\[ \alpha \overset{\text{def}}{=} (P(x,y,z) \land (\forall x (\exists y R(x,y,z)))) \]
\[ t \overset{\text{def}}{=} f(x,y) \]

State \( \alpha [t/z] \)
(Replace every free occurrence of \( z \) in \( \alpha \) by the term \( t \) without changing the meaning of \( \alpha \))

Step 1: Find free occurrences of \( z \) in \( \alpha \).

We need to replace both occurrences of \( z \) in \( \alpha \) by \( t \).

Step 2: Perform the substitution.

(Draw the tree for step 3)
Careful Substitution to Avoid Capture  2/3

Step 3: Did any variable in \( t \) get captured by a quantifier in \( \alpha \)?

- These quantifiers captured some variables in \( t \).
- \( x \) got captured by the quantifier \( \forall x \).
- \( y \) got captured by \( \exists y \).
- \( y \) got captured by \( \exists y \).
- Bound! Bound!

Step 4: Resolve capture by renaming variables in \( \alpha \).

In \( \alpha \), for each quantifier \( \forall \) \( v \) that captured a variable \( v \) in \( t \),
(a) Select a variable \( v' \) that is in neither \( \alpha \) nor \( t \).
(b) Replace \( v \) by \( v' \) beside the quantifier \( \forall \) and in the scope of the quantifier \( \forall \).

Let's call this new formula \( \alpha' \).
Step 5: Perform the substitution with $\alpha'$.

\[ \alpha' \overset{\text{def}}{=} (P(x, y, z) \land (\forall u \exists w R(u, w, z)))) \]
\[ t \overset{\text{def}}{=} f(x, y) \]
\[ \alpha'[t/z] = (P(x, y, f(x, y)) \land (\forall u \exists w R(u, w, f(x, y)))) \]

The parse tree after the substitution.

Exercise: $\beta = (\forall x (\exists y ((x+y) = z)))$
\[ \beta[y-1/z] = \beta'[y-1/z] \overset{\text{def}}{=} (\forall x (\exists w ((x+w) = (y-1)))) \]