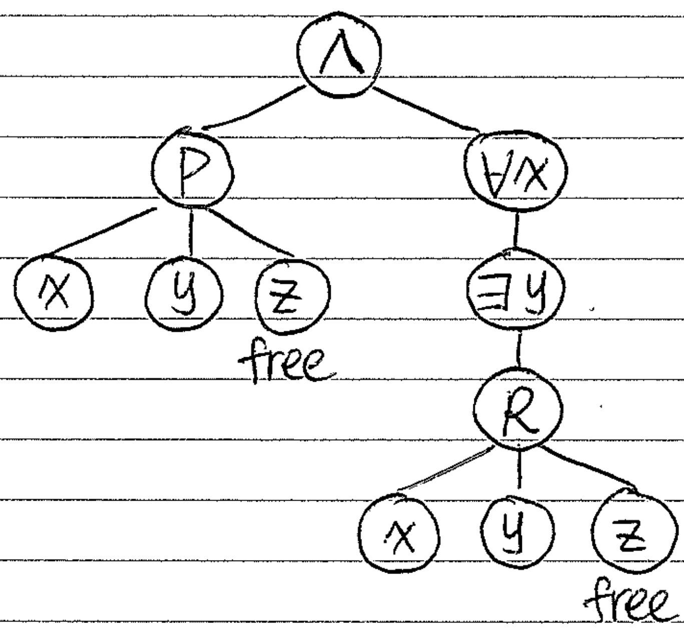
$\alpha \stackrel{\text{def}}{=} (P(x,y,z) \wedge (\forall x (\exists y\, R(x,y,z))))$
$t \stackrel{\text{def}}{=} f(x,y)$
State $\alpha[t/z]$
(Replace every free occurrence of z in α by the term t without changing the meaning of α.)

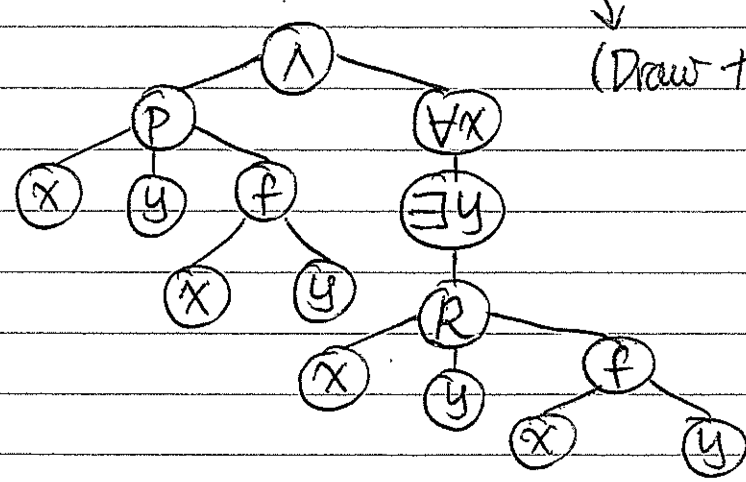
Step 1: Find free occurrences of z in α.



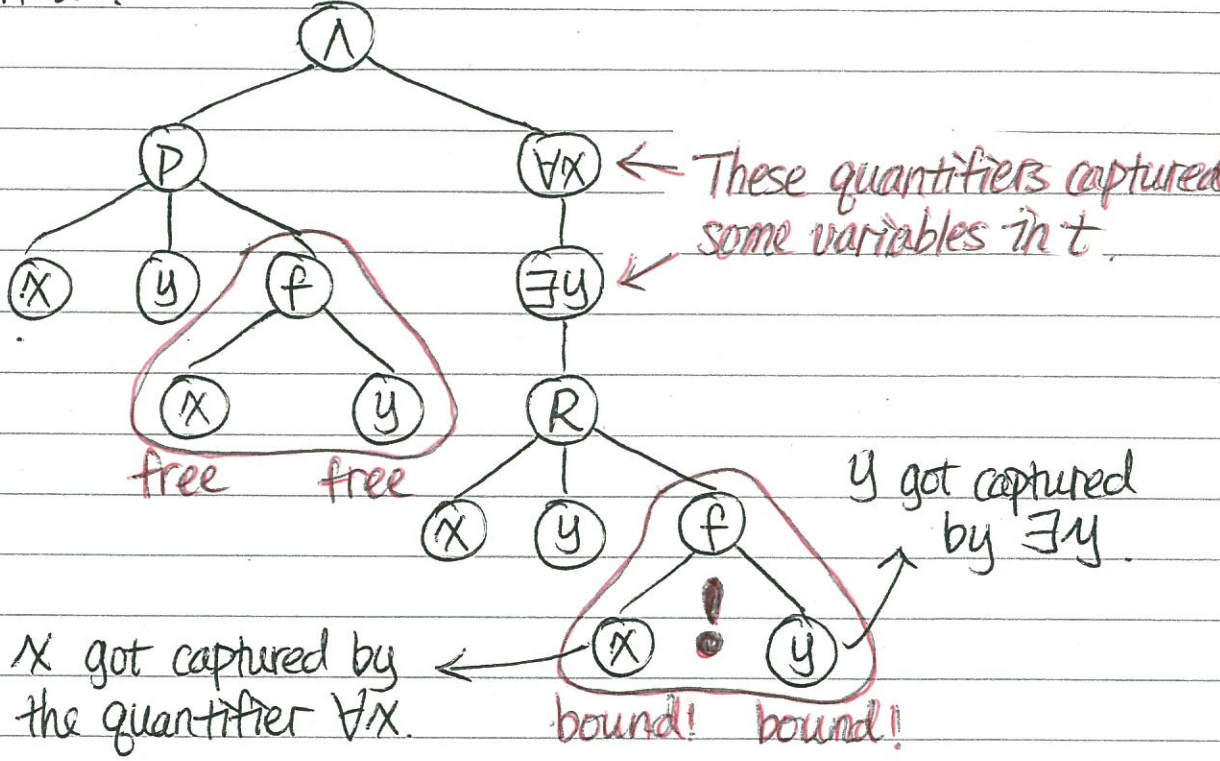
We need to replace both occurrences of z in α by t.

Step 2: Perform the substitution.

(Draw the tree for step 3)



Step 3: Did any variable in t get captured by a quantifier in α?

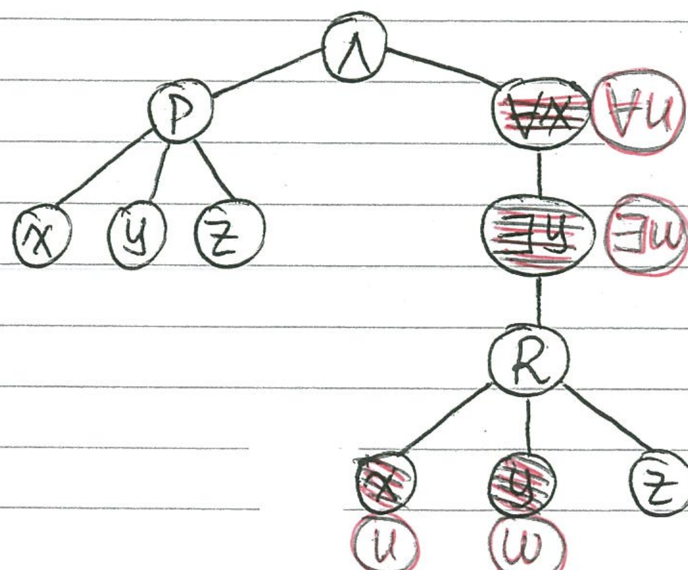


Step 4: Resolve capture by renaming variables in α.

In α, for each quantifier Qv that captured a variable v in t,

(a) Select a variable v' that is in neither α nor t.

(b) Replace v by v' beside the quantifier Q and in the scope of the quantifier Q.



Let's call this new formula α'.

Careful Substitution to Avoid Capture 3/3 Oct 19

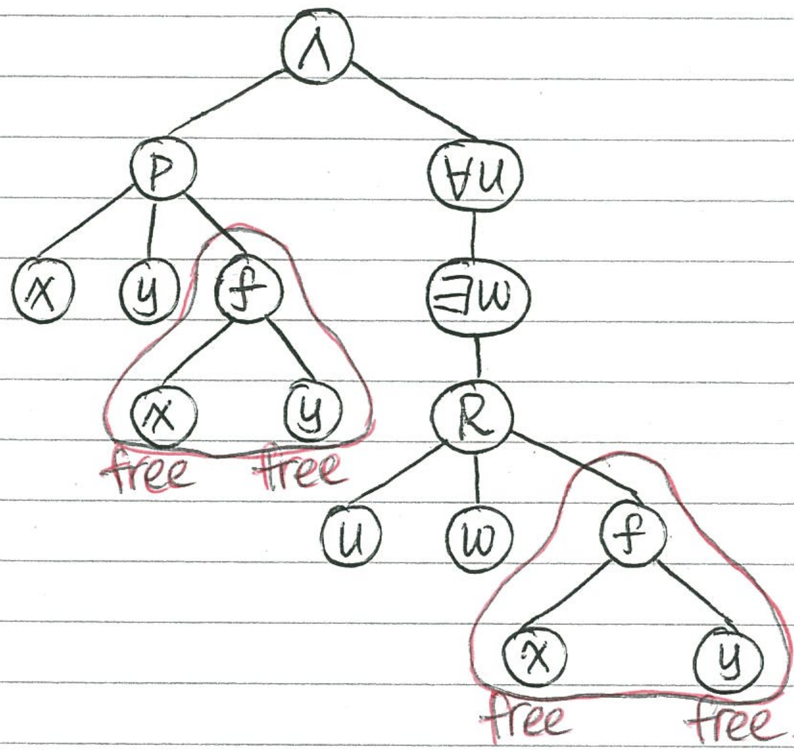
Step 5: Perform the substitution with α'.

$$\alpha' \stackrel{def}{=\!=} (P(x,y,z) \wedge (\forall u \cdot (\exists w\, R(u,w,z))))$$

$$t \stackrel{def}{=\!=} f(x,y)$$

$$\alpha'[t/z] == (P(x,y,f(x,y)) \wedge (\forall u (\exists w\, R(u,w,f(x,y)))))$$

The parse tree after the substitution.



Exercise : $\beta = (\forall x (\exists y ((x+y) = z)))$ $\quad\quad$ $\beta[(y-1)/z]$

$$\beta' = (\forall x (\exists w ((x+w) = z)))$$

$$\beta[(y-1)/z] = (\forall x (\exists w ((x+w) = (y-1))))$$