Translate English sentences with no ambiguity into compound propositions.

1. Eleanor is clever but not hard working.
   \[ C: \text{Eleanor is clever} \quad h: \text{Eleanor is hard working} \]
   \[ (C \land \neg h) \]

2. Sean will eat an apple or an orange but not both.
   \[ a: \text{Sean will eat an apple} \quad o: \text{Sean will eat an orange} \]
   There are many solutions: (It's an exclusive or.)
   \[
   ((a \lor o) \land \neg (a \land o)) \\
   \equiv ((a \lor o) \land ((a) \lor (o))) \\
   \equiv ((a \land \neg o) \lor ((a) \land o)) \\
   \equiv \neg (a \leftrightarrow o)
   \]

3. (If) Tom does not study hard, then he will fail.
   \[ S: \text{Tom studies hard}. \quad f: \text{Tom will fail}. \]
   \[
   (\neg S \rightarrow f) \equiv ((\neg (\neg S)) \lor f) \equiv (S \lor f) \\
   \text{definition of } \rightarrow \quad \text{double negation.}
   \]

4. Tom will fail unless he studies hard.
   We use the same definitions as 3.
   \[ (f \lor S) \equiv (S \lor f) \quad \text{(by commutativity.)} \]

5. Tom will not fail only if he studies hard.
   We use the same definitions as 3.
   \[
   (\neg f \rightarrow S) \equiv ((\neg (\neg f)) \lor S) \equiv (f \lor S) \equiv (S \lor f) \\
   \text{definition of } \rightarrow \quad \text{double negation} \quad \text{commutativity.}
   \]
   \[ a \text{ only if } b \equiv (a \rightarrow b) \]
1. Pigs can fly and the grass is red or the sky is blue.

\[ p: \text{Pigs can fly} \quad g: \text{the grass is red} \quad s: \text{the sky is blue} \]

2 possible translations:

\[ ((p \land g) \lor s) \quad (p \land (g \lor s)) \]

These two formulas are not logically equivalent.

Assume that \( p \equiv F \), \( g \equiv F \), and \( s \equiv T \).

\[ ((p \land g) \lor s) \equiv ((p \land g) \lor T) \equiv T \]

\[ (p \land (g \lor s)) \equiv (F \land (g \lor s)) \equiv F \]

2. If it is sunny tomorrow, then I will play golf, provided that I do not feel stressed.

\[ s: \text{It is sunny tomorrow} \]

\[ g: \text{I will play golf} \]

\[ f: \text{I feel stressed} \]

2 possible translations:

\[ (s \rightarrow ((\neg f) \rightarrow g)) \quad ((\neg f) \rightarrow (s \rightarrow g)) \]

These two formulas are logically equivalent, but syntactically different.

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Translating English sentences with logical ambiguity / Sept 12, 2017
Examples of well-formed formulas

(a) \neg a \quad \text{Not well-formed.} \quad (\neg a)

(b) \, (a \to b) \quad \text{well-formed}.

(c) \, (a \land b \land c) \quad \text{Not well-formed.}

\quad \text{2 possible fixes:} \quad (a \land b) \land c \quad \text{or} \quad (a \land (b \land c))

\quad \text{These two are logically equivalent.}

(d) \, (a \to b \to c) \quad \text{Not well-formed}

\quad \text{2 possible fixes:} \quad (a \to b) \to c \quad \text{or} \quad (a \to (b \to c))

\quad \text{These are NOT logically equivalent.}

If \ a = \text{F}, \ b = \text{T}, \ c = \text{F}, \ \text{then}

\quad (a \to b) \to c \quad \text{is false whereas} \quad (a \to (b \to c)) \quad \text{is true.}

(e) \, (a \lor b \land c) \quad \text{Not well-formed}

\quad \text{2 possible fixes:} \quad (a \lor b) \land c \quad \text{or} \quad (a \lor (b \land c))

\quad \text{There are NOT logically equivalent.}

If \ a = \text{T}, \ b = \text{F}, \ c = \text{E}, \ \text{then}

\quad (a \lor b) \land c \quad \text{is false whereas} \quad (a \lor (b \land c)) \quad \text{is true.
Parse tree of well-formed formulas

Write the parse tree of \(((\neg P) \land Q) \rightarrow (P \land (Q \lor \neg R)))\).

Write the parse tree of \(((a \lor b) \land (\neg (a \land b)))\).