Program Verification
Reversing an array

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Lecture 22

Based on work by J. Buss, L. Kari, A. Lubiw, B. Bonakdarpour, D. Maftuleac, C. Roberts, R. Trefler, and P. Van Beek
Outline

Program Verification: Reversing an array
Learning Goals
Introducing the array assignment rule
Revisiting the Learning Goals
Learning Goals

By the end of this lecture, you should be able to:
Partial correctness for array assignments
  ▶ Prove that a Hoare triple is satisfied under partial correctness for a program containing array assignment statements.
Let $A$ be an array of $n$ integers.

First, write down the sequence of changes. Resolve all of the changes when we prove the implied’s.

\[
\begin{align*}
&\langle Q[A\{e_1 \leftarrow e_2\}/A]\rangle \\
&A[e_1] = e_2; \\
&\langle Q\rangle & \text{array assignment}
\end{align*}
\]

- $A$ is the original array.
- $A\{e_1 \leftarrow e_2\}$ is the new array, which is identical to array $A$ except that the $e_1^{th}$ element is $e_2$. 

The array re-assignment notation:

The array reassignment notation:
\[
A\{e_1 \leftarrow e_2\}[i] = \begin{cases} 
e_2, & \text{if } i = e_1 \\ A[i], & \text{if } i \neq e_1 \end{cases}
\]

Note that \(e_1\) is an index whereas \(e_2\) is an array element.

We apply assignments from left to right.

**Examples:**
- \(A\{1 \leftarrow 3\}[1] = 3\)
- \(A\{1 \leftarrow 3\}\{1 \leftarrow 4\}[1] = 4\)
Consider an array $R$ of $n$ integers, $R[1], R[2], \ldots, R[n]$. We want to reverse the order of its elements.

Our algorithm:

For each $1 \leq j \leq \lfloor n/2 \rfloor$, we will swap $R[j]$ with $R[n + 1 - j]$. 
Reversing an array

R is an array of n integers, R[1], R[2], ..., R[n]. Prove that the following triple is satisfied under partial correctness.

\[
(\forall x \ ((1 \leq x \leq n) \rightarrow (R[x] = r_x))) \quad \left\vert \begin{array}{c}
\text{j = 1;}
\text{while (2 * j <= n) {}
\quad t = R[j];
\quad R[j] = R[n+1-j];
\quad R[n+1-j] = t;
\quad j = j + 1;
\text{}}
\end{array} \quad (\forall x \ ((1 \leq x \leq n) \rightarrow (R[x] = r_{n+1-x})))\]
\]
Reversing an array

$R$ is an array of $n$ integers, $R[1], R[2], \ldots, R[n]$. Prove that the following triple is satisfied under partial correctness.

Let $Inv(j)$ denote our invariant.

\[
\begin{align*}
(p ((\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_x)))) \land j = 1; \\
\textbf{while} (2 * j \leq n) \{ \\
\quad t = R[j]; \\
\quad R[j] = R[n+1-j]; \\
\quad R[n+1-j] = t; \\
\quad j = j + 1; \\
\}\end{align*}
\]

\[
(p ((\forall x ((1 \leq x \leq n) \rightarrow (R[x] = r_{n+1-x})))) \land \neg \text{Inv}(j))
\]
CQ 1: Consider the premise of implied (A). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 2: Consider the conclusion of implied (A).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.
(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 3: Consider the premise of implied (C). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 4 Reversing an array

CQ 4: Consider the conclusion of implied (C). Which of the following is an accurate description of the formula?

(A) No swap has occurred.

(B) Elements in $[1, j - 1]$ have been swapped, and elements in $[j, (n + 1)/2]$ have NOT been swapped.

(C) Elements in $[1, j]$ have been swapped, and elements in $[j + 1, (n + 1)/2]$ have NOT been swapped.

(D) All swaps have been completed.

(E) None of the above
CQ 5:  Consider the premise of implied (B). Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in \([1, j - 1]\) have been swapped, and elements in \([j, (n + 1)/2]\) have NOT been swapped.
(C) Elements in \([1, j]\) have been swapped, and elements in \([j + 1, (n + 1)/2]\) have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
CQ 6: Consider the conclusion of implied (B).
Which of the following is an accurate description of the formula?

(A) No swap has occurred.
(B) Elements in [1, j − 1] have been swapped, and elements in [j, (n + 1)/2] have NOT been swapped.
(C) Elements in [1, j] have been swapped, and elements in [j + 1, (n + 1)/2] have NOT been swapped.
(D) All swaps have been completed.
(E) None of the above
Revisiting the learning goals

By the end of this lecture, you should be able to:
Partial correctness for array assignments
  ▶ Prove that a Hoare triple is satisfied under partial correctness for a program containing array assignment statements.