Consider the following story:

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. A knight invited a newcomer to the island and told the newcomer the following facts about five inhabitants: Alice, Bob, Peggy, Rex and Ted.

It is not the case that Bob is a knight or Peggy is a knight.
Ted and Bob are both knights if and only if Rex is a knight.
If Rex is a knave, then Alice is a knave.

Using the power of logic, the newcomer concluded that Alice is a knave.

Note: This is a work of fiction. If your name appeared in the story, it is because Alice Gao liked it too much! ☺

Questions:

1. Translate the premises and the conclusion into propositional logic formulas. Define your own propositions.

Proposition definitions:

a: Alice is a knight.
b: Bob is a knight.
p: Peggy is a knight.
r: Rex is a knight.
t: Ted is a knight.

Premises:

1. $(\neg(\boldsymbol{b} \vee \boldsymbol{p}))$
2. $((\boldsymbol{b} \wedge \boldsymbol{t}) \leftrightarrow \boldsymbol{r})$
3. $((\neg \boldsymbol{r}) \rightarrow (\neg \boldsymbol{a}))$

Conclusion:

$$(\neg \boldsymbol{a})$$

Check your answer with us before proceeding to the following questions.

2. Do you believe that the premises semantically entail the conclusion? Why or why not? Could you give some intuitive reasoning to justify your belief?

For premise 1 to be true, it has to be that b and p are both false.
If b is false, then for premise 2 to be true, r has to be false.
If r is false, then a has to be false for premise 3 to be true.

3. Prove or disprove that the semantic entailment holds using a truth table or by reasoning in English.

Proof by contradiction:

Assume that the entailment does not hold. All three premises are true and the conclusion is false.

Since the conclusion is false, a is true.
Premise 1 is true so b and p are both false.
The left-hand side of premise 2 is false because b is false. If premise 2 is true, then r has to be false.
Since premise 3 is true and r is false, a has to be false. This contradicts with our assumption that a is true.

Therefore, our assumption is false and the entailment holds.

Check your answer with us before proceeding to the following questions.

If you proved that the semantic entailment holds, proceed to question 4. Otherwise, if you proved that the semantic entailment does not hold, proceed to question 5.

4. If you proved that the semantic entailment holds, give a natural deduction proof to show that you can derive the conclusion from the premises.

5. If you proved that the semantic entailment does not hold...
    a. Could you modify the argument to make the semantic entailment hold? Hint: try removing a part of a premise or the conclusion. If you can, prove that the semantic entailment holds in the modified argument.
    b. Give a natural deduction proof to show that you can derive the conclusion from the premises in the modified argument.

You may use the additional rules including Modus Tollens, De Morgan's law, and equivalence.

See a separate handout for the natural deduction proof.