

Structural Induction

Nov 1.

Theorem: A well-formed predicate formula φ has ^{the} property P .

Proof by structural induction:

Base case: Prove that a predicate $P(t_1, \dots, t_n)$ has the property P .

Induction step:

Case 1: φ is $(\neg\alpha)$ where α is a WFPF.

Induction hypothesis: assume that α has property P .

We need to show that $(\neg\alpha)$ has property P .

Case 2: φ is $(\alpha \star \beta)$ where α and β are WFPFs and \star is a binary connective.

Ind. Hyp.: assume that α and β have property P .

We need to show that $(\alpha \star \beta)$ has property P .

Case 3: φ is $(Qx\alpha)$ where Q is one of \forall and \exists , and α is a WFPF.

Ind. Hyp.: assume that α has property P .

We need to show that $(Qx\alpha)$ has property P .

By the principle of structural induction, every WFPF φ has property P .

QED.