Propositional Logic: Formal Deduction

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Lecture 7
Outline

Learning goals

Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Outline

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Motivation for formal deduction

Rules of formal deduction

Revisiting the Learning Goals
Learning goals

By the end of this lecture, you should be able to

▶ Describe rules of inference for natural deduction.
▶ Prove that a conclusion follows from a set of premises using rules of formal deduction.
Outline

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Revisiting the Learning Goals
Why study formal deduction?

- Want to prove that a conclusion can be deduced from a set of premises.
- Want to generate a proof that can be checked mechanically.
Formal Deducibility

Let the relation of formal deducibility be denoted by

\[ \Sigma \vdash A, \]

which means that \( A \) is formally deducible (or provable) from \( \Sigma \).

Comments:
- \( \Sigma \) is a set of formulas, which are the premises.
- \( A \) is a formula, which is the conclusion.
- Formal deducibility is concerned with the syntactic structure of formulas.
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Rules of Formal Deduction

- **Reflexivity (Ref):**
  
  \[ A \vdash A. \]

- **Addition of premises (+):**
  
  if \( \Sigma \vdash A, \)
  
  then \( \Sigma, \Sigma' \vdash A. \)

- **(\in):**
  
  if \( A \in \Sigma, \)
  
  then \( \Sigma \vdash A. \)
Conjunction Rules

And introduction ($\&+$)

if $\Sigma \vdash A$,
$\Sigma \vdash B$,
then $\Sigma \vdash A \& B$.

And elimination ($\&-$)

if $\Sigma \vdash A \& B$,
then $\Sigma \vdash A$.

if $\Sigma \vdash A \& B$,
then $\Sigma \vdash B$. 
Disjunction Rules

Or introduction ($\lor^+$)

if $\Sigma \vdash A$,
then $\Sigma \vdash A \lor B$.

if $\Sigma \vdash B$,
then $\Sigma \vdash A \lor B$.

Or elimination ($\lor^-$)

if $\Sigma, A \vdash C$,
$\Sigma, B \vdash C$,
then $\Sigma, A \lor B \vdash C$. 
Negation Rules

Negation introduction (¬+)  Negation elimination (¬−)

if \( \Sigma, A \vdash B \),
    \( \Sigma, A \vdash \neg B \),
then \( \Sigma \vdash \neg A \).

if \( \Sigma, \neg A \vdash B \),
    \( \Sigma, \neg A \vdash \neg B \),
then \( \Sigma \vdash A \).
Implication Rules

**Implication introduction**

\[(\rightarrow +)\]

if \(\Sigma, A \vdash B\), then \(\Sigma \vdash A \rightarrow B\).

**Implication elimination**

\[(\rightarrow -)\]

if \(\Sigma \vdash A\), \(\Sigma \vdash A \rightarrow B\), then \(\Sigma \vdash B\).
Equivalence Rules

Equivalence introduction
(\leftrightarrow +)

if \Sigma, A \vdash B,
\Sigma, B \vdash A,
then \Sigma \vdash A \leftrightarrow B.

Equivalence elimination
(\leftrightarrow -)

if \Sigma \vdash A,
\Sigma \vdash A \leftrightarrow B,
then \Sigma \vdash B.

if \Sigma \vdash B,
\Sigma \vdash A \leftrightarrow B,
then \Sigma \vdash A.
Revisting the Learning Goals

By the end of this lecture, you should be able to

- Describe rules of inference for natural deduction.
- Prove that a conclusion follows from a set of premises using rules of formal deduction.