Propositional Logic: Formal Deduction

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Lecture 7

Outline

Outline

Learning goals

By the end of this lecture, you should be able to

- Describe rules of inference for natural deduction.
- Prove that a conclusion follows from a set of premises using rules of formal deduction.

Outline

Why study formal deduction?

- Want to prove that a conclusion can be deduced from a set of premises.
- Want to generate a proof that can be checked mechanically.

Formal Deducibility

Let the relation of formal deducibility be denoted by

$$\Sigma \vdash A,$$

which means that A is formally deducible (or provable) from Σ.

Comments:

- Σ is a set of formulas, which are the premises.
- A is a formula, which is the conclusion.
- Formal deducibility is concerned with the syntactic structure of formulas.

Outline

Rules of Formal Deduction

- Reflexivity (Ref):

$$A \vdash A.$$

- Addition of premises (+):

$$\text{if } \Sigma \vdash A,$$
$$\text{then } \Sigma, \Sigma' \vdash A.$$

- ($\in$):

$$\text{if } A \in \Sigma,$$
$$\text{then } \Sigma \vdash A.$$

Conjunction Rules

And introduction ($\wedge+$)

$$\begin{aligned} \text{if } \Sigma &\vdash A, \\ \Sigma &\vdash B, \\ \text{then } \Sigma &\vdash A \wedge B. \end{aligned}$$

And elimination ($\wedge-$)

$$\begin{aligned} \text{if } \Sigma &\vdash A \wedge B, \\ \text{then } \Sigma &\vdash A. \\ \text{if } \Sigma &\vdash A \wedge B, \\ \text{then } \Sigma &\vdash B. \end{aligned}$$

Disjunction Rules

Or introduction ($\vee+$)

$$\text{if } \Sigma \vdash A,$$
$$\text{then } \Sigma \vdash A \vee B.$$
$$\text{if } \Sigma \vdash B,$$
$$\text{then } \Sigma \vdash A \vee B.$$

Or elimination ($\vee-$)

$$\text{if } \Sigma, A \vdash C,$$
$$\Sigma, B \vdash C,$$
$$\text{then } \Sigma, A \vee B \vdash C.$$

Negation Rules

Negation introduction ($\neg+$)

$$\begin{aligned} \text{if } &\Sigma, A \vdash B, \\ &\Sigma, A \vdash \neg B, \\ \text{then } &\Sigma \vdash \neg A. \end{aligned}$$

Negation elimination ($\neg-$)

$$\begin{aligned} \text{if } &\Sigma, \neg A \vdash B, \\ &\Sigma, \neg A \vdash \neg B, \\ \text{then } &\Sigma \vdash A. \end{aligned}$$

Implication Rules

Implication introduction $(\to +)$

$$\text{if } \Sigma, A \vdash B,$$
$$\text{then } \Sigma \vdash A \to B.$$

Implication elimination $(\to -)$

$$\text{if } \Sigma \vdash A,$$
$$\Sigma \vdash A \to B,$$
$$\text{then } \Sigma \vdash B.$$

Equivalence Rules

Equivalence introduction ($\leftrightarrow +$)

$$\begin{aligned} \text{if } & \Sigma, A \vdash B, \\ & \Sigma, B \vdash A, \\ \text{then } & \Sigma \vdash A \leftrightarrow B. \end{aligned}$$

Equivalence elimination ($\leftrightarrow -$)

$$\begin{aligned} \text{if } & \Sigma \vdash A, \\ & \Sigma \vdash A \leftrightarrow B, \\ \text{then } & \Sigma \vdash B. \\ \text{if } & \Sigma \vdash B, \\ & \Sigma \vdash A \leftrightarrow B, \\ \text{then } & \Sigma \vdash A. \end{aligned}$$

Revisting the Learning Goals

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- Prove that a conclusion follows from a set of premises using rules of formal deduction.