrow \big((\neg \boldsymbol{i}) \wedge (\neg \boldsymbol{m})\big))$

Conclusion:

$$(\neg \boldsymbol{e})$$

Check your answer with us before proceeding to the following questions.

2. Do you believe that the premises semantically entail the conclusion? Why or why not? Could you give some intuitive reasoning to justify your belief?

3. Prove or disprove that the semantic entailment holds using a truth table or by reasoning in English.

Proof by contradiction: Assume that there is a valuation t under which all the premises are true and the conclusion is false.

Since the conclusion is false, e is true under t.

By premise 4, p is false under t. By premise 1, a is false or w is false.

If a is false and w is true, then i has to be true for premise 2 to be true, and premise 3 is vacuously true. Since i is true, e has to be false for premise 5 to be true.

If a is true and w is true, them m has to be true for premise 3 to be true, and premise 2 is vacuously true. Since m is true, e has to be false for premise 5 to be true.

If a and w are both false, then i has to be true for premise 2 to be true and m has to be true for premise 3 to be true. Since i and m are both true, e has to be false for premise 5 to be true.

All of these cases contradict with the assumption that e is true.

Therefore, our assumption is false and the entailment holds.

Check your answer with us before proceeding to the following questions.

If you proved that the semantic entailment holds, proceed to question 4. Otherwise, if you proved that the semantic entailment does not hold, proceed to question 5.

4. If you proved that the semantic entailment holds, give a natural deduction proof to show that you can derive the conclusion from the premises.

5. If you proved that the semantic entailment does not hold...
    a. Could you modify the argument to make the semantic entailment hold? Hint: try removing a part of a premise or the conclusion. If you can, prove that the semantic entailment holds in the modified argument.
    b. Give a natural deduction proof to show that you can derive the conclusion from the premises in the modified argument.

You may use the additional rules including Modus Tollens, De Morgan's law, and equivalence.

See a separate handout for the natural deduction proof.