Evaluating a predicate formula.

Oct 24

Q: Consider the formula $\alpha = (\exists x (\forall y\ R(x, y, h(z, c))))$.

(c is a constant symbol. x, y, and z are variable symbols.
$h^{(2)}$ is a function symbol. $R^{(3)}$ is a predicate symbols.

① Give an interpretation I_1 and an environment E_1 such that $I_1 \models_{E_1} \alpha$.

I_1: $\text{dom}(I_1) = \{1, 2\}$ $\quad c^{I_1} = 2$.
h^I: $h(x, y) = y \ \forall x, y \in \text{dom}(I)$.
$R^I = \{\langle 1, 1, 2\rangle, \langle 1, 2, 2\rangle\}$.

E_1: $E_1(x) = 1 \quad E_1(y) = 1 \quad E_1(z) = 2$.

I will describe two ways of justifying our choices of I_1 and E_1.

Justification ① (similar to tutorial 5).

Under I_1 and E_1, α becomes.

there exists $x \in \{1, 2\}$ such that for all $y \in \{1, 2\}$,

$R(x, y, h(z, c))$ is true.

Note that $h(z, c)$ is $h(2, 2) = 2$ by I_1 and E_1.

When $x \mapsto 1$, if $y \mapsto 1$, $R(1, 1, 2)$ is true.
if $y \mapsto 2$, $R(1, 2, 2)$ is true.
This shows that $I_1 \models_{E_1} \alpha$ holds.

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Justification ②

To show that $\alpha^{(I_1, E_1)} = T$, it is sufficient to show that

$$(\forall y\ R(x, y, h(z, c)))^{(I_1, E_1[x \mapsto 1])} = T.$$

Under I_1 and E_1, $h(z,c)^{(I_1, E_1)} = h(2,2)^{(I_1, E_1)} = 2$.

Consider all possible values for y.

case 1: $R(x, y, h(z,c))^{(I_1, E_1[x \mapsto 1][y \mapsto 1])} = T$ because

$$\langle E_1[x \mapsto 1][y \mapsto 1](x),\ E_1[x \mapsto 1][y \mapsto 1](y),\ h(z,c)^{(I_1, E_1[x \mapsto 1][y \mapsto 1])} \rangle$$
$$= \langle 1, 1, 2 \rangle \in R^I.$$

case 2: $R(x, y, h(z,c))^{(I_1, E_1[x \mapsto 1][y \mapsto 2])} = T$ because.

$$\langle E_1[x \mapsto 1][y \mapsto 2](x),\ E[x \mapsto 1][y \mapsto 2](y),\ h(z,c)^{(I_1, E_1[x \mapsto 1][y \mapsto 2])} \rangle$$
$$= \langle 1, 2, 2 \rangle \in R^I.$$

Therefore, $I_1 \vDash_{E_1} \alpha$.

Both justifications are correct. Please choose one that you are more comfortable with. However, you should understand both justifications.