Sept 7, 2017

Translate English sentences with no ambiguity into compound propositions

1. Eleanor is clever but not hard working.

c : Eleanor is clever. h : Eleanor is hard working.

$(c \wedge (\neg h))$

2. Sean will eat an apple or an orange but not both.

a : Sean will eat an apple. o : Sean will eat an orange.

There are many solutions: (It's an exclusive or.)

$((a \vee o) \wedge (\neg (a \wedge o)))$

$\equiv ((a \vee o) \wedge ((\neg a) \vee (\neg o)))$ "$\equiv$" means "is logically equivalent to"

$\equiv ((a \wedge (\neg o)) \vee ((\neg a) \wedge o))$

$\equiv (\neg (a \leftrightarrow o))$

3. If Tom does not study hard, then he will fail. if a then b $\equiv (a \rightarrow b)$

s : Tom studies hard. f : Tom will fail.

$$((\neg s) \rightarrow f) \underset{\text{definition of } \rightarrow}{\equiv} ((\neg (\neg s)) \vee f) \underset{\text{double negation}}{\equiv} (s \vee f)$$

4. Tom will fail unless he studies hard. unless $\equiv$ or

We use the same definitions as 3.

$(f \vee s) \equiv (s \vee f)$ (by commutativity.)

5. Tom will not fail only if he studies hard.

We use the same definitions as 3.

$$((\neg f) \rightarrow s) \underset{\text{definition of } \rightarrow}{\equiv} ((\neg (\neg f)) \vee s) \underset{\text{double negation}}{\equiv} (f \vee s) \underset{\text{commutativity}}{\equiv} (s \vee f)$$

a only if b $\equiv (a \rightarrow b)$

Translating English sentences with logical ambiguity / Sept 12, 2017

1. Pigs can fly and the grass is red or the sky is blue.

p: Pigs can fly. g: the grass is red.
s: the sky is blue.

2 possible translations:
$((p \wedge g) \vee s)$ $(p \wedge (g \vee s))$

These two formulas are not logically equivalent.
Assume that $p \equiv F$, $g \equiv F$, and $s \equiv T$.

$$((p \wedge g) \vee s) \equiv ((p \wedge g) \vee T) \equiv T$$
$$(p \wedge (g \vee s)) \equiv (F \wedge (g \vee s)) \equiv F$$

2. If it is sunny tomorrow, then I will play golf, provided that I do not feel stressed.

s: It is sunny tomorrow
g: I will play golf
f: I feel stressed.

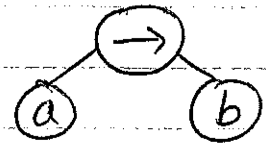
2 possible translations:
$(s \rightarrow ((\neg f) \rightarrow g))$ $((\neg f) \rightarrow (s \rightarrow g))$

These two formulas are logically equivalent, but syntactically different.

Examples of well-formed formulas Sept 12, 2017

(a) $\neg a$ Not well-formed. $(\neg a)$

(b) $(a \to b)$ well-formed.



(c) $(a \wedge b \wedge c)$ Not well-formed.
2 possible fixes: $((a \wedge b) \wedge c)$ or $(a \wedge (b \wedge c))$

These two are logically equivalent.

(d) $(a \to b \to c)$ Not well-formed.
2 possible fixes: $((a \to b) \to c)$ or $(a \to (b \to c))$

These are NOT logically equivalent.

If $a \equiv F$, $b \equiv T$, $c \equiv F$, then
$((a \to b) \to c)$ is false whereas $(a \to (b \to c))$ is true.

(e) $(a \vee b \wedge c)$ Not well-formed.
2 possible fixes: $((a \vee b) \wedge c)$ or $(a \vee (b \wedge c))$

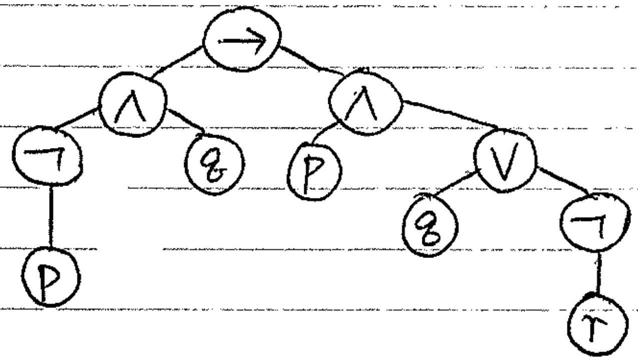
There are NOT logically equivalent.

If $a \equiv T$, $b \equiv F$, $c \equiv F$, then
$((a \vee b) \wedge c)$ is false whereas $(a \vee (b \wedge c))$ is true.

Parse tree of well-formed formulas

Sept 12, 2017

Write the parse tree of $(((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$.



Write the parse tree of $((a \vee b) \wedge (\neg (a \wedge b)))$