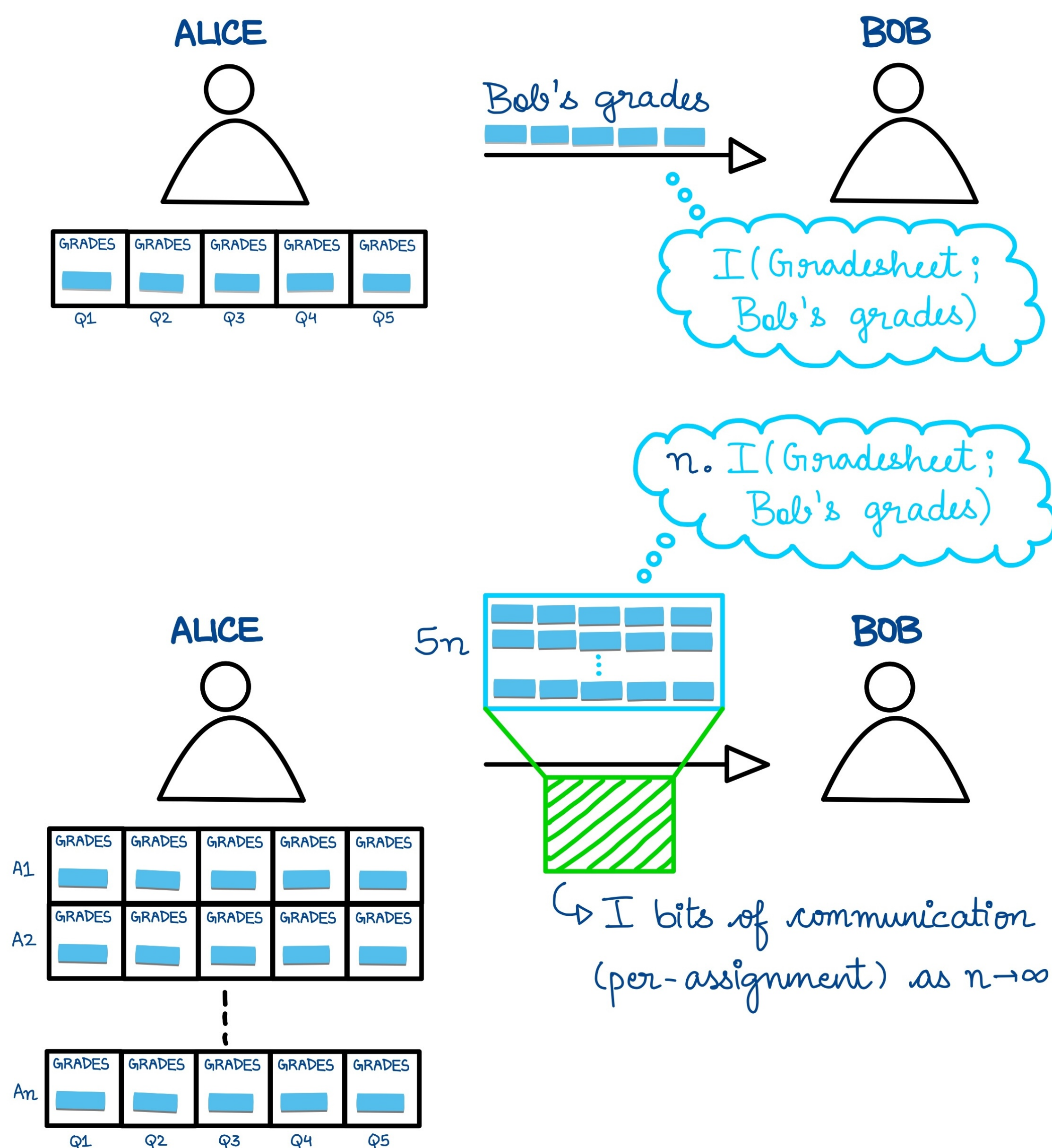


The Information Complexity of Decision Trees

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MUTUAL INFORMATION

$$I(X; Y) = H(X) - H(X|Y) \\ = H(Y) - H(Y|X)$$

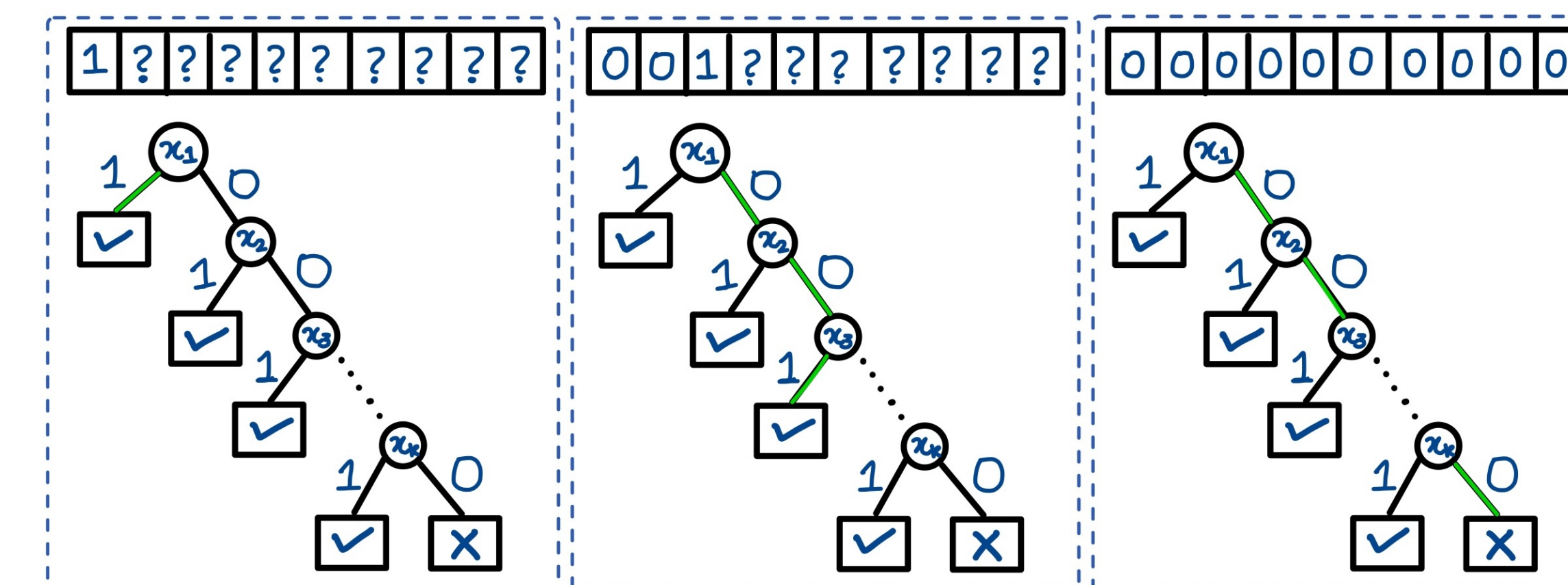
Amount of information
Y reveals about X
= Amount of information
X reveals about Y

Ques. What can information theory
tell us about decision trees?

Ans. Close relationship to tree-size
Insights into compression of size
of decision trees.

REFERENCES

- [TC05] Elements of Information Theory
- [BR11] Information Equals Amortized Communication
- [BB25] Direct-Product Theorems for Randomized Query Complexity
- [ABB26] The Information Complexity of Decision Trees



Depth = $\Theta(k)$

Information = $O(\log k)$

$\log(\text{Size}) = O(\log k)$

Information $\ll$ Depth

Information^u(T)
||
I(hidden input;
path on tree)

$IC_T^u(f) = \min.$
expected info. of T
achieving success γ

