The Environment Re-assignment Notation:

Oct 24

Variable symbols: x, y Predicate symbols: $P^{(2)}$

I: $dom(I) = \{a, b\}$ $P^I = \{\langle a,a\rangle, \langle a,b\rangle, \langle b,b\rangle\}$
E: $E(x) = a$, $E(y) = b$

$E[y \mapsto a]$ produces an environment that is different from E.

$E[y \mapsto a]$ preserves all the mappings in E EXCEPT that it re-assigns y to a.

$E[y \mapsto a]$ maps x to a, same as E. but it maps y to a.

Examples:

$E[x \mapsto b](x) = b$ $E[x \mapsto b](y) = b$
$E[x \mapsto b][y \mapsto a](x) = b$ $E[x \mapsto b][y \mapsto a](y) = a$.

$\langle E[y \mapsto a](y), E[y \mapsto a](x)\rangle = \langle a, a\rangle$.

$P(x, y)^{(I, E[y \mapsto a])}$ means the value of $P(x, y)$ under I and $E[y \mapsto a]$

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E: $E(x) = a$ $E(y) = b$

Q1: What is $\alpha = (\exists x\ P(x,y))^{(I,E)}$?

- y is a free variable and has the value b by def of E.
- To make α true, we need to find one value for x to make $P(x,y)$ true. "b" is such a value.

Formally, $\langle E[x \mapsto b](x), E[x \mapsto b](y)\rangle = \langle b, b\rangle \in P^I$

Thus, $P(x,y)^{(I,E[x\mapsto b])} = T$ and $(\exists x\ P(x,y))^{(I,E)} = T$

Q2: What is $\beta = (\forall x\ P(x,y))^{(I,E)}$?

- y is free and takes the value b by def. of E.
- To make β true, we need to verify that $P(x,y)$ is true for every possible value of x.

Formally,

$$\langle E[x\mapsto a](x), E[x\mapsto a](y)\rangle = \langle a, b\rangle \notin P^I.$$

so $P(x,y)^{(I,E[x\mapsto a])} = F$

We found one value of x for which $P(x,y)$ is false, so β must be false. $(\forall x\ P(x,y))^{(I,E)} = F$.