Array Assignment

Tue Nov 21

A is an array of n integers $A[1], A[2], \ldots, A[n]$.

$$
\begin{array}{l}
\{\ ???\ \} \\
\quad A[x] = 1; \\
\{\ A[y] = 0\ \}
\end{array}
$$

if $x = y$, ??? should be $1 = 0$.
if $x \neq y$, ??? should be $A[y] = 0$.

When we use variables as indices into arrays, we need to account for multiple cases for many possible values that the variables can take.

Solutions: Write down the sequence of changes first and resolve them when we need to prove any implied conditions.

$$
\begin{array}{l}
\{\ Q[A\{e_1 \leftarrow e_2\}/A]\ \} \\
\quad A[e_1] = e_2; \\
\quad \{\ Q\ \}
\end{array}
$$

array assignment.

For an assignment to an array value $A[e_1] = e_2$, assume that the assignment produced a new array $A\{e_1 \leftarrow e_2\}$.

input: array A $\quad$ index $\quad$ value

output: array $A\{e_1 \leftarrow e_2\}$, which is identical to A except the e_1^{th} element is changed to have the value e_2.

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Prove that the following program satisfies the given triple under partial correctness.

$(\!| (A[x] = x_0) \wedge (A[y] = y_0) |\!)$

$(\!| (A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = y_0)$
$\wedge (A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = x_0) |\!)$ implied

t = A[x];

$(\!| (A\{x \leftarrow A[y]\}\{y \leftarrow t\}[x] = y_0)$
$\wedge (A\{x \leftarrow A[y]\}\{y \leftarrow t\}[y] = x_0) |\!)$ assignment

A[x] = A[y];

$(\!| ((A\{y \leftarrow t\}[x] = y_0) \wedge (A\{y \leftarrow t\}[y] = x_0)) |\!)$ array assignment

A[y] = t;

$(\!| ((A[x] = y_0) \wedge (A[y] = x_0)) |\!)$ array assignment.

To prove the "implied" condition, we need to prove the following:

① $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[x] = A[y]$, and

② $A\{x \leftarrow A[y]\}\{y \leftarrow A[x]\}[y] = A[x]$

Proof of ①: The first assignment "$x \leftarrow A[y]$" assigns $A[y]$ to the x^{th} element of A. Consider 2 cases for y.

(1) If $y \neq x$, then the second assignment does not change the x^{th} element of A. Thus, the x^{th} element of A is $A[y]$ after the assignments.

(2) If $y = x$, the second assignment can be rewritten as $x \leftarrow A[y]$, which is the same as the first assignment. Thus, the x^{th} element of A is $A[y]$ after the assignments.

Proof of ②: Disregard the first assignment. The second assignment assigns $A[x]$ to the y^{th} element of A, and this is the desired result.

- solutions