Program Verification
Reversing an array

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Lecture 22

Based on work by J. Buss, L. Kari, A. Lubiw, B. Bonakdarpour, D. Maftuleac, C. Roberts, R. Trefler, and P. Van Beek
Outline

Program Verification: Reversing an array
Learning Goals
Introducing the array assignment rule
Revisiting the Learning Goals
Learning Goals

By the end of this lecture, you should be able to:
Partial correctness for array assignments
▶ Prove that a Hoare triple is satisfied under partial correctness for a program containing array assignment statements.
The array assignment inference rule

Let $A$ be an array of $n$ integers.

First, write down the sequence of changes.
Resolve all of the changes when we prove the implied’s.

\[ Q[A\{e_1 \leftarrow e_2\}/A] \]
\[ A[e_1] = e_2; \]
\[ Q \text{ array assignment} \]

- $A$ is the original array.
- $A\{e_1 \leftarrow e_2\}$ is the new array, which is identical to array $A$ except that the $e_1^{th}$ element is $e_2$. 
The array re-assignment notation:

The array reassignment notation:

\[ A\{e_1 \leftarrow e_2\}[i] = \begin{cases} e_2, & \text{if } i = e_1 \\ A[i], & \text{if } i \neq e_1 \end{cases} \]

Note that \( e_1 \) is an index whereas \( e_2 \) is an array element.

We apply assignments from left to right.

**Examples:**

- \( A\{1 \leftarrow 3\}[1] = 3 \)
- \( A\{1 \leftarrow 3\}\{1 \leftarrow 4\}[1] = 4 \)
Consider an array $R$ of $n$ integers, $R[1], R[2], ..., R[n]$. We want to reverse the order of its elements.

Our algorithm:

For each $1 \leq j \leq \lfloor n/2 \rfloor$, we will swap $R[j]$ with $R[n + 1 - j]$. 

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Reversing an array
Reversing an array

$R$ is an array of $n$ integers, $R[1], R[2], \ldots, R[n]$. Prove that the following triple is satisfied under partial correctness.

$$\left( \forall x \ ((1 \leq x \leq n) \rightarrow (R[x] = r_x)) \right)$$

\[ j = 1; \]
\[ \textbf{while} \ (2 \ast j <= n) \ {\} \]
\[ \begin{align*}
& t = R[j]; \\
& R[j] = R[n+1-\ j]; \\
& R[n+1-\ j] = t; \\
& j = j + 1;
\end{align*} \]

$$\left( \forall x \ ((1 \leq x \leq n) \rightarrow (R[x] = r_{n+1-x})) \right)$$
Reversing an array

$R$ is an array of $n$ integers, $R[1], R[2], ..., R[n]$. Prove that the following triple is satisfied under partial correctness.

Let $Inv(j)$ denote our invariant.

$$(\forall x ((1 \leq x \leq n) \to (R[x] = r_x)))$$

j = 1;
while (2 * j <= n) {
    t = R[j];
    R[j] = R[n+1-j];
    R[n+1-j] = t;
    j = j + 1;
}

$$(\forall x ((1 \leq x \leq n) \to (R[x] = r_{n+1-x})))$$
CQ 1: Consider the premise of implied (A). Which of the following is an accurate description of the formula?

(A) No swap has occurred.

(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.

(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.

(D) All swaps have been completed.

(E) None of the above
CQ 2: Consider the conclusion of implied (A).

Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 3: Consider the premise of implied (C).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 4: Consider the conclusion of implied (C).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 5: Consider the premise of implied (B).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 6: Consider the conclusion of implied (B). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
By the end of this lecture, you should be able to:
Partial correctness for array assignments
   ▶ Prove that a Hoare triple is satisfied under partial correctness for a program containing array assignment statements.