① Show that $\{((p \wedge q) \rightarrow r)\} \vdash (p \rightarrow (q \rightarrow r))$

Proof :

1	$((p \wedge q) \rightarrow r)$	premise
2	p	assumption
3	q	assumption
4	$(p \wedge q)$	$\wedge i$: 2,3
5	r	$\rightarrow e$: 1,4
6	$(q \rightarrow r)$	$\rightarrow i$: 3-5
7	$(p \rightarrow (q \rightarrow r))$	$\rightarrow i$: 2-6

QED

Fill in the missing justifications in the proof.
For each justification, indicate the name of the rule (e.g. $\wedge i$) and the numbers for the lines the rule applied to.

spare

② Show that $(p \to (q \to r)) \vdash ((p \land q) \to r)$

Proof:

1	$(p \to (q \to r))$	premise.
2	$(p \land q)$	assumption.
3.	p	$\land e: 2$
4	$(q \to r)$	$\to e: 1, 3$
5	q	$\land e: 2.$
6.	r	$\to e: 4, 5$
7.	$((p \land q) \to r)$	$\to i: 2-6$

QED.

Write down the premises and the conclusion.
Can I apply an elimination rule to a premise?
Can I apply an introduction rule to get to the conclusion?

③ Show that $\{(p \wedge (q \vee r))\} \vdash ((p \wedge q) \vee (p \wedge r))$.

Proof:

1	$(p \wedge (q \vee r))$	premise
2	p	$\wedge e$: 1
3	$(q \vee r)$	$\wedge e$: 1
4	q	assumption
5	$(p \wedge q)$	$\wedge i$: 2,4
6	$((p \wedge q) \vee (p \wedge r))$	$\vee i$: 5
7	r	assumption
8	$(p \wedge r)$	$\wedge i$: 2,7
9	$((p \wedge q) \vee (p \wedge r))$	$\vee i$: 8
10	$((p \wedge q) \vee (p \wedge r))$	$\vee e$: 3, 4-6, 7-9

QED.

Fill in the missing justifications in the proof.
For each justification, indicate the name of the rule (e.g. $\wedge i$) and the numbers for the lines the rule applied to.

spare

④ Show that $\{(p \to q)\} \vdash (r \lor p) \to (r \lor q)$.

Proof:

1	$(p \to q)$	premise.
2	$(r \lor p)$	assumption
3	r	assumption
4	$(r \lor q)$	$\lor i$: 3
5	p	assumption.
6	q	$\to e$: 1, 5
7	$(r \lor q)$	$\lor i$: 6.
8	$(r \lor q)$	$\lor e$: 2, 3-4, 5-7
9	$((r \lor p) \to (r \lor q))$	$\to i$: 2-8.

QED

① Show that $\{(\alpha \to (\neg\alpha))\} \vdash (\neg\alpha)$.

Proof:

1.	$(\alpha \to (\neg\alpha))$	premise.
2	α	assumption.
3	$(\neg\alpha)$	$\to$e: 1,2
4.	$\perp$	$\perp$i: 2,3
5.	$(\neg\alpha)$	$\neg$i: 2-4

QED.

Fill in the missing justifications in the proof.

② Show that $\{(P \to q), (\neg q)\} \vdash (\neg P)$ (modus tollens).

Proof:

1.	$P \to q$	premise
2.	$(\neg q)$	premise
3.	P	assumption
4.	q	$\to$e: 1,3
5.	$\perp$	$\perp$i: 2,4
6.	$(\neg P)$	$\neg$i: 3-5

QED

③ Show that $\{(p \to (q \to r)), p, (\neg r)\} \vdash (\neg q)$.

Proof:

1	$(p \to (q \to r))$	premise
2	p	premise.
3	$(q \to r)$	$\to e: 1,2$
4	q	assumption.
5	r	$\to e: 3,4$
6	$(\neg r)$	premise.
7	$\bot$	$\bot i: 5,6.$
8	$(\neg q)$	$\neg i: 4-7$

QED.

Fill in the missing justifications in the proof.

④ Show that $\{((p \wedge (\neg q)) \to r), (\neg r), p\} \vdash q$.

Proof:

1.	$((p \wedge (\neg q)) \to r)$	premise
2.	$(\neg r)$	premise
3.	p	premise
4	$\neg q$	assumption
5	$(p \wedge (\neg q))$	$\wedge i: 3,4$
6	r	$\to e: 1,5$
7	$\bot$	$\bot i: 2,6.$
8	$\neg\neg q$	$\neg i: 4-7$
9	q	$\neg\neg e: 8$

QED

⑤ Show that $\{((\neg\alpha)\wedge(\neg\beta))\} \vdash (\neg(\alpha\vee\beta))$.

(This is one part of the De Morgan's law.)

Hints: you may want to use proof by contradiction ($\neg i$), and proof by cases ($\vee e$).

Proof:

1.	$((\neg\alpha)\wedge(\neg\beta))$	premise
2.	$\cancel{(\alpha\vee\beta)}$ $(\neg\alpha)$	$\wedge e$: 1
3	$(\neg\beta)$	$\wedge e$: 1
4	$(\alpha\vee\beta)$	assumption
5	α	assumption
6	$\perp$	$\perp i$: 2,5
7	β	assumption
8	$\perp$	$\perp i$: 3,7
9	$\perp$	$\vee e$: 4,5-6,7-8
10	$(\neg(\alpha\vee\beta))$	$\neg i$: 4-9.

QED