

# The Environment Re-assignment Notation:

Oct 24

Variable symbols:  $x, y$

Predicate symbols:  $P^{(2)}$

$I: \text{dom}(I) = \{a, b\}$

$P^I = \{\langle a, a \rangle, \langle a, b \rangle, \langle b, b \rangle\}$

$E: E(x) = a, E(y) = b$

$E[y \mapsto a]$  produces an environment that is different from  $E$ .

$E[y \mapsto a]$  preserves all the mappings in  $E$  EXCEPT that it re-assigns  $y$  to  $a$ .

$E[y \mapsto a]$  maps  $x$  to  $a$ , same as  $E$ .  
but it maps  $y$  to  $a$ .

Examples:

$E[x \mapsto b](x) = b$        $E[x \mapsto b](y) = b$

$E[x \mapsto b][y \mapsto a](x) = b$        $E[x \mapsto b][y \mapsto a](y) = a$

$\langle E[y \mapsto a](y), E[y \mapsto a](x) \rangle = \langle a, a \rangle$

$P(x, y)$   <sup>$(I, E[y \mapsto a])$</sup>  means the value of  $P(x, y)$   
under  $I$  and  $E[y \mapsto a]$

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$I: \text{dom}(I) = \{a, b\}$

$P^I = \{\langle a, a \rangle, \langle b, a \rangle, \langle b, b \rangle\}$

$E: E(x) = a \quad E(y) = b$

Q1: What is  $\alpha = (\exists x P(x, y))^{(I, E)}$ ?

- $y$  is a free variable and has the value  $b$  by def of  $E$ .
- To make  $\alpha$  true, we need to find one value for  $x$  to make  $P(x, y)$  true. " $b$ " is such a value.

Formally,  $\langle E[x \mapsto b](x), E[x \mapsto b](y) \rangle = \langle b, b \rangle \in P^I$

Thus,  $P(x, y)^{(I, E[x \mapsto b])} = T$  and  $(\exists x P(x, y))^{(I, E)} = T$ .

Q2: What is  $\beta = (\forall x P(x, y))^{(I, E)}$ ?

- $y$  is free and takes the value  $b$  by def. of  $E$ .
- To make  $\beta$  true, we need to verify that  $P(x, y)$  is true for every possible value of  $x$ .

Formally,

$\langle E[x \mapsto a](x), E[x \mapsto a](y) \rangle = \langle a, b \rangle \notin P^I$

So  $P(x, y)^{(I, E[x \mapsto a])} = F$

We found one value of  $x$  for which  $P(x, y)$  is false, so  $\beta$  must be false.  $(\forall x P(x, y))^{(I, E)} = F$ .