Propositional Logic: Formal Deduction

Alice Gao

Lecture 7
Outline

Learning goals

Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Outline

Learning goals

Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Learning goals

By the end of this lecture, you should be able to

- Describe rules of inference for natural deduction.
- Prove that a conclusion follows from a set of premises using rules of formal deduction.
Outline

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Revisiting the Learning Goals
Why study formal deduction?

- Want to prove that a conclusion can be deduced from a set of premises.
- Want to generate a proof that can be checked mechanically.
Formal Deducibility

Let the relation of formal deducibility be denoted by

$$\Sigma \vdash A,$$

which means that $A$ is formally deducible (or provable) from $\Sigma$.

Comments:
- $\Sigma$ is a set of formulas, which are the premises.
- $A$ is a formula, which is the conclusion.
- Formal deducibility is concerned with the syntactic structure of formulas.
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Revisiting the Learning Goals
Rules of Formal Deduction

- Reflexivity (Ref):
  \[ A \vdash A. \]

- Addition of premises (+):
  \[
  \text{if } \Sigma \vdash A, \\
  \text{then } \Sigma, \Sigma' \vdash A.
  \]

- (∈):
  \[
  \text{if } A \in \Sigma, \\
  \text{then } \Sigma \vdash A.
  \]
Conjunction Rules

And introduction ($\land^+$)

If $\Sigma \vdash A$,
$\Sigma \vdash B$,
then $\Sigma \vdash A \land B$.

And elimination ($\land^-$)

If $\Sigma \vdash A \land B$,
then $\Sigma \vdash A$.

If $\Sigma \vdash A \land B$,
then $\Sigma \vdash B$. 
Disjunction Rules

Or introduction ($\lor^+$)

- If $\Sigma \vdash A$,
  - Then $\Sigma \vdash A \lor B$.
- If $\Sigma \vdash B$,
  - Then $\Sigma \vdash A \lor B$.

Or elimination ($\lor^-$)

- If $\Sigma, A \vdash C$,
  - Then $\Sigma, B \vdash C$,
  - Then $\Sigma, A \lor B \vdash C$. 

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Negation Rules

Negation introduction (¬+)  
if \( \Sigma, A \vdash B \),  
\( \Sigma, A \vdash \neg B \),  
then \( \Sigma \vdash \neg A \).

Negation elimination (¬−)  
if \( \Sigma, \neg A \vdash B \),  
\( \Sigma, \neg A \vdash \neg B \),  
then \( \Sigma \vdash A \).
Implication Rules

Implication introduction ($\rightarrow +$)

\[
\text{if } \Sigma, A \vdash B, \\
\text{then } \Sigma \vdash A \rightarrow B.
\]

Implication elimination ($\rightarrow -$)

\[
\text{if } \Sigma \vdash A, \\
\Sigma \vdash A \rightarrow B, \\
\text{then } \Sigma \vdash B.
\]
Equivalence Rules

Equivalence introduction
\( (\leftrightarrow +) \)

- if \( \Sigma, A \vdash B \),
  \( \Sigma, B \vdash A \),
then \( \Sigma \vdash A \leftrightarrow B \).

Equivalence elimination
\( (\leftrightarrow -) \)

- if \( \Sigma \vdash A \),
  \( \Sigma \vdash A \leftrightarrow B \),
then \( \Sigma \vdash B \).
- if \( \Sigma \vdash B \),
  \( \Sigma \vdash A \leftrightarrow B \),
then \( \Sigma \vdash A \).
Revisting the Learning Goals

By the end of this lecture, you should be able to

- Describe rules of inference for natural deduction.
- Prove that a conclusion follows from a set of premises using rules of formal deduction.