Consider the following argument, drawn from an article by Julian Baggini. The onnagata are male actors portraying female characters in kabuki theatre.

Premise 1: If women are too close to femininity to portray women, then men must be too close to masculinity to play men, and vice versa.

Premise 2: And yet, if the onnagata are correct, women are too close to femininity to portray women and yet men are not too close to masculinity to play men.

Conclusion: Therefore, the onnagata are incorrect, and women are not too close to femininity to portray women.

Relevant articles:

http://www.butterfliesandwheels.org/2004/tu-quoque/

https://www.theguardian.com/stage/2004/aug/21/theatre

Questions:

1. Translate the premises and the conclusion into propositional logic formulas. Define your own propositions.

Proposition definitions:

w: women are too close to femininity to portray women.
m: men are too close to masculinity to play men.
o: the onnagata are correct.

Premises:

Premise 1: $(\boldsymbol{w} \leftrightarrow \boldsymbol{m})$
Premise 2: ($\boldsymbol{o} \rightarrow (\boldsymbol{w} \wedge (\neg \boldsymbol{m}))$)

Conclusion: $((\neg \boldsymbol{o}) \wedge (\neg \boldsymbol{w}))$

Check your answer with us before proceeding to the following questions.

2. Do you believe that the premises semantically entail the conclusion? Why or why not? Could you give some intuitive reasoning to justify your belief?

The premises do not semantically entail the conclusion. Here is a truth valuation which makes both premises true and the conclusion false.

$$w^t = T, m^t = T, o^t = F$$

Premise 1: since w and m have the same truth values under t, the bi-conditional is true.

Premise 2: Since o is false under t, the implication is vacuously true.

Conclusion: Since w is true under t, the conjunction is false.

3. Prove or disprove that the semantic entailment holds using a truth table or by reasoning in English.

o	w	m	$(w \leftrightarrow m)$	$o \rightarrow (w \wedge (\neg m))$	$((\neg o) \wedge (\neg w))$
0	0	0	1	1	1
0	0	1	0	1	1
0	1	0	0	1	0
0	1	1	1	1	0
1	0	0	1	0	0
1	0	1	0	0	0
1	1	0	0	1	0
1	1	1	1	0	0

The fourth row indicates that the semantic entailment does not hold.

Check your answer with us before proceeding to the following questions.

If you proved that the semantic entailment holds, proceed to question 4. Otherwise, if you proved that the semantic entailment does not hold, proceed to question 5.

4. If you proved that the semantic entailment holds, give a natural deduction proof to show that you can derive the conclusion from the premises.

5. If you proved that the semantic entailment does not hold...
   a. Could you modify the argument to make the semantic entailment hold? Hint: try removing a part of the conclusion. If you can, prove that the semantic entailment holds in the modified argument.
   b. Give a natural deduction proof to show that you can derive the conclusion from the premises in the modified argument.

You may use the additional rules including Modus Tollens, De Morgan's law, and equivalence.

5(a)

Change the conclusion from $((\neg o) \wedge (\neg w))$ to $(\neg o)$. Then the following truth tables proves that the semantic entailment holds.

o	w	m	$(w \leftrightarrow m)$	$o \rightarrow (w \wedge (\neg m)))$	$(\neg o)$
0	0	0	1	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	0	0	0
1	1	0	0	1	0
1	1	1	1	0	0

5(b)

See a separate handout for the natural deduction proof.