Propositional Logic:
Formal Deduction

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Lecture 7
Outline

Learning goals

Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Outline

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Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Learning goals

By the end of this lecture, you should be able to

▶ Describe rules of inference for natural deduction.
▶ Prove that a conclusion follows from a set of premises using rules of formal deduction.
Outline

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Revisiting the Learning Goals
Why study formal deduction?

- Want to prove that a conclusion can be deduced from a set of premises.
- Want to generate a proof that can be checked mechanically.
Let the relation of formal deducibility be denoted by

$$ \Sigma \vdash A, $$

which means that \( A \) is formally deducible (or provable) from \( \Sigma \).

Comments:
- \( \Sigma \) is a set of formulas, which are the premises.
- \( A \) is a formula, which is the conclusion.
- Formal deducibility is concerned with the syntactic structure of formulas.
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Rules of Formal Deduction

- **Reflexivity (Ref):**
  \[ A \vdash A. \]

- **Addition of premises (+):**
  \[
  \text{if } \Sigma \vdash A, \\
  \text{then } \Sigma, \Sigma' \vdash A.
  \]

- **(∈):**
  \[
  \text{if } A \in \Sigma, \\
  \text{then } \Sigma \vdash A.
  \]
Conjunction Rules

And introduction ($\land+$)

if $\Sigma \vdash A$, 
$\Sigma \vdash B$, 
then $\Sigma \vdash A \land B$.

And elimination ($\land-$)

if $\Sigma \vdash A \land B$, 
then $\Sigma \vdash A$. 
if $\Sigma \vdash A \land B$, 
then $\Sigma \vdash B$. 
Disjunction Rules

Or introduction ($\lor+$)

if $\Sigma \vdash A$,
then $\Sigma \vdash A \lor B$.

if $\Sigma \vdash B$,
then $\Sigma \vdash A \lor B$.

Or elimination ($\lor-$)

if $\Sigma, A \vdash C$,
$\Sigma, B \vdash C$,
then $\Sigma, A \lor B \vdash C$. 
Negation Rules

Negation introduction (¬+)  
if \( \Sigma, A \vdash B \), 
\( \Sigma, A \vdash \neg B \), 
then \( \Sigma \vdash \neg A \).

Negation elimination (¬−)  
if \( \Sigma, \neg A \vdash B \), 
\( \Sigma, \neg A \vdash \neg B \), 
then \( \Sigma \vdash A \).
Implication Rules

Implication introduction $(\rightarrow +)$

if $\Sigma, A \vdash B$,
then $\Sigma \vdash A \rightarrow B$.

Implication elimination $(\rightarrow -)$

if $\Sigma \vdash A$,
$\Sigma \vdash A \rightarrow B$,
then $\Sigma \vdash B$. 
Equivalence Rules

**Equivalence introduction**

(\(\leftrightarrow +\))

\[
\text{if } \Sigma, A \vdash B, \\
\Sigma, B \vdash A, \\
\text{then } \Sigma \vdash A \leftrightarrow B.
\]

**Equivalence elimination**

(\(\leftrightarrow -\))

\[
\text{if } \Sigma \vdash A, \\
\Sigma \vdash A \leftrightarrow B, \\
\text{then } \Sigma \vdash B.
\]

\[
\text{if } \Sigma \vdash B, \\
\Sigma \vdash A \leftrightarrow B, \\
\text{then } \Sigma \vdash A.
\]
Revisting the Learning Goals

By the end of this lecture, you should be able to

▶ Describe rules of inference for natural deduction.
▶ Prove that a conclusion follows from a set of premises using rules of formal deduction.