

Interpreting The Quantifiers

Let the domain be $\{Alice, Bob, Eve\}$

Let $C(x)$ mean x likes chocolates

Q1: Which of the following is equivalent to $(\forall x C(x))$?

(a) $((C(Alice) \wedge C(Bob)) \wedge C(Eve))$

(b) $((C(Alice) \vee C(Bob)) \vee C(Eve))$

(c) $((C(Alice) \rightarrow C(Bob)) \rightarrow C(Eve))$

(d) $((C(Alice) \leftrightarrow C(Bob)) \leftrightarrow C(Eve))$

(e) None of the above

Q2 Which of the following is equivalent to $(\exists x C(x))$?

(a) $((C(Alice) \wedge C(Bob)) \wedge C(Eve))$

(b) $((C(Alice) \vee C(Bob)) \vee C(Eve))$

(c) $((C(Alice) \rightarrow C(Bob)) \rightarrow C(Eve))$

(d) $((C(Alice) \leftrightarrow C(Bob)) \leftrightarrow C(Eve))$

(e) None of the above

Let $D = \{d_1, d_2, \dots, d_n\}$

\forall is a big AND:

$(\forall x P(x))$ is equivalent to $P(d_1) \wedge P(d_2) \wedge \dots \wedge P(d_n)$

\exists is a big OR:

$(\exists x P(x))$ is equivalent to $P(d_1) \vee P(d_2) \vee \dots \vee P(d_n)$

Negating a quantified formula:

$$\neg(\forall x P(x)) \equiv (\exists x (\neg P(x)))$$

$$\neg(\exists x P(x)) \equiv (\forall x (\neg P(x)))$$

$$\neg(\forall x (\exists y P(x, y))) \equiv (\exists x (\forall y (\neg P(x, y))))$$

$$\neg(\exists x (\forall y P(x, y))) \equiv (\forall x (\exists y (\neg P(x, y))))$$