Translate English Sentences Into Predicate Logic. Oct 12, 2017

Domain: set of animals.
Honey(x) means x likes honey.
Bear(x) means x is a bear.

1. All animals like honey. $(\forall x\ Honey(x))$
   each, every.
⑤ No animal dislikes honey. $(\neg(\exists x\ (\neg Honey(x))))$

2. At least one animal likes honey. $(\exists x\ Honey(x))$.
   some
⑥ Not every animal dislikes honey. $(\neg(\forall x\ (\neg Honey(x))))$.

3. Not every animal likes honey. $(\neg(\forall x\ Honey(x)))$
   It's not the case that every animal likes honey.
⑦ Some animal dislikes honey. $(\exists x\ (\neg Honey(x)))$.

4. No animal likes honey. $(\neg(\exists x\ Honey(x)))$.
   There does not exist an animal who likes honey.
⑧ Every animal dislikes honey. $(\forall x\ (\neg Honey(x)))$.

Each pair of sentences and formulas is logically equivalent.
However, when doing translations, always give the more direct and literal translation to avoid losing marks.

Translate English Sentences into Predicate Logic.

Domain: set of animals
Honey(x) means x likes honey.

5. No animal dislikes honey. $(\neg(\exists x\ (\neg \text{Honey}(x))))$.
There does not exist an animal who dislikes honey.

6. Not every animal dislikes honey. $(\neg(\forall x\ (\neg \text{Honey}(x))))$
It's not the case that every animal dislikes honey.

7. Some animal dislikes honey. $(\exists x\ (\neg \text{Honey}(x)))$.

8. Every animal dislikes honey. $(\forall x\ (\neg \text{Honey}(x)))$.

sample

Translate English Sentences into Predicate Logic. Oct 12.

Domain: the set of animals
Bear(x): x is a bear. Honey(x): x likes honey.

Translate "every bear likes honey" into predicate logic.

(a). $(\forall x\ (Bear(x) \land Honey(x)))$
Every animal is a bear and likes honey.

(b) $(\forall x\ (Bear(x) \lor Honey(x)))$
Every animal is a bear or likes honey or both.

(c) $(\forall x\ (Bear(x) \rightarrow Honey(x)))$
If an animal is a bear, then it likes honey.
If an animal is NOT a bear, cool, no problem!

(d) $(\forall x\ (Honey(x) \rightarrow Bear(x)))$
Every honey-liker is a bear.

Domain 1: { Bear X likes honey, Bear Y likes Honey, Rabbit K dislikes honey }
(a) is False because Rabbit K is NOT a bear.
(c) is True.
(b) is False, Rabbit K is NOT a bear and dislikes honey.

Domain 2: { Bear X likes honey, Rabbit K likes honey. }.
(c) is True.
(d) is False, K likes honey but is not a bear.

Domain D, and predicate P(x).
"All <things in D for which P is true> have the property Q."

$$\underbrace{(\forall x\ (P(x)}_{\text{sample}} \rightarrow Q(x)))$$

sample All <things in D for which P is true>.

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Domain: the set of animals.
Bear(x): x is a bear. Honey(x): x likes honey.

Translate "some bear likes honey." into predicate logic.
(Implicitly, it says "there exists a bear who ..... ".
Some animal is a bear and also likes honey.
There is an animal that is a bear and likes honey.

(a) $(\exists x\ (Bear(x) \wedge Honey(x)))$.
There is one bear who likes honey.

(b) $(\exists x\ (Bear(x) \vee Honey(x)))$.
There is an animal who is either a bear or likes honey or both.

(c) $(\exists x\ (Bear(x) \rightarrow Honey(x)))$.
There is an animal who is a bear and likes honey or who is NOT a bear.

(d) $(\exists x\ (Honey(x) \rightarrow Bear(x)))$.
There is an animal who likes honey and is a bear or who doesn't like honey.

Domain 1: {Rabbit}.
(a) is False. (c) is True.

Domain 2: {Tiger T doesn't like honey}.
(a) is False (d) is True.

Domain 3: {Bear X doesn't like honey}.
(a) is False (b) is True.

Domain D, predicate P(x).
Some <thing in D for which P is true> has the property Q.

$$\underbrace{(\exists x\ (P(x)}\ \wedge\ Q(x)))$$

sample. Some <thing in D for which P is true>.

Translate English Sentences into Predicate Logic Oct 12.

Domain: the set of people

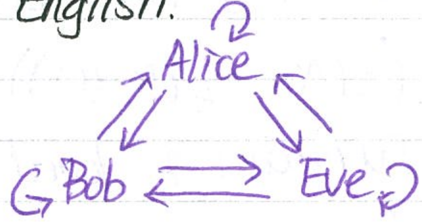
$L(x, y)$: person x likes person y. (directed relationship).

$L(x, y)$ is not necessarily the same as $L(y, x)$.

Translate the following formulas into English.

① $(\forall x (\forall y \; L(x, y)))$

Everyone likes everyone.



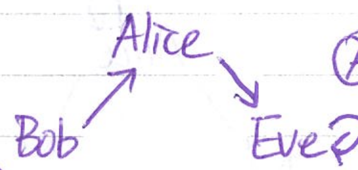
② $(\exists x (\exists y \; L(x, y)))$

Someone likes someone.

③ $(\forall x (\exists y \; L(x, y)))$

Everybody likes somebody.

(Everyone has at least one outgoing edge.)



(A).

④ $(\exists y (\forall x \; L(x, y)))$.

Somebody is liked by everybody.

(One person has an incoming edge from everyone.)

(a) Give an example for which ③ is true and ④ is false.

(b) Give an example for which ③ is false and ④ is true.

(A) is an answer for (a).

(b) is impossible, ④ → ③. If X is liked by everyone, then everyone likes X.

If the multiple quantifiers are the same, changing their order does NOT change the formula's meaning.

If the multiple quantifiers are different, changing their order WILL change the formula's meaning.

Order matters when the quantifiers are different.

sample

Oct 12.

③ $(\forall x \ (\exists y \quad L(x,y)))$

Everybody loves somebody.

⑤ $(\exists x \ (\forall y \quad L(x,y)))$

Somebody loves everybody.

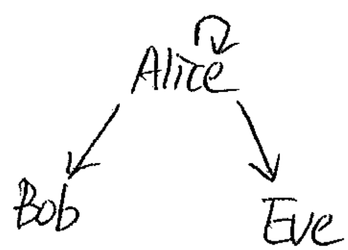
⑥ $(\forall y \ (\exists x \quad L(x,y)))$

Everybody is loved by somebody.

④ $(\exists y \ (\forall x \quad L(x,y)))$

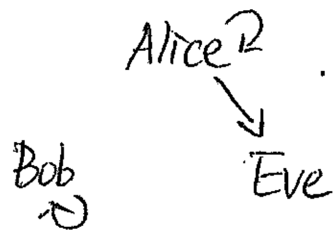
Somebody is loved by everybody.

⑤



- One person has an outgoing edge to everyone.

⑥



- Everyone has at least one incoming edge.

⑤ $\rightarrow$ ⑥.

If X loves everyone, then everyone is loved by X.