Theorem: $\{\wedge, \vee\}$ is NOT an adequate set of connectives for propositional logic.

Proof:

To prove that a set of connectives is not adequate, we only need to show that one other connective cannot be expressed using only the connectives in this set. In this case, we will show that $\neg$ is not definable in terms of $\wedge$ and $\vee$.

We prove this by contradiction. Suppose that $\neg$ is definable in terms of $\wedge$ and $\vee$.

Consider $(\neg \mathrm{x})$ where x is a propositional variable. Since $\neg$ is definable in terms of $\wedge$ and $\vee$, we can write $(\neg \mathrm{x})$ as an equivalent formula φ using only x, $\wedge$ and $\vee$. Consider a valuation t where $\mathrm{x}^{\mathrm{t}} = \mathrm{T}$. By the definition of negation, $(\neg \mathrm{x})^{\mathrm{t}} = \mathrm{F}$. We will show that $\varphi^{\mathrm{t}} = \mathrm{T}$ (proved as a lemma below).

($\neg$x) is logically equivalent to φ but they have different values under the same valuation t. This is a contradiction.

It remains to show that for any formula φ containing only the propositional variable x, $\wedge$ and $\vee$, if x is true under a valuation t, then φ is also true under t. We will prove the following lemma, which is stronger than this statement.

Lemma: For any formula φ using any number of propositional variables, $\wedge$ and $\vee$, if every variable is true under a valuation t, then φ is true under t.

QED