

Logical Equivalence Proofs

Sept 21

If it is sunny, I will play golf, provided that I am relaxed

s g r

$$(s \rightarrow (r \rightarrow g))$$

$$(r \rightarrow (s \rightarrow g))$$

$$((s \wedge r) \rightarrow g)$$

Theorem: $(r \rightarrow (s \rightarrow g)) \equiv ((s \wedge r) \rightarrow g)$

Proof: $(r \rightarrow (s \rightarrow g))$

$$\equiv (r \rightarrow ((\neg s) \vee g))$$

Implication

$$\equiv ((\neg r) \vee ((\neg s) \vee g))$$

Implication

$$\equiv (((\neg r) \vee (\neg s)) \vee g)$$

Associativity.

$$\equiv ((\neg(r \wedge s)) \vee g)$$

De Morgan's

$$\equiv ((r \wedge s) \rightarrow g)$$

Implication

$$\equiv ((s \wedge r) \rightarrow g)$$

Commutativity.

QED

Theorem: $(r \rightarrow (s \rightarrow g)) \equiv (s \rightarrow (r \rightarrow g))$

Proof: $(r \rightarrow (s \rightarrow g))$

$$\equiv (r \rightarrow ((\neg s) \vee g))$$

Implication

$$\equiv ((\neg r) \vee ((\neg s) \vee g))$$

Implication

$$\equiv (((\neg r) \vee (\neg s)) \vee g)$$

Associativity.

$$\equiv (((\neg s) \vee (\neg r)) \vee g)$$

Commutativity.

$$\equiv ((\neg s) \vee ((\neg r) \vee g))$$

Associativity.

$$\equiv ((\neg s) \vee (r \rightarrow g))$$

Implication

$$\equiv (s \rightarrow (r \rightarrow g))$$

Implication.

QED