Internalization on the Toronto Stock Exchange ⋆

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Abstract

This is a study of the impact of internalization on the Toronto Stock Exchange (TSX). We present an abstract model of the internalization process and discuss some observations and conclusions that can be drawn from the model, including results from a simulation study. We describe a system that we built in order to use actual order data from the Toronto Stock Exchange, and report our findings and experiences from working with this data.

1 Introduction

The Toronto Stock Exchange (TSX) is a fully electronic, limit-order, time-priority based trading system and is the primary Canadian exchange for senior stocks. [1] It is the world's largest source of equity financing for the mining industry, and is Canada's largest exchange [1]. The TSX has been fully computerized since 1997 and has led the way in incorporating new technologies and features into trading systems.

Like many other exchanges, the TSX uses a *central limit order book*. A limit buy or sell order specifies both the number of shares and the desired price. A trade will only be executed if there is a matching party on the opposing side, as defined by the matching procedure, and otherwise the order is added to the orderbook, which lists all limit orders awaiting possible future execution. Most exchanges use similar matching procedures. In general, for an order to be executed immediately it needs to *cross the spread*. That is, any bid to buy must be at a price equal to or greater than the lowest current listed sell price

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1 Junior stocks are traded on the TSX Venture Exchange.

Broker	Quantity	Price	Price	Quantity	Broker
85	300	15.45	15.50	500	7
2	1700	15.45	15.50	800	2
7	800	15.45	15.52	1500	79
11	1300	15.40	15.55	1200	9

Table 1
An example orderbook. On the left are the Bids and on the right are the Offers.

Broker	Quantity	Price	Price	Quantity	Broker
2	1500	15.45	15.50	500	7
7	800	15.45	15.50	800	2
11	1300	15.40	15.52	1500	79
			15.55	1200	9

Table 2
The orderbook after a trade has been executed. On the left are the Bids and on the right are the Offers.

on the book, and any offer to sell must be equal to or less than the highest current listed buy price listed on the book. If an incoming order crosses the spread then it is matched with an order on the books immediately, taking into account price, and the time an order has been on the books.

Example 1 *Table 1 provides an example of an orderbook. Broker 85 wants 300 shares and is willing to pay $15.45 for each share. On the other side of the book, broker 7 has 500 shares that it is willing to sell for $15.50 each. The orders are listed in order of price, and if there are multiple orders at the same price, then orders are listed by their arrival time. That is, older orders are listed first. If a new order arrives, looking to sell 500 shares at $15.45 per share, then a trade would be executed. The order at the top of the bid list (i.e. broker 85's order) would be filled first, and then the remaining 200 shares would be sold to the second broker on the list (broker 2). After this transaction, the orderbook is updated as shown in Table 2.*

While limit orderbooks have existed since the start of many exchanges, it has only been recently that these books have been made visible to all brokers in real time. As such, brokers have a great deal of information, and can speculate about the actions of other brokers based on their orders in the books. This also implies that brokers have to be wary when placing orders, as doing so may divulge information that the broker may wish to keep private.

The TSX is fairly unique in affording dealers the choice of advertising their identity when they place in order. In accordance with Canadian regulations,

Broker	Quantity	Price	Price	Quantity	Broker
85	300	15.45	15.50	500	7
2	1700	15.45	15.50	800	2
7	800	15.45	15.52	1500	79
11	1300	15.40	15.55	1200	9

Table 3
An example orderbook. On the left are the Bids and on the right are the Offers.

the choice of whether to attribute an order to a particular dealer must remain with the dealer.[2] Orders traded anonymously are marked with the placeholder identifier "01". While regulators such as Market Regulation Services Inc. and the TSX manage the identities and owners of the anonymous trades, the market itself never knows the actual identity of the brokers using the identifier "01". This has interesting repercussions for information trading; while many models of trader activity account for a trader's unwillingness to divulge information, the anonymous trading system easily allows traders to alternately trade publicly or anonymously.

Another aspect of the TSX that is unique is its *internalization mechanism* for matching orders. In general, the mechanism for matching orders follows the procedure that was described earlier. However, if there are multiple orders at the same price, then priority is given, not by the time an order was entered, but whether the orders' broker numbers are the same. Orders with matching broker numbers are executed first, and only then is time priority used. If a broker uses the placeholder identifier "01", then the internalization mechanism is never used. As before, we provide an example to illustrate the process.

Example 2 *Assume that the orderbook is shown in Figure 3.*

If a new order arrives, offering to sell 500 shares at $15.45 and has broker number 7, then these 500 shares would be sold to broker 7 on the bids side of the book, skipping over brokers 85 and 2, even though their orders have been on the book longer than broker 7's. The book would now look like the one in Figure 4.

The goal of this project is to study internalization on the TSX in order to gain an understanding of its benefits and disadvantages. While the TSX has implemented internalization presumably to lower transaction costs for certain trades and encourage crosses, the effect of internalization on the market has yet to be studied. To this end we have designed and developed the first simulator capable of analyzing compiled data and generating statistics based on this data. While basic statistics such as volumes traded and average prices are

[2] This is since 2001.

Broker	Quantity	Price	Price	Quantity	Broker
85	300	15.45	15.50	500	7
2	1700	15.45	15.50	800	2
7	300	15.45	15.52	1500	79
11	1300	15.40	15.55	1200	9

Table 4
The orderbook after internalization has take place. On the left are the Bids and on the right are the Offers.

regularly published on many financial web sites, more complex questions such as the average time an order sits on the books or quantifications of the depths of the orderbooks can now be addressed. Our simulator also allows us to *modify* the rules of the exchange and then observe the impact that the modification has on brokers. In particular, with our simulator we are able to turn on and off the internalization mechanism in order to study its impact. We have also developed an abstract model of internalization, which allows us to identify and isolate different factors related to internalization. We are able to conduct a set of simulations, isolating different parameters in order to understand internalization impact trends.

In this report we describe out simulator, model, and findings from our study. In the next section (Section 2) we discuss previous research which is relevant to the study of limit-order trading and internalization. We then describe the simulator we developed and present an analysis of the statistics we were able to extract from TSX trading data (Section 3). In Section 3.3 we describe a series of experiments we conducted where we *turned on and off* the internalization mechanism in order to determine what types of brokers benefit from internalization. We then present an abstract model of internalization and include simulation results which allow us to isolate parameters of interest in the system (Section 4). In Section 5 we conclude with a summary of our findings.

2 Related Research

Both the economics and computer science communities have investigated stock trading systems. There have been significant developments in analyzing stock markets from a computational perspective, using simulators that use rational agents to model the actions of brokers in a market. Raberto *et al.* present a multi-agent based simulation of an artificial financial market [7]. Agents are given a finite starting amount of cash and equity, and for some number of discrete time steps they execute a series of buy and sells. The generation of orders is probabilistic, but several factors can affect what orders are placed (buys vs. sells) as well as how they are placed (pricing). Through manipulation of these parameters, the authors create a model that replicates the volatility clustering and trading distributions that are often noticed when analyzing real data. The authors also allow for the manipulation of several other variables in their model, including the number of traders, length of time allowed for trades, probability of making a buy or sell order, and so on. Their final system is an effective yet analyzable multi-agent based system which can be modified to offer insight into the activities of traders.

Another simulator that has been developed is the Penn-Lehman Automated Trading Project, developed jointly by Lehman Brothers and the University of Pennsylvania [5]. Their project uses information from the NASDAQ as the basis for their simulator. The NASDAQ is a limit order based system, but consists of several markets where trades can occur, complicating price-setting. Their simulator additionally consists of an API which allows researchers to easily insert agents into the system, so that testing of agents' algorithms can take place. The primary focus of the work is on agent-trading strategies and algorithms rather than market microstructure, and as such do not describe results on the microstructure of the NASDAQ.

Kakade *et al.* examine strategies for moving large volumes effectively over the course of a day [4]. Comparisons are made to the One-Way Trading problem as well as to simplified systems based on sequences of executed trades rather than buy and sell orders. However, the most interesting results come analysis of limit orderbooks. The authors seek to add a series of buy or sell orders into the existing stream of orders so as to generate a revenue competitive with the VWAP (Volume Weighted Average Price). Their algorithm initially guesses some conservative price, p, and places a limit order for all the shares at that price at the beginning of the day. The authors then show that no algorithm is competitively better then an algorithm that tries to track the VWAP. They demonstrate how errors made in guessing the VWAP incur only algorithmic losses, which are insignificant compared to the benefits of approximating the VWAP.

Theoretical frameworks for evaluating trading strategies have only limited applications on the trading floor. One never has all the information required to make the best decision, and a stream of orders may deviate from the expected stream significantly. Whether or not a trader is using the orderbook to affect their decision will affect the amount of this variance. Understanding the affects of variations in a stream of orders can yield fruitful information. Even-dar *et al.* examine how small variations in the sequence of orders that arrive during the course of a day vary based on whether people place their bids independently of orderbook state or dependent on the orderbook state [2]. When brokers place orders independently of orderbook state, these variations cause only small fluctuations in the result of the day's activities. Even-dar *et al.* also present examples to demonstrate how, if all brokers depend solely on the orderbook to determine how to trade, then a small fluctuation early in the day can cause arbitrarily large fluctuations in the above mentioned variables.

In modern markets, traders make judgments and decisions based both on the information from the orderbook, as well as activities that occur off the book (changes in industry, press announcements, etc) which would indicate that traders to not neatly fit into either category. Even-dar's simulations show that having even a small number of traders who ignore the orderbook offer great stability to the situation: If as few as 10% of the traders ignore the orderbook, this still offers 90% of the stability.

While we approach the TSX using computational tools, an understanding of the economics behind trading is also valuable. Nevmyvaka *et al.* present a methodology for assessing various trading strategies for orderbook based systems [6]. Their system considers not only execution time, order size, order price and time window (as are frequently considered in other evaluations) but also accounts for various market conditions such as relative daily volume and orderbook depth. They also account for the risk involved with placing a limit order, *e.g.* that your trade may not execute within a given time limit, resulting in a market order trade at a higher price. By combining issues of risk and the expected execution price, they create what they call an *efficient pricing frontier* which describes relationships between risk and price under varying market conditions.

Nevmyvaka's analysis is based on trying to minimize execution costs, calculated as the difference between the mid-point of the bid-ask spread at the time the order is placed, and the price at which an order executes. Their analysis examines three possible trading strategies:

(1) execute immediately as a market order,
(2) execute as a market order later, or
(3) place a limit order now and execute any untraded shares at the end of some time limit as a market order.

Their analysis provides guidelines on minimizing execution costs while placing limit orders.

One area of great interest is the determination of prices. As discussed by Even-dar *et al.*, prices can be derived from the order book, accounting for the behaviors of other, or from information available about the company. Flemina and Remolona thoroughly examine price formation in the treasury market in response to public information [3]. The treasury market is unique in that public information plays a much greater role than private information does, allowing a greater examination of the new information's impact on liquidity. The minimal impact of market makers and the high volumes of trading combine to make this market an excellent specimen for study. They note two stages of changes in the trading dynamics after the release of information. The first is a sudden price shift, accompanied by an increase in the bid-ask spread, as well as a drop on trading volumes. The second is a prolonged stage of increased trading volume, volatile prices and a moderately large bid-ask spread.

As noted earlier, traders have the option to register a trade as being anonymous rather than placing it under their institution's identification number. Doing so within the TSX provides the benefit that the trader does not leak information they might not want to, but the trader also loses out on potential internalization. A thorough analysis of anonymous trading is done by Reiss and Werner, focusing on the London Stock Exchange [9]. Trading on the London exchange can take place in a variety of markets, some anonymous and others not, giving traders the opportunity to trade anonymously if they so choose. Non-anonymous trades require the broker to additionally declare if the trade is for the brokerage or on behalf of a customer.

Anonymous trades in the London exchange have two additional factors to consider, both of which revolve around the fact that the main trading floor has market makers and the third-party exchanges do not. Due to the lack of market makers, both the buyer and seller can execute the trade cheaper than otherwise. However, the lack of a market maker also reduces the level of liquidity outside the main exchange. If a trader wishes to be more certain of a fast and efficient trade, they must give up the price improvements available outside of the main trading floor. The end result is the rather counter-intuitive observation that uninformed trades tend to take place anonymously, whereas trades based on private information tend to occur non-anonymously. Whether the same dynamics would occur on the TSX would be worthy of investigation.

Ranaldo examines orderbooks of the Swiss Stock Exchange where there are no market makers or specialists [8]. Specifically, Ranaldo examines the factors that influence the decision-making processes of traders. He sampled fifteen highly liquid stocks over a period of two months and attempted to quantify the level of aggressiveness of both buy and sell orders and both market and limit orders, to see how different order types vary based on the depth of the book, size of spread, and volatility. His results show, among other things, that wider spreads induce increased limit order trading compared to market trading, that market orders and limit orders react in opposite ways to changes in the market, and that thickness on one side of an orderbook will encourage trading on that side of the book. He also noted several asymmetries on the two sides of the book, such as how buy orders are placed with greater awareness of the order book, in particular the depth of the opposite side of the orderbook, than are sell orders. He challenges several assumptions made elsewhere, such as that market orders can be characterized as limit orders and that buy and sell orders show complete symmetry.

One great difficulty in examining stock market dynamics is that the dynamics change over the course of the day. Trading during the first few minutes after a stock exchange opens is known to have different dynamics than trading at midday. Stoll and Whaley examine the opening of the stock market on the NYSE over several years' worth of data to examine volatility, pricing, timing and volume during the open of the markets, as well as the effect of the specialist on the market opening process [10]. The specialist, when present, determines the clearing price. The clearing price is the price at which all orders at the market open trade. This value is selected as the specialist desires, which may not be the optimal clearing price. On the TSX, it should be noted that the clearing price is picked via a Walrasian auction, which maximizes the volume of stock that can be traded at the open. The opening price should be selected so as to minimize volatility that may occur after the market has opened. The authors further examine the sources of volatility at the market open. Factors such as awareness of booked orders and the heightened power of the specialist at this time can increase volatility, and mitigating factors such as numbers of other traders or regulation of the specialist are required to reduce this volatility. It is also interesting to note that the gap between market open and the time of first transaction is significant, although it has decreased over the years. Currently, on the TSX, there is no significant delay between the market open and the time of the first transaction. The volume at the market open has also increased year over year, and is greater for high-value stocks which would have more overnight orders booked than low-value stocks.

3 Studying Internalization on the TSX

In this section we describe our system we designed to analyze data from the TSX, as well as our findings from this data. We start with a description of the simulator we developed for this problem (Section 3.1). We then discuss the statistics on internalization that we were able to extract using our simulator system (Section 3.2), followed by our attempt to quantify the benefits of internalization through a series of experiments (Section 3.3).

3.1 The Simulator

Our simulator of the TSX works by taking in sequences of orders and recreating the orderbooks and transactions. As processing occurs, various statistics can be recorded about the TSX, the behaviour of individual brokers, and the market as a whole can be analyzed. It is possible to use the collected statistics to characterize brokers in terms of aggressiveness of trading and volume of trading. It is also possible to categorize different equities by average price and volume traded.

Orders for a given equity are processed individually, under the assumption that actions for one stock will not affect others. Each record of data represents one of the following order types:

- a recorded buy order,
- a recorded sell order,
- a change to a buy order,
- a change to a sell order,
- a cancelled buy order,
- a cancelled sell order, or
- a recorded trade between two brokers.

A cancel order takes the order off the book. A change order might change the price or volume of the order. Change orders generally re-set the time priority of the order, unless the change was solely to decrease the volume of the trade. The change orders are generally preceded in the data by cancel orders. It should also be noted that a single buy order may be recorded in the data as separate orders if part (or all) of the order traded against the book before it was recorded. As an example, say a sell order for 1000 shares is placed. If 500 shares can trade with one broker, 300 shares with a second broker, and the last 200 shares must be written in the book, then the single 1000 share order will be recorded as spread over three separate records.

The data we were provided was not guaranteed to be in order. Time stamps are rounded off to the nearest second, which is insufficient to determine the order of executions. As such, orders for an entire second are gathered up before processing. This gathering permits analysis to determine which orders are independent, and which are separate. This allows aggregation of related cancel and buys requests, as well as related trades and buy/sell orders. This way, we recreate the sequences of buys and sell that were placed by brokers, rather than the sequence of orders that was placed on the book.

A model of the orderbook determines when trades are to occur. Orders are placed on the book as buys and sells, and are converted into trades when an order across the book matches the stated price. It should be noted that orders are only placed on the book as buys or sells. Even though the data lists trades, and the orders get executed as trades, it is important to convert the data into a stream of only buys and sells, to allow for future modifications of the market microstructure and design of the market. Orders are matched and traded using internalization. That is, when an order crosses the book (and can be traded), the order will match first against orders that are from the same broker, even if that broker does not have the highest time priority. Only after the matching price priority is clear of any orders from the same broker to transactions occur based on time priority. Note that the role of the potential market maker is not taken into account here.

Each price point is a recursive data structure that holds orders for a particular price priority. The orderbook thus refers solely to the bucket of buy orders with the highest price and the bucket of sell orders with the lowest price. All orders, be they removals (in the case of a cancel or trade), insertions (when an order is recorded), or modifications (in the case of a change order) are all done recursively.

A separate statistics object is responsible for gathering information about the processing. The orderbook channels information, as appropriate, to the statistics object. This object accumulates and manages all data, so that it can be displayed during processing and formatted for file output after processing is complete. A variety of statistics are recorded including overall statistics for the stock, such as number of buys, sell, changes to buys, changes to sells, cancellations of buys, cancellations of sells, volume traded, average and standard deviation of the volume per trade, and the average and standard deviation of the price per trade.

For each broker, the number of trades, number of internalized trades (trades where both the buyer and seller had the same broker identifier), volume of all trades, volume of all internalized trades, and the score of potential internalization were all recorded. Potential internalization is scored as follows: Periodically (every 30 trades), the orderbook would be examined. For each broker, the number of shares that have higher time priority but the same price priority is calculated for every entry a broker has in the books. This number is then averaged across all entries the broker has on the books to create an internalization score for that examination. Averaging this value across all such examinations provides a broker's internalization score for a particular day. This score provides a gauge for the amount of internalization that might occur, across the entire orderbook. To be used as a meaningful statistic on its own, we would only consider orders that are near the top of the book, as orders low in the book are unlikely to be executed as part of a trade. However, in order to combine this score with other values, it must be kept independent.

A moving window provides the total volume for groups of trades at different times of the day, as well as the weighted average price of trades that executed at different times of the day. This allows us to track the moving average of the prices over the course of the day.

A more detailed description of the simulator's architecture is included in Appendix A.

3.1.1 A Comment on the Data

Our data consisted of all orders and trades for all equities on the TSX for the months of March, April and May 2007, for all symbols that were members of the S&P/TSX Composite Index as of late April 2007. There were several issues with the data which had to be addressed either by the simulator itself, or by a pre-processing phase.

As we mentioned earlier, the data we were provided was not guaranteed to be in order due to the fact that the time stamps on the orders were rounded to the nearest second. This was insufficient to determine the actual orders of execution. The simulator had to be aware of this and track and aggregate related cancel and buy requests in order to recreate the sequence of buys and sells placed by brokers, rather than the sequence of orders listed in the data.

Since the data we were provided contained all orders and trades, it contained information from both the market open and close. It is well known that the market open and close have dynamics that differ from the market during the day, and can be modeled by various auctions and optimal price-finding techniques. We, however, are interested in the market during the day. As such, we did not record trades that occur at the market open for further analysis, though we did process them in an operational sense. Any trades that were designated to trade through the TSX's Market-On-Close mechanism were not recorded onto the orderbooks. There were other orders that needed to be removed from the order stream, as they were not processed using the standard orderbook rules. For example, odd lots, orders with a volume that was not a multiple of 100, were removed since market makers play a significant role in filling odd lots.[3] Orders which were negotiated off the market but were recorded on the market for regulatory reasons (i.e. the *crosses*) were also removed from the data since these did not actually pass through the market mechanism itself. Short sells were treated as regular sells and were not handled using short-sell price restrictions, as this was computationally infeasible to track.

[3] In Canada, market makers are assigned to each stock listed on the TSX (but not the TSX Venture) and are responsible for maintaining a bid-ask spread, filling client orders through the Minimum Guaranteed Fill (MGF) facility, and filling tradable odd-lot orders. Market makers also enjoy certain other (technical) benefits. For the purpose of this study, market marker activity is accounted for in the study because all market-maker trades are reflected similarly to trades executed by other parties. For the purposes of this study, the market maker is a market participant like any other (though the peculiarities of some of the automatic trade execution done by market makers create complications for data processing). However, when injecting additional order flow into historical data we are not explicitly simulating market-maker trades based on the minimum guaranteed fill facility or participation.

3.2 Internalization on the TSX

We attempted to generate statistics for 278 different equities selected from the S&P/TSX Composite as of late April 2007 for 63 business days from March. 1, 2007 to May, 31, 2007. We made use of publicly available information as was provided on the Toronto Broadcast Feed. Some basic information about the volume, liquidity, message traffic and pricing of each equity can be viewed in Appendix D. Ninety seven brokerages participating in trading on the equities samples, and some basic information about numbers of trades, internalized trades, and trade volumes can be viewed in Appendix E.

The simulator was run on a 16-node SunFire X4100 server cluster, where each node had two dual-core AMD Opteron 280 CPUs for a total of four cores per node. Usable disk space was approximately 2.8 TB.

3.2.1 General Findings

As the Toronto Stock Exchange (TSX) has not previously been studied using a re-creation of the exchange from real trading data, we present some results pertaining to trading on the TSX. We examined whether or not stocks tend to fall into certain ranges or bins based on either volumes traded during the day or by the prices that the stocks trade around. We plotted, for each equity, both the total accumulated volume and the price, to see if there is any interaction. This information is presented in Figure 1. No obvious grouping or clustering is evident.

We also examined the distributions of volumes traded. Please note that this does not correlate directly to the sizes of the submitted orders, as these orders are frequently broken down into fragments when they trade. Figure 2 presents the empirical distribution of trades. Statistics were kept for eleven discrete bins. This distribution seems to follow a log normal distribution. Note that the fat right-tail is due to the increased sizes of the bins, rather than an actual increase in trade frequency.

Trades can be initiated by either a sell order or a buy order that crosses the spread. The simulator tracked what proportions of trades were triggered by a sell order being recorded, compared to being triggered by a buy order. The result is that 72% of all trades were initiated by a sell order. This result can likely best be explained as an artifact of the simulator rather than as a feature of the market mechanism. When two orders are received and trade against each other in the same second, the record in the data only shows the trade – not which came first. The simulator processed all such records as a buy order being recorded on the book, followed by a sell order which crosses the book and causes a trade to execute. This would artificially inflate the number of

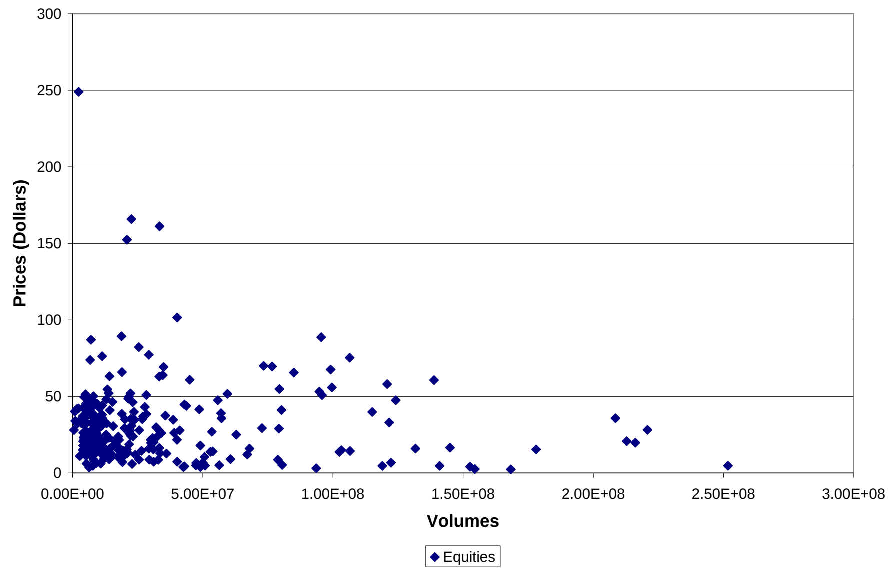

Fig. 1. *Prices of equities as a function of their volume traded.*

trades driven by sell orders.

As a form of comparison, the internalized trades were also examined, to see what proportion of internalized trades were driven by sell orders compared to buy orders. When only this subset was examined, it appeared that 40% of trades are sell driven. This is the opposite of what occurs for a general trade. We speculate that this occurs because the initial trades are predominantly sell-driven trades, which may only partially execute, leaving many small partial orders on the sell side of the book. Those few trades which are buy-driven would then have a greater opportunity to internalize.

3.2.2 Internalization Properties on the TSX

We start our study by recording the proportion of trades that occur between two parties from the same brokerage. These trades may or may not have involved a violation of time priority, but even if time priority is not violated we still consider the trade internalized. Figure 3 depicts the proportion of trades that are internalized.

Each data point in Figure 3 represents all trading activity for a particular brokerage of a particular equity for a particular day. As can be seen, internal-

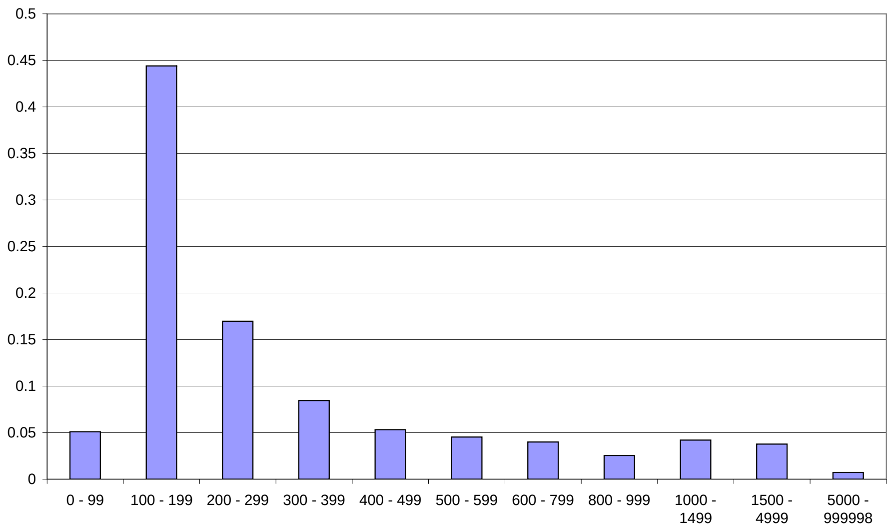

Fig. 2. *Empirical distribution of trades as a function of trade size.*

ization increases when the trading activity increases. Overall, 15.05% of trades were internalized by count, and 5.89% of trades were internalized by volume. It is interesting to note that the percentage of internalization when counting by number of trades is higher than when counting by volume. This implies that internalized trades might be of lower volume that normal trades. We investigated this further. To make the data clearer, we aggregated all trades that occurred under a single broker identifier. Each data point shown in Figure 4 represents all trading activity across all equities and all three months of data for a particular broker.

The average sizes for the internalized trades do tend to be lower than the average size for the average trade. The size of the average trade is 434.68 shares, while the size of the average internalized trade is 340.26 shares. We speculate that frequently when time priority is violated, only part of the order is able to be filled. This causes the instigating buy or sell order to be divided up more than it might have otherwise been, and thus the internalized trades have a tendency to be smaller. In scenarios where there is an execution cost for each trade, this may actually be a disadvantage for internalization, as more trades would be required to move the same amount of stock.

Figures 3 and 4 present results for all internalization, including both trades

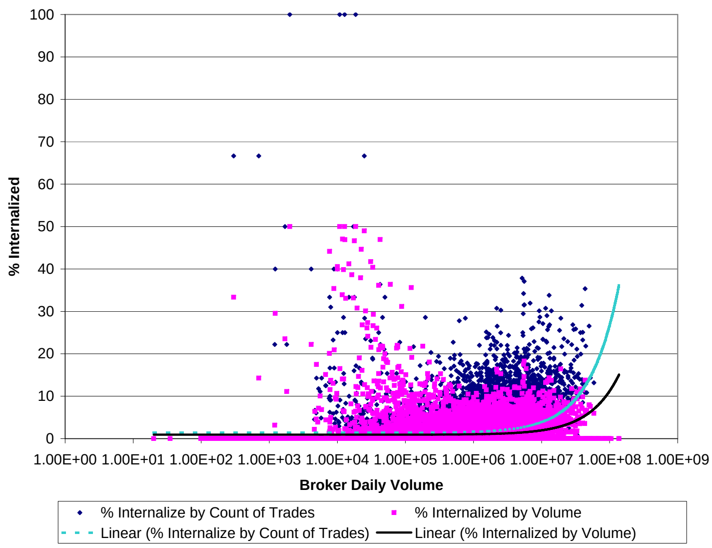

Fig. 3. *Proportion of trades that were internalized.*

that violated time priority and those that did not. The area of interest, however, is purely those trades that did violate time priority. To investigate this, we ran the simulator without the ability to violate time priority. This yielded statistics only concerning *passive internalization*, or those trades which are between the same brokerage but did not violate time priority. We then compared these results to the results obtained from *active internalization*, when time priority can be violated.

First, we present the overall results for passive internalization in Figure 5. Here, 13.16% of all trades were passively internalized by count (compared to 15.05% of all trades that were internalized), or 5.06% of trades by volume (compared to 5.89% of all trades that were internalized). This is a decrease of 12.5% by count or 14.0% by volume. Thus, we conclude that of all internalization, most of it is by happenstance. That is, the trade would have occurred whether the identifiers of the orders matched or not.

The same overall trend of internalized trades being smaller is still visible. This is due to the way crosses are handled by the simulator. Crosses are large trades which occurred and were agreed upon outside the exchange, but were registered with the exchange. These crosses are counted statistically for normal trading, but are removed from counts in internal trading. As such, any

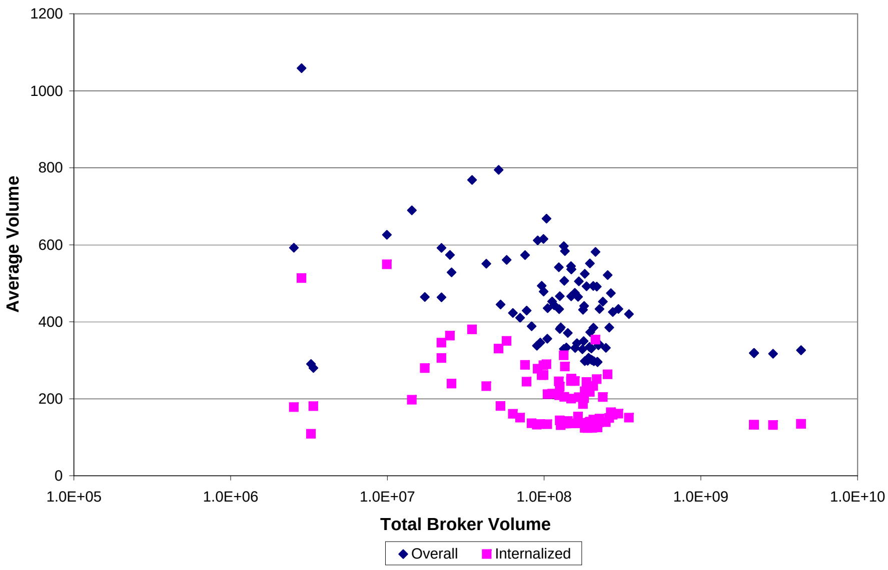

Fig. 4. *A comparison of the size of internalized trades with overall trades.*

difference in trade size that appears in the passive internalization scenario can be attributed to the crosses. The average reported normal trade size is 431.73 shares, where the average reported internalized trade size is 332.20 shares. This is a similar difference in average trade size to what was seen above. As such, we can dismiss the difference in trade size between internalized and regular trades as being an artifact of the simulation.

We examined the gains provided by internalization, and whether they are distributed equally across all brokerages. Figure 6 displays the proportion of internalization that occurs for each equity. There is no trend for internalization based on the liquidity of the equity. Levels of internalization varied widely, and the variability increased as equities became more liquid, but the dependency on levels of trading is negligible.

We examined the levels of internalization enjoyed by each brokerage, under both passive and active internalization. [4] Figure 7 depicts, for each brokerage, what level of internalization occurred under both passive and active inter-

[4] Note that the active internalization numbers include all passively internalized trades in addition to trades where the time priority was violated. Thus the numbers will always be higher for active internalization compared to passive internalization. What is of interest is the *difference* between the two types.

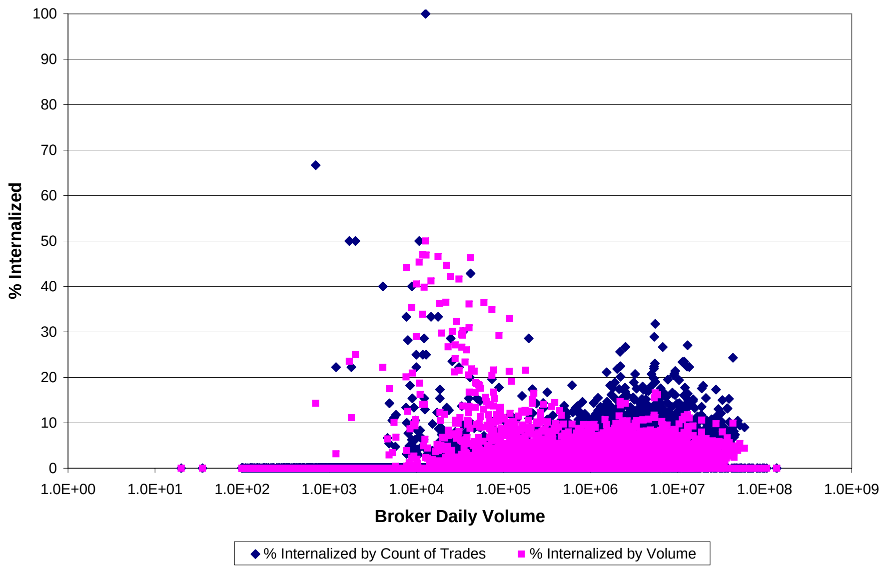

Fig. 5. *Percentage of trades which were passively internalized.*

nalization. The brokers are sorted by volume traded, where the data points furthest to the right indicate the busiest brokers. As can be seen from the graph, internalization increases as the trading volume of the brokerage grows. Both components of internalization, the active and passive portions, are linear trends. The passive internalization is best fit by the linear equation

$$7 \cdot 10^{-8} x + 0.76, \tag{1}$$

which means that as average daily-traded volume rises, so does the internalization that occurs naturally. The percentage gain on internalization from active internalization is best fit by the equation

$$5 \cdot 10^{-11} x + 32. \tag{2}$$

Figure 8 illustrates the difference between active and passive internalization as a function of volume traded. Values close to zero indicate that most internalization is passive (i.e. it occurs by happenstance) while numbers above zero indicate that time priority is being violated.

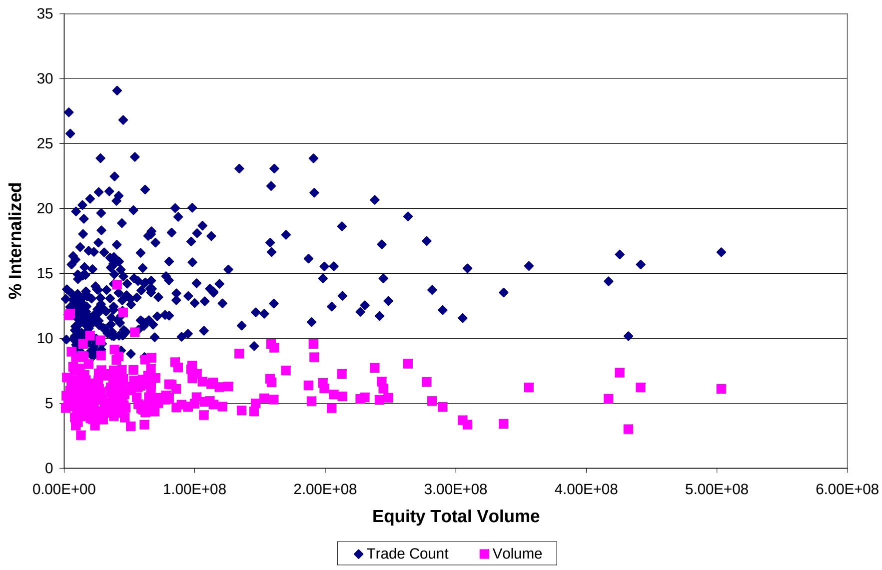

Fig. 6. *Percentage of internalization as a function of an equity's trading volume.*

3.3 Quantifying the Benefits from Internalization

While internalization offers some financial benefit to brokerages large enough to take advantage of it, the exact benefit remains unmeasured. A description of how much time or money can be saved by trading as part of a large brokerage, compared to trading anonymously, would be of great interest. One way to measure this benefit is by tracking the price paid to acquire a large block of shares of a particular equity. Such acquisitions are generally broken down into a series of smaller trades executed over the course of the day, so as not to adversely affect the market. By comparing the price paid to acquire the block of shares when trading as a large brokerage when compared to executing the same trades anonymously (where active internalization never occurs), we can observe the actual gain of internalization.

There are a variety of ways to move a large block of shares over a day. We chose a fairly simple algorithm. The algorithm has two phases, and is executed every 5 minutes. The algorithm purchases exactly 500 shares within every five minute period. We simulate the purchase of a block of 36,000 shares per day, acquiring shares between 10am and 4pm. The algorithm works as follows:

(1) At time t, place an order to purchase 500 shares at the best bid price.

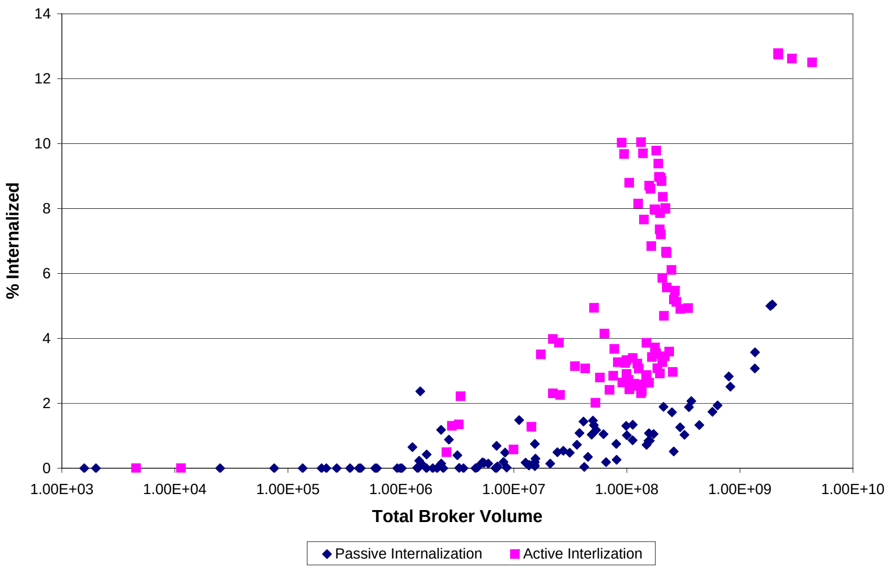

Fig. 7. *The percentage of volume that brokers passively and actively internalized as a function of their trading volume.*

(2) At time $t + 60$ seconds, if the order is still on the books (even partially), replace it with an order to purchase at $1.05p^*$ where p^* is the best ask price. This will execute as a market order.
(3) At time $t + 300$ seconds, repeat.

We compared the results across a range of brokers. In particular, fifteen brokers, including a broker trading under the anonymous identifier, were selected to represent both high and low volume traders. The brokers were selected since they were representative of different types of firms trading on the TSX. [5] In particular we chose a subset of the major banks (for example, CIBC *etc.*), some smaller brokerages (for example, Desjardins *etc.*) as well as some other shops (for example, Penson *etc.*). The brokers selected are displayed in Table 5. Data about these brokers, along with others operating on the TSX can be found in Appendix E. The procedure we just described was added into the order stream for every trading day from March 1, 2007 to May 31, 2007, for a selection of 221 equities from the TSX.

For each equity and each buying broker, the average price paid for the eq-

[5] In Appendix F we list data for a larger set of brokers. Our results and conclusions do not change when we include additional brokers.

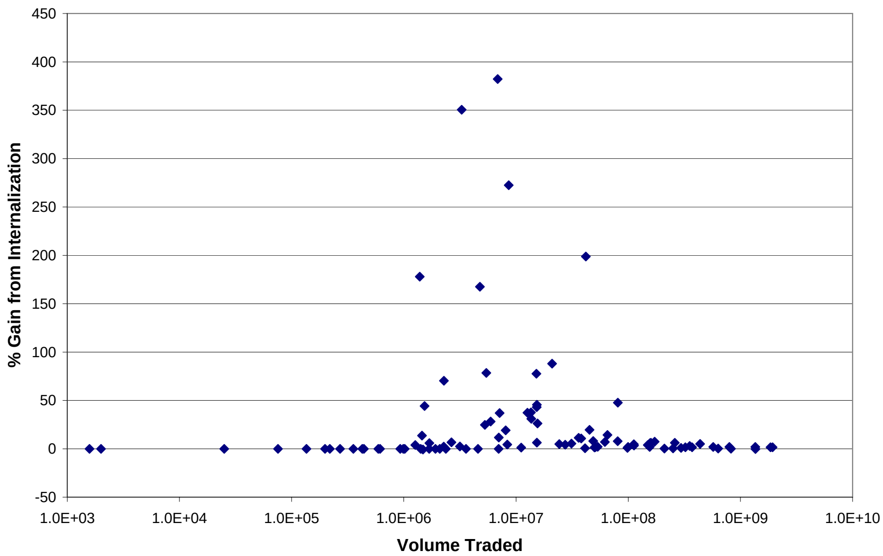

Fig. 8. *The difference between the amount of active and passive internalization as a function of volume traded.*

01 - Anonymous	101 - FIMAT Derivatives
14 - ITG Canada Inc.	19 - Desjardins Securities Inc.
2 - RBC Capital Markets	36 - W.D. Latimer Co. Ltd.
39 - Merrill Lynch Canada Inc.	3 - Tristone Capital Inc.
4 - Versant Partners Inc.	5 - Penson Financial
79 - CIBC World Markets Inc.	7 - TD Securities Inc.
80 - National Bank Financial Inc.	8 - Maple Securities
9 - BMO Nesbitt Burns Inc.	

Table 5
Brokers selected for testing.

uity was calculated through repeated execution of the simulator. We used the anonymous broker as a reference point. For each equity we calculated

$$100.0(\frac{P_{\text{non-anon}}}{P_{\text{anon}}} - 1.0)$$

where $P_{\text{non-anon}}$ is the price the broker trading non-anonymously paid, while P_{anon} is the price that the anonymous broker paid. The maximum, minimum and average ratios are shown in Table 6. Positive values represent situations where a broker who potentially internalized paid *more* than an anonymous broker, while negative values represent situations where a broker outperformed. That is, lower numbers indicate better performance. Our findings are also shown in Figure 9.

Broker	Ave	Max	Min	Volumes
14	-0.00671	-0.10040	0.00407	1.73E+09
7	-0.00393	-0.14253	0.01354	2.70E+09
2	-0.00378	-0.03262	0.01431	2.15E+09
79	-0.00358	-0.03237	0.01308	3.98E+09
9	-0.00281	-0.05386	0.02241	2.39E+09
80	-0.00155	-0.04125	0.01595	9.92E+08
39	-0.00150	-0.02237	0.00901	6.11E+08
101	-0.00068	-0.01599	0.00211	5.28E+08
19	-0.00066	-0.02497	0.00539	3.49E+08
5	-0.00019	-0.01482	0.00268	6.26E+07
36	-0.00008	-0.00412	0.00159	77666946
8	-0.00004	-0.00287	0.00000	1.32E+06
4	-0.00001	-0.00291	0.00005	6.45E+06
3	-0.00001	-0.00065	0.00011	29014758
01	0	0	0	NA

Table 6
Summary results for the injection experiments. Negative values indicate that a broker is benefiting from internalization.

On average, most brokers showed no benefit. Only extremely large brokers, where total volumes traded range near the hundreds of millions, show any benefit, which never exceeds 0.004% on average. The maximum and minimum values are also displayed, which describe the equity for which each broker had the best and worst performance. Never is there a penalty for trading under a brokerage identifier that exceeds 0.023%, nor is there a gain that ever exceeds 0.1426%. The five largest brokers show an average benefit of 0.0042%. The fact that some brokers under-performed with respect to the anonymous broker may be due to the presence of cancellation orders which can only take effect if the order is not yet executed.

Further investigation was conducted to see if other factors also influenced the benefit of internalization. First, each equity was examined separately, to see

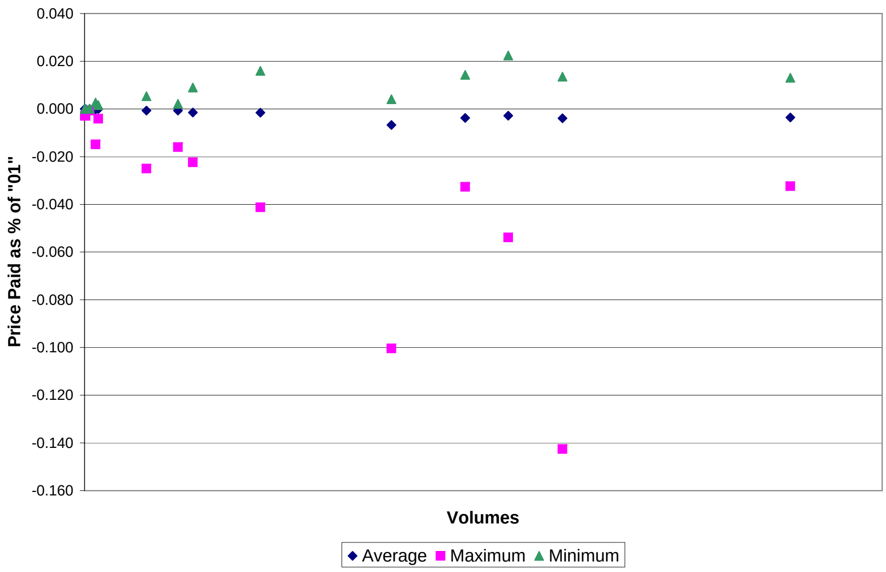

Fig. 9. *Percentage difference between the payment between a broker trading openly and one trading anonymously, averaged across all equities, sorted by broker volume. Numbers below 0.0 indicate that the broker is benefiting from internalization. As the volume increases, there is a slight benefit from internalization.*

if the liquidity of an equity affects the average price paid. The results are displayed in Figure 10. There is no upwards or downwards trend in the data; increasing or decreasing levels of liquidity have no effect.

Another potentially confounding factor is the proportion of the day's trading for a particular equity that is done by a particular brokerage, that is, the broker's market share. It is possible for a large brokerage firm to make up almost half of the trading for certain equities, and have a much smaller take of other equities. In Figure 11, we plot the percentage of the anonymous price, as compared to the proportion of that equity's trading that consisted of a particular broker. As can be seen, there is no particular trend, and thus we conclude that market share for a particular equity does not necessarily lead to benefits due to internalization.

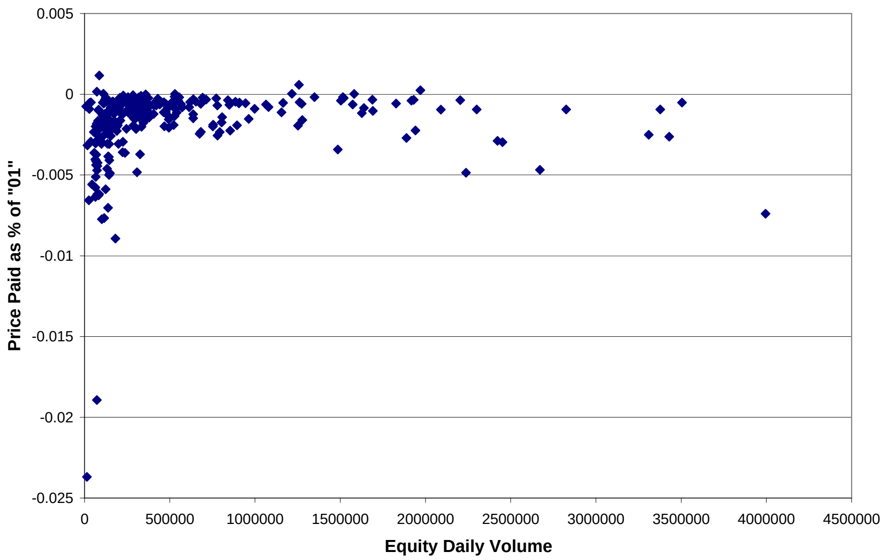

Fig. 10. *Impact that liquidity of an equity has on the price paid by a broker who internalizes compared to an anonymous broker.*

3.4 Summary

The potential to violate time priority does increase the amount of internalization that occurs within the system, however if we define internalization to be either passive (the trade would have occurred even without the internalization mechanism) or active (time priority was violated during the trade) then we note that a significant fraction of all internalization is actually passive.

We did note that the ability to internalize did favor larger brokerages over smaller ones, so that large brokers will be able to internalize a greater proportion of their trades. The liquidity of the equity being traded appeared to have no effect, however. If there is an execution cost per trade that is greater for trades with other brokerages than for within-brokerage trades, internalization may yield significant savings the larger firms. This seems to be the only significant savings, however.

When trying to move a large order, there is little monetary benefit derived from internalization. Even for large brokerages, the average benefit is only approximately 0.002%. When purchasing 50,000 shares of a $20.00 stock, that is a savings of only $2,000 on a one million dollar purchase. Whether this

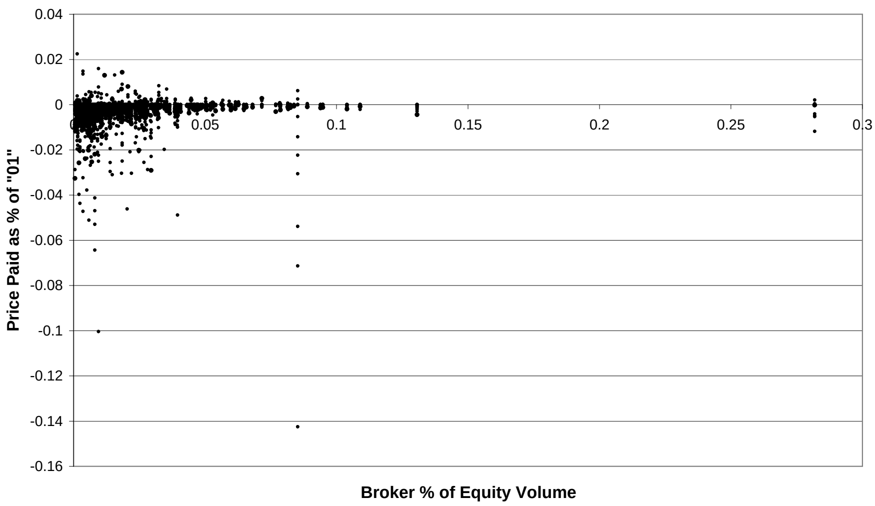

Fig. 11. *Impact that brokerage market-share of a particular equity has on the price paid by a broker who internalizes compared to an anonymous broker.*

is worth the information loss for trading under your declare brokerage rather than anonymously will depend on the investment banker's personal judgement.

4 An Abstract Model of Internalization

Even though we have analyzed internalization using actual data from the TSX, there is value in developing an abstract model of the mechanism. An abstract model allows us to find, isolate and control different parameters in order to gain a theoretical understanding of the underlying properties of internalization. In this section we describe our model for internalization, and make observations and predictions. We conduct a series of simulations where we control different parameters of the model to gain further understanding of their affect.

We model the internalization using a *Markov chain* with rewards. For readers unfamiliar with Markov chains, we have included basic background information in Appendix B. There are several reasons why a Markov chain is an appropriate representation for studying internalization:

(1) the orderbook fully describes the current trading state of a particular equity since it lists all limit orders awaiting possible future execution,[6]
(2) transitions between one orderbook listing to another depend *only* on incoming orders,
(3) rewards in the system are naturally related to transitions from one orderbook listing to another given an incoming order, and
(4) attitudes toward risk (in particular, risk-aversion) can be modelled with discount factors.

4.1 Model Description

The model is defined from the perspective of an arbitrary broker trading under brokerage identifier β. We will assume, for the sake of clarity, that the broker is attempting to sell some number of shares of a particular equity. The model is easily modified to fit the situation where the broker is a buyer.

A key component of our model is the *orderbook view* of broker β. Given an orderbook, this representation captures all information that is relevant to the study of internalization from the perspective of broker β.

Definition 1 (Orderbook View) *An orderbook view from the perspective of broker β, is a vector*

$$OB_\beta = \left\langle \begin{pmatrix} p_1 & v_1 \\ p_2 & v_2 \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \right\rangle$$

where

- $p_i < p_{i+1}$,

[6] If "iceberging" is allowed then this is not precisely true. We discuss this in Section 4.2.

- *for* $1 \leq i < n$, v_i *is the total volume of shares offered at price* p_i*,*
- *at price* p_n*,* β*'s order price,*
  - v_n *is the total volume of shares at price* p_n *with higher time priority than* β*'s order, and*
  - *for beta's order and all other orders at price* p_n *and with lower time priority, the broker identifier and order volume is listed.*
- P^* *is the highest price on the Buy side of the book.*

An orderbook view is similar to *market-by-price* in that for prices less than β's we are only interested in the volume being offered, and do not care about which particular orders are involved. At the price of broker β's order, more detailed information is needed. In particular we require knowledge of the actual order details for all orders at price p_n with lower time priority than broker β's order since these may become relevant if and when internalization occurs. Note, however, that we can ignore any sell order with price higher than p_n since this will never directly affect broker β's possible future transactions involving the order listed in the book. The order of broker β has priority over all Sell orders with higher price, even if internalization occurs. Similarly, we store no information about the Buy orders in the orderbook since they will never be executed with broker β's order at the current set of prices.

Example 3 *Assume there is an orderbook as pictured in Table 7.*

Broker	Quantity	Price	Price	Quantity	Broker
85	300	15.45	15.50	500	6
2	1700	15.45	15.50	800	2
7	300	15.45	15.52	1500	79
11	1300	15.40	15.52	400	7
2	500	15.39	15.52	200	3
11	400	15.39	15.52	600	12
3	1000	15.38	15.55	1200	9

Table 7
On the left are the Bids and on the right are the Offers.

Then

$$OB_7 = \langle \begin{pmatrix} 15.50 & 1300 \\ 15.52 & \begin{pmatrix} & 1500 \\ 7 & 400 \\ 3 & 200 \\ 12 & 600 \end{pmatrix} \end{pmatrix}, 15.45 \rangle$$

and

$$OB_9 = \langle \begin{pmatrix} 15.50 & 1300 \\ 15.52 & 2700 \\ 15.55 & \begin{pmatrix} & 0 \\ 9 & 1200 \end{pmatrix} \end{pmatrix}, 15.45\rangle.$$

Our definition of an order coincides with an order on the TSX.

Definition 2 (Order) *An* order*, o, is a tuple*

$$o = \langle b, p, v, id\rangle$$

where

- *b specifies the type or order, $b \in \{\text{Buy}, \text{Sell}, \text{Change}\}$,*
- *p is the price*
- *v is the volume*
- *id is the id of the brokerage firm of the broker placing the order.*

When an order arrives it triggers a transition from the current orderbook view to a new orderbook view. This transition from one view to another depends solely on the orders which appear. This observation allows us to specify the transition probabilities as a function of the incoming order. assuming that the brokers are non-strategic. Thus, the transition probability matrix depends on the probability that a particular order will arrive. The full details of the transition model are described in Appendix C.

We define the *reward* that the broker receives to be a function of its current orderbook view and the most recent order. That is

$$R_\beta(OB_\beta, o) = F(p_n, v)$$

where $F(\cdot,\cdot)$ is some function of the number of shares sold by broker β at its asking price p_n. We also define a reward of the orderbook view to be the expected reward. That is

$$R_\beta(OB_\beta) = E_{o\in O}[R_\beta(OB_\beta, o)]$$

where O is the set of all possible orders. In Appendix C we provide further details of the rewards for each orderbook view – order pair.

Given the transition model and reward structure outlined in Appendix C we are able to fully specify the value of a particular orderbook view by computing the discounted sum of future rewards, given an initial orderbook view.

$$V(OB_\beta) = \sum_o Pr(o)(R((OB_\beta, o)) + \gamma V(OB'_\beta)) \tag{3}$$

where

$$(OB_\beta, o) \rightarrow OB'_\beta$$

and

$$V(OB_\beta) = 0,\ \forall\ OB_\beta \text{ with } v_\beta = 0.$$

Equation 3 can be expanded further since we have a clear model of orderbook view transitions;

$$\begin{aligned} V(OB_\beta) = & \sum_{o \in O_{\text{sell}}} Pr(o)(R((OB_\beta, o)) + \gamma V(OB'_\beta)) && (4) \\ & + \sum_{o \in O_{\text{buy}}} Pr(o)(R((OB_\beta, o)) + \gamma V(OB'_\beta)) && (5) \\ & + \sum_{o \in O_{\text{change}}} Pr(o)(R((OB_\beta, o)) + \gamma V(OB'_\beta)) && (6) \end{aligned}$$

where O_{buy} are the set of possible orders with b = Buy, O_{sell} are the set of possible orders with b = Sell, and O_{change} are the set of possible orders with b = Change.

We can further expand the expressions 4, 5, and 6. Line 4 is equal to

$$\gamma Pr(b = \text{Sell}) \Bigg[V(OB_\beta)(\sum_{p=0}^{P^*} Pr(p) + \sum_{p=p_n}^{p_{\max}} Pr(p)) + \\ \sum_{v=0}^{v_{\max}} Pr(p_n, v) V(OB_\beta^{(\text{Sell}, p_n, v)}) + \\ \sum_{p_i=p_1}^{p_{n-1}} \sum_{v=0}^{v_{\max}} Pr(p_i, v) V(OB_\beta^{(\text{Sell}, p_i, v)}) \Bigg].$$

Line 5 can be expanded as

$$Pr(b = \text{Buy})[\gamma \sum_{p=0}^{P^*} Pr(p)V(OB_\beta) + \gamma \sum_{p=P^*}^{p_1} Pr(p)V(OB_\beta^{P^*\to p}) +$$
$$\gamma \sum_{p=p_1}^{p_{n-1}} \sum_{v=0}^{v_{\max}} Pr(p,v)V(OB_\beta^{(\text{Buy},p,v)}) + \gamma \sum_{p=p_n}^{p_{\max}} \sum_{v=0}^{\sum_{j=1}^{n-1} v_j} Pr(p,v)V(OB_\beta^{(\text{Buy,v}\leq\sum_{\text{j=1}}^{\text{n}-1}\text{v}_\text{j})}) +$$
$$\sum_{p=p_n}^{p_{\max}} \sum_{v=\sum_{j=1}^{n-1}}^{v_{\max}} Pr(p,v)[Pr(id=\beta)\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n-1} v_k]) + \gamma V(OB_\beta^{(\text{Buy},\beta,v)})\right) +$$
$$\sum_{i=1}^{m} Pr(id = id_i)\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n} v_k - v_{id_i}]) + \gamma V(OB_\beta^{(\text{Buy},id_i,v)})\right) +$$
$$(1 - (Pr(id=\beta) + \sum_{i=1}^{m} Pr(id = id_i))\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n} v_j]) + \gamma V(OB_\beta^{(\text{Buy,NoInt},v)})\right)]].$$

Line 6 can be expanded is a similar way as lines 4 and 5;

$$\gamma Pr(b = \text{Change})[\sum_{p=p_n}^{p_{\max}} \sum_{v=0}^{v_{\max}} \sum_{v'=0}^{v} Pr(p\to p', v \to v')V(OB_\beta^{\text{Change},p,v\to v')}) +$$
$$\sum_{p=0}^{p_n} \sum_{p'=0}^{p} \sum_{v=0}^{v_{\max}} \sum_{v'=0}^{v} Pr(p, v\to v')V(OB_\beta^{\text{Change},p\to p',v\to v')}) +$$
$$\sum_{p=p_n}^{p_{\max}} \sum_{p'=0}^{p_n} \sum_{v=0}^{v_{\max}} Pr(p\to p', v)V(OB_\beta^{(\text{Sell},p_i,v)})\ ].$$

4.2 Assumptions and Limitations

We make three important assumptions which have varying degrees of impact on the model. First, we assume that the transition from one orderbook view to another depends solely on the orders which appear, and that these orders are appearing due to some underlying probability distribution (which could change over time). If orders are placed by *strategic* brokers then this property might disappear since an incoming order could depend on the orderbook view. While studying strategic behavior of brokers is beyond the scope of this project, our belief is that our general conclusions with respect to internalization also apply to strategic settings. Strategizing on the part of brokers might have some impact on the frequency that internalization occurs, but it will not change the underlying internalization mechanism.

The second assumption we make concerns the probability of a particular order arriving. In particular, as can be seen from the description of the transition model in Appendix C, we make certain independence assumptions. We first assume that

$$Pr(\langle b, p, v, id \rangle) = Pr(b)Pr(id)Pr(p, v).$$

That is, we assume that order type (b) and brokerage identifier (id) are independent of price (p) and volume (v). We could relax this assumption without changing the model, though keeping it allows us to isolate some of the parameters of interest. A more insidious assumption we make is that an order arriving at time t does not depend on orders that have previously arrived We are aware that both assumptions do not precisely capture the real nature of the TSX, and that in reality there is likely to be correlations between, for example, a particular brokerage and the size or price of an order. However, assumptions about independence of parameters, often called the *Naive Bayes assumption* or independent feature model, has been shown, in a wide range of applications, to work very well in complex real-world settings. [7] We believe that this is also true for the internalization mechanism, since the goal of our investigation is to understand trends as opposed to precisely capture all aspects of the TSX.

The third assumption we make is that the orderbook views contain the full volume of shares that are listed in the orderbook above and at the price of broker β's order. That is, our model does not capture *iceberging*, where icebergs are orders on the orderbook where only part of the order is shown. Since we do not see how iceberging would change the internalization mechanism in any qualitative fashion, and since incorporating iceberging would come at a cost of great computational complexity, we stand by our assumption of full observability of the orderbook views.

4.3 Observations

Given the derivation of Equation 3 it is possible to make some observations and draw some conclusions about the internalization mechanism, and when it is beneficial or not beneficial for a particular broker.

Our first observation is that the only time a broker β receives a non-zero reward are in states where a transaction involving its order is executed. While the reward depends on the function $F(\cdot, \cdot)$, for many reasonable functions the reward from internalization is always as least as high as it would have been if broker β had been trading anonymously.

Observation 1 *Assume that broker β can choose between trading using brokerage identifier β and trading anonymously. If $F(p, v)$ is monotonically increasing in v, then, all other things being equal, the reward received by par-*

[7] For example, many text classifiers and spam-filters make this assumption in assuming that words in a document are independent of each other. Even though this is clearly not true, the classifiers are still highly effective.

ticipating in the internalization mechanism is at least as great as the reward received if the broker traded anonymously.

If a broker identifies itself in order to use internalization when possible, then all other things being equal, it is able to trade at least as many shares as if it had not internalized (*i.e.* participated anonymously). This is due to the fact that occasionally it might be able to participate in a transaction by internalizing, that it would not have been able to if it had traded anonymously. We can conclude that if other brokers are using the internalization mechanism (*i.e.* others are not trading anonymously), then it is never in broker β's best interest to trade anonymously if it is only concerned about its reward in *non-strategic settings*. There are likely *strategic* reasons for hiding ones identify, and thus strategic value may be obtained from anonymized trading. In particular, anonymous trading reduces information leaks which might influence the price of an equity. However, these strategic considerations are outside the scope of this project.

An immediate conclusion that can be drawn from Observation 1 is that the more internalization opportunities a broker has, the more benefit it receives from the internalization mechanism, assuming that other brokers are potentially internalizing.

Observation 2 *All other things being equal, the benefit broker β receives from trading under brokerage identifier β increases as its market share increases.*

Observations 1 and 2 state that it is beneficial for a particular broker to trade non-anonymously, assuming that other traders in the market are trading non-anonymously. These observations do not imply that the internalization mechanism is beneficial in general.

Observation 3 *Any broker can be harmed by internalization.*

Consider the expansion of Equation 5;

$$
\begin{aligned}
&Pr(b = \text{Buy})[\gamma \sum_{p=0}^{P^*} Pr(p)V(OB_\beta) + \gamma \sum_{p=P^*}^{p_1} Pr(p)V(OB_\beta^{P^* \to p}) + \\
&\gamma \sum_{p=p_1}^{p_{n-1}} \sum_{v=0}^{v_{\max}} Pr(p,v)V(OB_\beta^{(\text{Buy},p,v)}) + \gamma \sum_{p=p_n}^{p_{\max}} \sum_{v=0}^{\sum_{j=1}^{n-1} v_j} Pr(p,v)V(OB_\beta^{(\text{Buy,v}\leq\sum_{\text{j=1}}^{\text{n}-1}\text{v}_\text{j})}) + \\
&\sum_{p=p_n}^{p_{\max}} \sum_{v=\sum_{j=1}^{n-1}}^{v_{\max}} Pr(p,v)[Pr(id=\beta)\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n-1} v_k]) + \gamma V(OB_\beta^{(\text{Buy},\beta,v)})\right) + \\
&\sum_{i=1}^{m} Pr(id = id_i)\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n} v_k - v_{id_i}]) + \gamma V(OB_\beta^{(\text{Buy},id_i,v)})\right) + \\
&(1 - (Pr(id=\beta) + \sum_{i=1}^{m} Pr(id = id_i))\left(F(p_n, \min[v_\beta, v - \sum_{k=1}^{n} v_j]) + \right. \\
&\left.\gamma V(OB_\beta^{(\text{Buy},\text{NoInt},v)})\right)]].
\end{aligned}
$$

The third and four lines describe what happens if an incoming Buy order is internalized involving brokers who have orders at price p_n (*i.e.* the price of broker β's order). First, we note for broker β to be affected by this particular incoming order, it must be the case that the order volume must be greater than $\sum_{i=1}^{n-1} v_i$ since these orders all have price priority which is never violated during internalization. That is, the incoming order must be *large enough.* Second, we note that if the incoming order is *large enough*, then if broker β is able to internalize, up to an additional amount of v_n of its order can be filled by the incoming order compared to the situation where there was no internalization. If however, internalization allows another broker with identifier $id_i \in \{id_1, \ldots, id_m\}$ to violate time-priority, then broker β can *lose* up to an amount v_{id_i} of its order compared to if there was no internalization. If broker β is unable to re-coup this potential loss of v_{id_i} every time another broker with an order at price p_n violates the time priority via the internalization mechanism, then broker β would have been better off in a system where no internalization was allowed.

In the worst case, a broker might face *starvation conditions* where the internalizing of other brokers leads to a situation where β *never* gets its order filled (though it would have been filled if internalization was not allowed). The only way that a broker in this situation could escape the starvation conditions is to change the order's price to be more aggressive.

Interestingly, when we try to isolate the characteristics of brokers who will be harmed in general by the internalization mechanism, there is little that can be said. Brokers with high volume are not more protected against missed opportunities due to internalization than low-volume brokers. Similarly, the liquidity of the equity being traded does not appear to directly factor into the determination as to whether internalization is beneficial for a broker or not. The key feature is the volume of orders at price p_n which have higher time priority, as well as the orders and their broker identifiers at price p_n that have lower time priority. While these features can be correlated with volume or liquidity in some situations, in other settings there may be (almost) no relation.

Observation 4 *The following features are the most important when determining whether the internalization mechanism is beneficial for a broker β with order with price p_n;*

- *the volume of orders at price p_n with higher time priority, and*
- *the volume and identities of brokers for orders at price p_n with lower time priority.*

Volume and liquidity are only important in how they affect the above features.

If internalization is allowed in the market, then, as we discussed in Observation 1, a broker is best off taking advantage of the mechanism (ignoring strategic gains that hiding information may bring). This is particularly true for brokers who place value on minimizing the amount of time their orders are listed on the orderbook.

Our model of internalization incorporates a measure of *aversion to opportunity costs*. In particular, we interpret the discount factor, γ, as a broker's risk tolerance. If γ is set to be 1.0, then a broker is risk-neutral – it does not distinguish between selling (or buying) x shares immediately and selling x shares at some point in the future. If, however, $\gamma < 1.0$ then a broker places more value on selling shares now compared to selling them later. The broker values having its orders spend less time listed on the orderbook.

Taking the perspective of a broker trying to sell some volume of shares, by studying Equation 5 one observes that if the internalization mechanism is used by other brokers, then for any incoming order our broker (β) will sell a larger fraction of the shares in its listed order if it is not trading anonymously, compared to if it is trading anonymously. This means that *more* (or, at least no fewer) of its shares will be sold faster. Thus, if the broker has risk-aversion $\gamma < 1.0$, it will gain additional value by having those shares sold quickly. Thus, any broker who assigns value to executing its order quickly will benefit from internalization.

Observation 5 *Assume that there exists at least one broker who is using the internalization mechanism (*i.e. *at least one broker is not trading anonymously). Then, risk-averse brokers benefit from internalization. In particular, by taking advantage of internalization, a broker may reduce the time its order is listed on the orderbook.*

In summary, given our model of the internalization mechanism, we can draw the following conclusions;

- The internalization mechanism can harm brokers. No particular broker type is immune against the harm.
- If a broker has an order, o, at price p_n, then volume and liquidity are only important, with respect to internalization, in how they affect the total volume of orders at price p_n with higher time priority than o, and the volume and identities of brokers for orders at price p_n with lower time priority.
- If other brokers are using the internalization mechanism, then a broker is best off also using internalization. That is, there are no non-strategic benefits from trading anonymously.
- A broker that places value on executing trades quickly (in particular, risk-averse brokers) will benefit from internalizing (assuming other brokers are potentially internalizing also).

4.4 Simulations

We conducted a series of simulations using our abstract model in order to gain a deeper understanding as to what factors, if any, impact the benefit brokers could gain from using the internalization mechanism. *We stress that the goals of our simulations were to understand trends and properties of the internalization mechanism, and not to provide fully accurate simulations of the TSX.*

The basic setup of all our experiments was the same. Given a set of parameters (which are discussed later) we generated an initial orderbook view. We then generated a stream of thirty thousand orders, using the same parameters. We calculated the reward that our broker would have received, allowing for internalization. We then repeated the procedure, this time making our broker trade using an anonymous identifier. We compared the rewards in both situations. This procedure was repeated one thousand times, re-initializing the orderbook each time.

Unless otherwise stated, we used the following parameters to generate the order stream and initial orderbook view;

Volume

The volume, v, of an order was drawn from the interval $[100, 10000]$ using a log-normal distribution with mean of 300. This distribution and range were chosen based on statistical analysis of actual order data from the TSX between March 1, 2007 to May 31, 2007. Volumes were rounded to the nearest one hundred.

That is, we did not generate odd or mixed lots.

Prices

Prices were drawn from three distributions. For the first distribution, we defined the normal distribution with mean equal to 4.00 and standard deviation equal to 0.333 (LOW). We then drew prices from this distribution, rounding them to the nearest cent. We repeated this procedure with mean equal to 22.50 and standard deviation of 0.833 (MED), and with mean equal to 175.00 and standard deviation of 8.333 (HIGH). The means and standard deviations were used since they were representative of different equity price levels derived from actual trading data from the spring of 2007.

Order Type

An order was equally likely to be a Buy or Sell order. In our simulations we did not consider Change orders.

Risk Aversion

We set $\gamma = 1.0$. That is, unless otherwise stated we assumed that our broker was risk-neutral.

Broker Identifiers

An order's brokerage identifier was drawn uniformly from the interval [2, 100]. We reserved the identifier 1 for an anonymous trader. We emphasize that these identifiers in no way are related to identifiers used on the TSX. For example, any appearance of the identifier "7" in the simulations does not imply that we are executing orders as TD. We could have equivalently used identifiers from the set $\{a, b, ...\}$.

For each simulation, we focused on a particular parameter, modifying it while keeping all the other parameter settings the same. We calculated the reward of an agent for a given transaction to be

$$R_\beta = F(p, v) = p \times v,$$

that is, the price of the transaction multiplied by the total number of shares sold during that transaction. The total reward for the agent was the weighted sum of rewards from the transactions. For each stream of orders we compared the total reward the agent would have received if it could potentially internalize compared to the total reward it would have received if it had been anonymous. We report the ratio

$$K = \frac{R_\beta^{\text{internalization}}}{R_\beta^{\text{no-internalization}}}.$$

If K is greater than 1.0, then internalization benefits the agent, if it is equal to 1.0 then internalization neither harms nor hurts the agent, and if it is below 1.0 then internalization harms the agent.

We note that there are other possible ways of calculating R_β. For example, the VWAP measure could be used. However, since we use the reward when the agent does not internalize as a benchmark, and report K, we are abstracting away from the actual details as to how the reward is calculated.

Finally, as will be seen, in some of our simulations the magnitude of difference between when the broker is anonymous and when it is not is very small. However, in all of our experiments we found that the difference was statistically significant. The p-values, which we calculated using a Fischer Sign Test, were all less than 0.000001.

4.4.1 Brokerage Market Share

In our first set of experiments we studied what impact brokerage market share had. We generated data sets where the market share of our broker of interest ranged from 0% to 100%. All other parameters were fixed. Our results are found in Figure 12.

As we expected, as the broker market share increases, we observe an increased benefit from internalization. It is interesting to note, however, that the benefit for the broker when trading a high-priced equity was very small – the maximum benefit according to our simulations was a 0.083% increase over what it would have received while participating anonymously.

We note that on the TSX no brokerage has a market share of 80% or 90%. We included this range in order to understand how the increase *would* affect brokers. Realistically, for most equities the maximum market share of any particular brokerage is at most 20%. In our simulations, with low-priced equities we observed a benefit as market share increased, but for medium and high-priced equities there was no significant difference between 10% and 20% market share.

4.4.2 Offsetting Orders

We investigated how the percentage of possible offsetting orders impacted a broker's reward when it could internalize compared to if it traded anony-

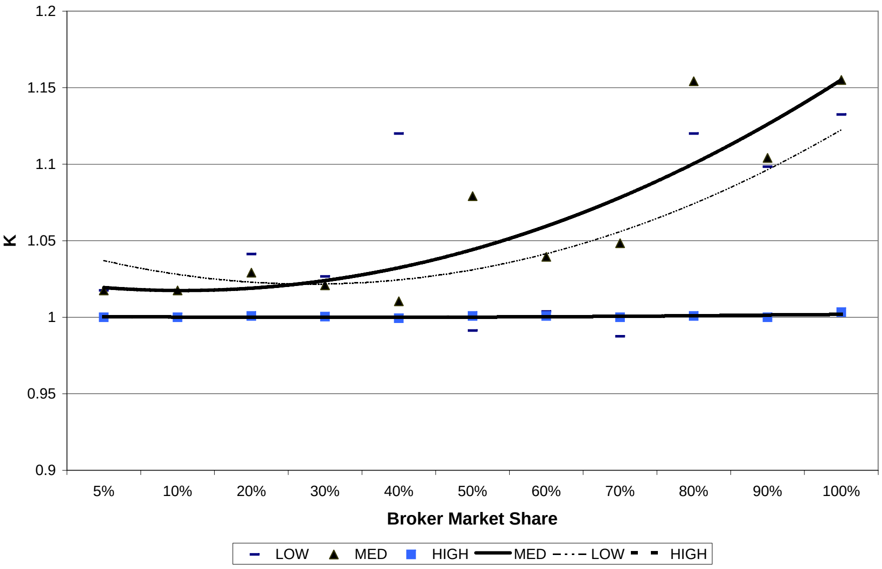

Fig. 12. *The benefits of internalization as the broker market share increases. Values above 1.0 indicate that there is a benefit to an agent from internalization.*

mously. We conducted this set of experiments by varying the global percentage of Buy orders in the order stream. Our findings are presented in Figure 13.

As can be seen from the graph, for all three price-levels, as the percentage of potential offsetting orders increased, on average the benefit the broker experienced from not trading anonymously also increased. However, if the percentage of potential offsetting orders was low (less than 50%) the internalization mechanism actually harmed the broker when prices were drawn from the medium-level distribution. Even at unreasonably high levels of potential offsetting orders, the benefit from the internalization mechanism, compared to trading anonymously, was quite small. On average a broker's reward was at most 10.0% higher if it internalized when possible. At the more reasonable 50% point, the benefits for all price distributions were within 6.0% of the reward, $R_{\beta}^{\text{no}-\text{internalization}}$, the broker would have received from trading anonymously.

4.4.3 Order Volumes

Given our abstract model for internalization, we were able to conclude that volume would indirectly affect whether or not internalization was beneficial for a broker. However, our model was unable quantify the indirect impact

Fig. 13. *As the overall fraction of offsetting orders increases, a broker benefits more from taking advantage of the internalization mechanism.*

(or lack of impact). Thus we conducted a series of simulations where we controlled volume. To this end we systematically shifted the volume distributions so that the probability of observing an order for a large number of shares increased while the probability of observing an order for a small number of shares decreased. Figure 14 shows our findings. The x-axis lists the smallest order-size in terms of number of shares, for each averaged data point. As always, the y-axis is the K measurement described earlier in this section. Values of K which are greater than 1.0 indicate that internalizing is beneficial to our broker compared to the situation where it trades anonymously.

We observe that changing the volume-range of the orders has little impact. For orders with prices from the MED distribution we noted the largest improvement as the volume size increased. However, the improvement was less than 2.0% compared to trading anonymously. These findings are in line with our observation that stated that any benefit a broker might gain by potentially internalizing was only indirectly related to order volume.

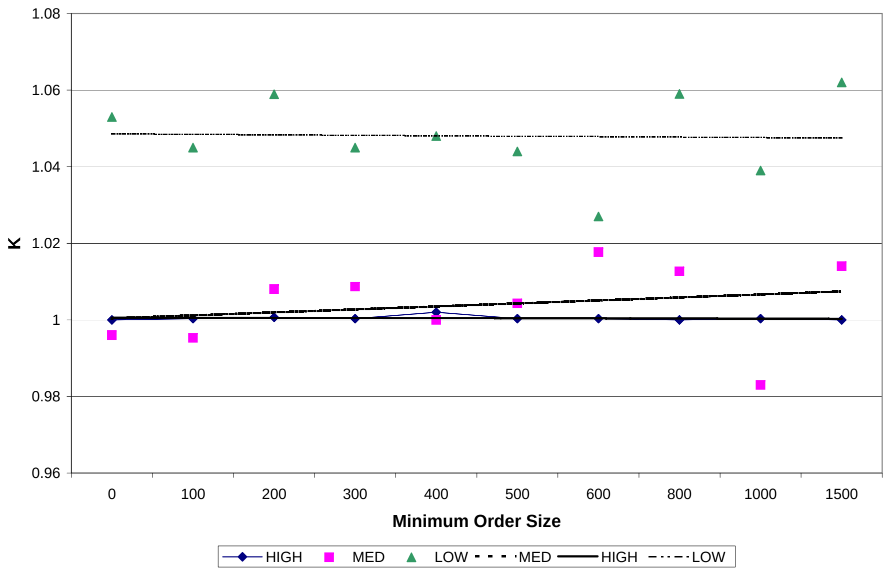

Fig. 14. *Change in benefit from the internalization mechanism as order volume increases.*

4.4.4 Risk Aversion

In our final set of simulations we experimented with the risk attitude, or impatience, of the broker by changing γ. In particular, we varied γ between 1.0 and 0.9 and measured how measure of internalization benefit changed.[8] Our findings can be seen in Figure 15.

Of all the simulations we conducted, controlling the risk-attitude (or impatience-level) caused the most dramatic improvement for an internalizing broker compared to an anonymous broker. In particular, as γ decreased, the broker became more impatient and placed increased value on selling its shares sooner as opposed to later. Any delay in selling shares hurt a broker substantially. Thus, our simulation results allow us to deduce that if not trading anonymously, the broker sold its shares sooner than if it traded anonymously. This means that the overall risk of having its order listed on the book for longer periods of time

[8] We ran additional experiments where γ was varied between 1.0 and 0.8. The results for the lower values of γ were even more pronounced than those reported here. We present only the results for what we believe are the most *reasonable* values for γ.

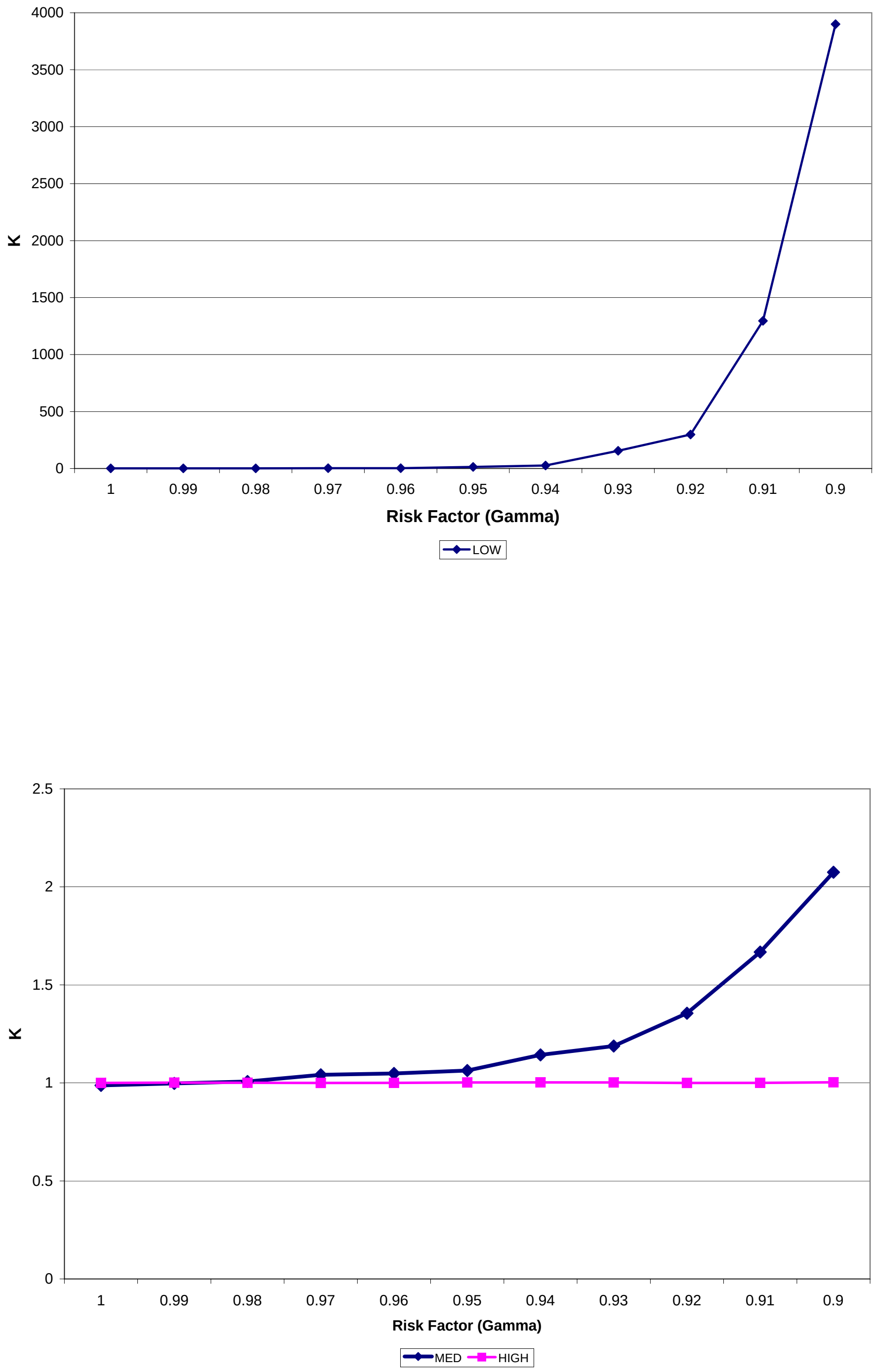

Fig. 15. *As a broker's risk aversion increases, it benefits more from taking advantage of internalizing compared to trading anonymously. The top graph shows the results when prices of orders were drawn from the LOW distribution. The bottom graph shows the results when prices were drawn from the MED and HIGH distributions.*

is also reduced.[9]

4.5 Summary

In this section we presented an abstract model for internalization based on a Markov system. Given this model we were able to draw several conclusions about the internalization mechanism. We concluded that the internalization mechanism is sometimes beneficial for a particular broker, but at other times it can prove to be disadvantageous. That is, brokers might be better off if internalization was never permitted. There is no explicit characterization of brokers who will be put at a disadvantage by the mechanism. Both high- and low-volume brokers can be harmed by the process. The properties of the equity being traded also do not provide any direct information as to whether brokers trading that equity are placed at an advantage or disadvantage by having the internalization mechanism in the market.

If, however, the market is using internalization, then we were able to show that a broker is always better off choosing to identify itself when placing orders, as opposed to trading anonymously. This claim, however, does ignore *strategic concerns*, such as information leakage, which may cause a broker to hide its identity. Brokers who benefit particularly from the internalization mechanisms are those who prefer to have their orders executed quickly. The more impatient (or risk-averse) a broker is, the more it will perceive the benefits of internalization.

We conducted a series of simulations using our model where we isolated different parameters. In particular, we ran experiments where we controlled market-share, the percentage of potential offsetting orders, volume distributions of orders, price distributions of orders, and risk-aversion of brokers. The findings from the simulations supported both our theoretical conclusions, and agreed with our findings from our study using actual TSX data. Our simulations indicated that a broker who traded under its own identifier would see increased gains compared to trading anonymously as its market share increased, as well as if the percentage of offsetting orders increased. We noted that increasing

[9] We believe that the results for the orders with prices drawn from the LOW distribution are exaggerated due to the setup of the experiment. Since any delay in trade execution quickly reduced the reward to close to zero, and since there was a tendency to be more delay in trade execution if the broker was trading anonymously, the denominator in our ratio for K was often very small, thus leading to inflated K values. That said, the overall trend, which is also observed for the MED and HIGH price distributions where the vanishing denominator was not an issue, is that as risk-aversion increases, a broker does much better if it uses the internalization mechanism when able.

the size of orders in terms of volume made almost no difference to a broker's performance, and that the most significant benefits were for brokers who were highly risk-averse (or impatient).

We note that in all our simulations we observed a difference between the three price distributions, LOW, MED, and HIGH (see Section 4.4 for a description of these distributions). Though the trends for each price distribution was similar in all our simulations, the percentage improvement over trading anonymously did differ. We do not believe that this difference is directly related to the prices, but instead is a feature of the standard deviations of the distributions. In particular, the standard deviation of the LOW distribution was only 0.333, while for the HIGH distribution it was 8.333. This resulted in the number of possible price-points for the LOW distribution experiments to be quite small, while for the HIGH distribution there were many more possible price points. This meant that orderbooks that were generated with the LOW distribution tended to have, for any given price, multiple orders listed. The orderbooks generated for the HIGH distribution, on the other hand, were more likely to have only a single order listed at a certain price. If there are several different orders at a price point, then a broker can gain from the internalization mechanism, as it allows it to "jump the queue" when it violates time-priority. If a broker has the only order at a price point, then internalization is irrelevant since there is no opportunity to violate time-priority. We believe that this explanation, and not the fact that the actual prices are different, is the rational for the difference between LOW, MED, and HIGH price distribution behavior.

5 Conclusion

Our goal for this project was to understand the internalization mechanism found on the TSX. To this end we:

(1) Developed an abstract model based on a Markov chain with rewards and drew conclusions about the benefits of internalization from the model.
(2) Conducted a series of simulations in order to gain an understanding as to what parameters affect internalization.
(3) Designed and developed a simulator for analyzing actual TSX order streams.
(4) Conducted controlled experiments with our simulator on actual order data in order to quantify the benefits of internalization under realistic conditions.

Given our findings we are now able to answer a set of questions that were posed to us at the start of the project.

Does the internalization procedure used by the TSX benefit any particular type of firm?

From our simulations and abstract model we determined that internalization is, on average, most beneficial to risk-averse firms. That is, traders who strongly prefer to execute their trades as quickly as possible. From our experiments with TSX order data, using our simulator, we noted a slight benefit for high volume brokers compared to low volume brokers, but this benefit was very small.

Does it put particular types of brokers at a disadvantage? For example, does internalization benefit high volume brokers more than low volume brokers?

Our experiments using the TSX data indicate that very high volume brokers benefit slightly from internalization. We did not discover any broker type that was placed at a significant disadvantage.

Is it possible to quantify the advantage?

- In terms of dollar savings?
  Yes, our experiments using the simulator indicated that very high volume brokers saw an average benefit of 0.0042%. No brokerage had a penalty from internalization more than 0.023%.
- In terms of speed-up in execution of orders?
  We are unable to quantify this directly. Our simulation results can be interpreted as there being an observable speed-up in the execution of orders. However, we are unable to translate this into a precise value to be applied to the TSX.

Do the benefits of internalization depend on the type of stock being traded (for example, high liquidity vs. low liquidity)?

Interestingly, it appears as though the benefits of internalization *do not* depend on the type of stock being traded. This is supported by both our model and our findings using the simulator.

It is possible to place orders anonymously, but then one does not benefit from internalization. Other than keeping the information about who is placing an order secret, are there any other quantifiable benefits of trading anonymously?

In theory there is no benefit of trading anonymously (ignoring strategic concerns), if other brokers are trading non-anonymously. In practice, our analysis indicates that there is little penalty in trading anonymously (the maximum penalty we observed was 0.023%). It is likely that strategic considerations would make this small penalty meaningless.

Acknowledgements

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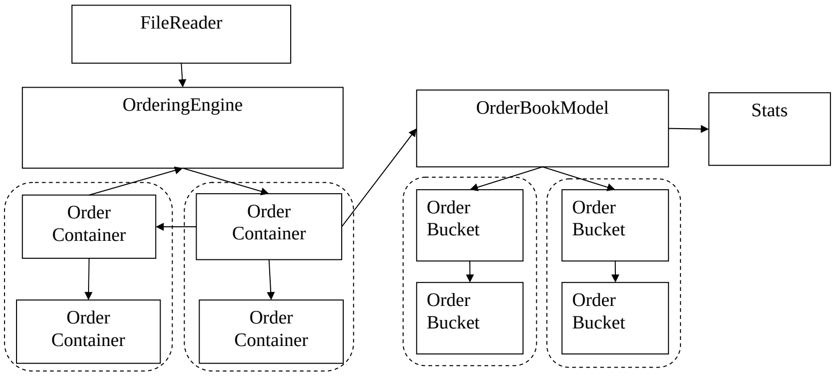

Fig. A.1. *The architectural design of the simulator.*

A Simulator Architecture

In this section we provide an overview of the architecture of the simulator.

The following classes are used to manage the flow and storage of data during processing:

- FileReader,
- OrderingEngine,
- OrderContainer,
- OrderBookModel,
- OrderBucket,
- Stats.

The Order class represents a single order throughout the flow of the program, although what is considered an order changes as the Order flows through the program. A visual diagram relating the components is shown in Figure A.1.

A single FileReader object exists for each data file being processed. This object reads in a single line of the file and creates an Order object containing that data. At this stage, certain orders are removed from the system, such as odd lots and market-on-close orders. Successive calls to the FileReader return one Order after another until the file has been processed completely.

The OrderingEngine contains a reference to the FileReader. The OrderingEngine is created by the main program, instantiates the FileReader, and then begins getting Orders from the FileReader. Each Order corresponds to a line in the file, and is stored in an OrderContainer. OrderContainers are recursive data structures, where each container accumulates all Orders that should be considered a single transaction and recursively points to other OrderContainers that contain other transactions that occurred at the same time. For example, a sell request which had immediately traded as two trades would here be placed into the same OrderContainer. All Orders that occurred in a given second are accumulated; as soon as an Order is received which occurs in the next second, the OrderContainer that recursively contains all accumulated Orders are processed into the OrderBookModel. After processing, the old cache is purged, and the OrderContainer is moved into the cache. The cache contains all orders from the previous second, allowing us to match Orders which occurred as a single transaction but are erroneously recorded as occurring at different times.

Orders are recorded into the OrderBookModel as orders would be placed in the market by a broker: As change orders, buy or sell orders, or cancellations. The OrderBookModel persists through the duration of the program, although the buy and sell side halves of the book are each purged at the beginning of a new day. The OrderBookModel does not contain data; it contains a reference to the top-most OrderBucket on the buy and sell sides. Each OrderBucket contains all requests buys or sells for at a particular price, as well as a reference to the OrderBucket at the next price point. For example, one OrderBucket might contain all buy requests at $15.50, as well as a reference to the OrderBucket containing all buy requests at $15.47. Change and cancellation requests affect the data contained in the OrderBuckets, or in the deletion of an OrderBucket. Change requests or new orders may result in the creation of a new OrderBucket, and/or a trade being conducted. Trades occur when a buy (sell) Order's price is larger (smaller) than the lowest (highest) price recorded for a sell (buy) request. A trade then occurs between the order being processed and the order in the other half of the book with the greatest time priority (unless active internalization is in place, in which case this priority may be violated). Trades result in the volumes of an order being decrease by the greatest amount possible without sending either Order below zero. Zero volume Orders are then removed from the OrderBuckets.

As trades are made and other requests are being processed, the OrderBookModel calls methods on the Stats object. This object records information over the lifetime of this OrderBookModel, calculating all required information and eventually writing the statistics out as a comma-delimited file.

B Background: Markov Chains

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Definition 3 (Markov Chain) *A Markov Chain is defined by*

- *A set of states* $\{S_1, S_2, \ldots, S_N\}$ *and*
- *A transition probability matrix*

$$\mathbf{P} = \begin{pmatrix} P_{1,1} & P_{1,2} & \ldots & P_{1,N} \\ P_{2,1} & & & \\ \vdots & & & \\ P_{N,1} & & & P_{N,N} \end{pmatrix}$$

such that the Markov property holds,

$$P_{i,j} = \text{Prob}(\textit{Next State} = S_j | \textit{This State} = S_i).$$

We further expand the definition to include *rewards*. In particular, for each state S_i we define R_i to be the reward that is collected while in that state. We also introduce a *discount factor* γ, $0 < \gamma \leq 1$ which is used to discount future rewards.

To solve a Markov chain with rewards, we calculate the value of being in each of the possible states in the system. This value, $V(S_i)$, is defined to be the expected discounted sum of future rewards starting at state S_i. Potential rewards, which may be realized only in the distant future, are not viewed as favorably as rewards which can be achieved sooner. In particular, a reward now of amount x is considered to be equal in value to a reward of $\frac{x}{\gamma^t}$ t steps in the future. For example, if the system is deterministic (that is, if we are in state S_i currently, then in the next step it is known with certainty that we would be in state S_{i+1}), then

$$V(S_i) = R_i + \gamma R_{i+1} + \gamma^2 R_{i+2} + \gamma^3 R_{i+3} + \ldots$$

In vector notation, if we define

$$\mathbf{R} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{pmatrix}$$

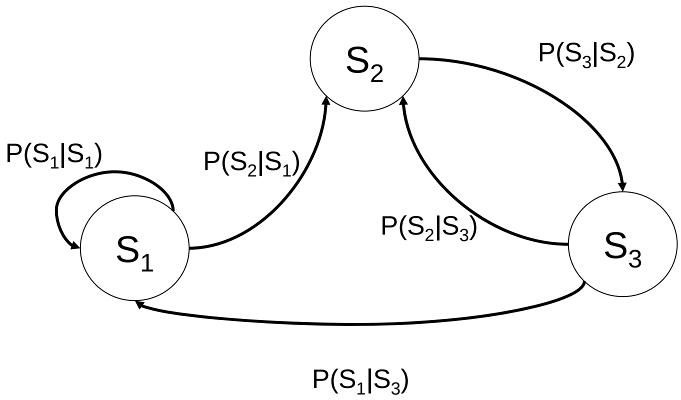

Fig. B.1. A small example of a Markov chain represented as a finite state machine.

and

$$\mathbf{V} = \begin{pmatrix} V(S_1) \\ V(S_2) \\ \vdots \\ V(S_N) \end{pmatrix}$$

Then

$$\mathbf{V} = \mathbf{R} + \gamma\mathbf{P}\mathbf{V}$$

or equivalently

$$\mathbf{V} = (\mathbf{I} - \gamma\mathbf{P})^{-1}\mathbf{R}$$

where $\mathbf{I}$ is the $N \times N$ identity matrix.

On a final note, one useful way of envisioning a Markov chain is via a finite state machine representation. Figure B.1 shows a small example with three states. The transitions between states are labelled with the probability of that transisition occuring, and if there is no transition then it is assumed that the probability is zero. The matrix representation of this problem would be

$$\mathbf{R} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix},$$

$$\mathbf{P} = \begin{pmatrix} P(S_1|S_1) & P(S_2|S_1) & 0 \\ 0 & 0 & P(S_3|S_2) \\ P(S_1|S_3) & P(S_2|S_3) & 0 \end{pmatrix},$$

with

$$\begin{pmatrix} V(S_1) \\ V(S_2) \\ V(S_3) \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} + \gamma \begin{pmatrix} P(S_1|S_1) & P(S_2|S_1) & 0 \\ 0 & 0 & P(S_3|S_2) \\ P(S_1|S_3) & P(S_2|S_3) & 0 \end{pmatrix} \begin{pmatrix} V(S_1) \\ V(S_2) \\ V(S_3) \end{pmatrix}.$$

C Transition Model and Rewards

In this appendix we provide the details of the transition model along with the rewards that a broker would receive from each transition. The probability of a particular order $o = \langle b, p, v, id \rangle$ being placed is $Pr(\langle b, p, v, id \rangle)$. In this report, when referring to the abstract model we will make independence assumptions about the features of a specific order since this allows us to highlight and isolate parameters of interest with respect to internalization. In particular, we assume that both the order type (b) and the broker identifier (β) are independent of the price and volume of the order (p and v). This means that

$$Pr(\langle b, p, v, id \rangle) = Pr(b)Pr(id)Pr(p, v)$$

where $Pr(p, v)$ is the joint distribution over the price and volume. Recall that

$$Pr(p) = \sum_v Pr(p, v)$$

and

$$Pr(v) = \sum_p Pr(p, v).$$

It is possible that the distributions change over time and our model allows for this. While we will not explicitly attach a time index to the probability distributions in an attempt to reduce the notational overhead, it can be thought of as being there implicitly.

When describing the transition model we assume that we start with an arbitrary initial order-book view;

$$OB_\beta = \langle \begin{pmatrix} p_1 & v_1 \\ p_2 & v_2 \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle.$$

We present the transition model by cases.

C.1 *Order* $o = \langle \text{Sell}, p, v, id \rangle$

If the incoming order is a sell order then there are four cases which we must consider.

C.1.1 *Order o Crosses the Spread*

If a sell order arrives and crosses the spread then it is executed immediately and is never listed in the order book. Thus, the order book-view does not change. This means that

$$(OB_\beta, o) \to OB_\beta.$$

Since broker β was not involved in a trade execution, we define its reward to be zero. That is

$$R((OB_\beta, o)) = 0$$

where $R((OB_\beta, o))$ is the reward in the state (OB_β, o).

The probability that this occurs is

$$Pr(b = \text{Sell}) Pr(p \leq P^*).$$

It is possible that an order comes in, crosses the spread, but is not filled entirely due to volume constraints on the buyer side of the book. This will cause an update in the order-book view. We handle this case as though there were two incoming orders – one of which is filled and the other that is placed on the book. This second situation is covered by one of the cases presented in C.1.2, C.1.3, or C.1.4.

C.1.2 $p > p_n$

If the order is such that $p > p_n$ then it will be placed in the order book *below* all orders with price equal to p_n. This means that the order book view will not change. Therefore,

$$(OB_\beta, o) \rightarrow OB_\beta$$

and

$$R((OB_\beta, o)) = 0.$$

The probability that this occurs is

$$Pr(b = \text{Sell})(1 - Pr(p \leq p_n))$$

C.1.3 $p = p_n$

If the incoming order has $p = p_n$ the order book view will be updated to reflect this new order. In particular,

$$(OB_\beta, o) \rightarrow OB_\beta^{(\text{Sell}, p_n, v)}$$

where

$$OB_\beta^{(\text{Sell}, p_n, v)} = \langle \begin{pmatrix} p_1 & v_1 \\ p_2 & v_2 \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \\ id & v \end{pmatrix} \end{pmatrix}, P^* \rangle.$$

The reward is

$$R_\beta(OB_\beta, o) = 0$$

and the probability of this transition is

$$Pr(b = \text{Sell})Pr(p_n, v).$$

C.1.4 $p < p_n$

If the price of the order is less than p_n but the order does not cross the spread, then it will be entered into the order book above the order of β. Thus, there exists some i such that $p = p_i$, and

$$(OB_\beta, o) \to OB_\beta^{(\text{Sell}, p_i, v)}$$

where

$$OB_\beta^{(\text{Sell}, p_i, v)} = \langle \begin{pmatrix} p_1 & v_1 \\ \vdots & \vdots \\ p_i & v_i + v \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle.$$

The reward is

$$R_\beta(OB_\beta, o)) = 0$$

and the probability of this transition is

$$Pr(b = \text{Sell})Pr(p_i, v).$$

C.2 Buy Orders

As in the sell-order case, we start with an initial order-book view and analyze the cases that can arise when a buy-order appears. Let

$$o = \langle \text{Buy}, p, v, id \rangle.$$

C.2.1 $p \leq P^*$

If the incoming Buy order has a price p which is less than P^* then no trade will be executed. The order will be placed on the Bid side of the order book, and thus not affect the order-book view. Therefore

$$(OB_\beta, o) \rightarrow OB_\beta$$

and

$$R((OB_\beta, o)) = 0.$$

The probability of this occurring is

$$Pr(b = \text{Buy})Pr(p \leq P^*).$$

C.2.2 $P^* < p < p_1$

If the incoming order has price p which is greater than P^* but less than p_1, then the order will not cross the spread but will change the order-book view. Thus

$$(OB_\beta, o) \rightarrow OB_\beta^{P^* \rightarrow p}$$

where

$$OB_\beta^{P^* \rightarrow p} = \langle \begin{pmatrix} p_1 & v_1 \\ p_2 & v_2 \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, p\rangle$$

and

$$R((OB_\beta, o)) = 0.$$

The this transition will occur with probability

$$Pr(b = \text{Buy})Pr(P^* < p < p_1).$$

C.2.3 $p_1 \leq p < p_n$

If the order price is greater than the lowest offer-price then a trade will occur. However, since $p < p_n$ only orders listed *above* broker β's in the order-book view will be involved. Assume that $p = p_i$ for some $i < n$. Then,

$$(OB_\beta, o) \rightarrow OB_\beta^{(\text{Buy}, p<p_n, v)}$$

where

$$OB_\beta^{(\text{Buy}, p<p_n, v)} = \langle \begin{pmatrix} p_1 & \max[0, v_1 - v] \\ p_2 & \max[0, v_2 - \max[0, v - v_1]] \\ \vdots & \vdots \\ p_i & \max[0, v_i - \max[0, v - \sum_{k=1}^{i-1} v_k]] \\ p_{i+1} & v_{i+1} \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle$$

Since the transaction does not involve broker β,

$$R((OB_\beta, o)) = 0.$$

The probability of this transition is

$$Pr(b = \text{Buy}) Pr(p_1 \leq p < p_n, v).$$

C.2.4 $p_n \leq p$

If a Buy order arrives with a price which is greater than or equal to p_n there is a possibility that broker β will be involved in a transaction. However, the volume of shares that broker β eventually sells depends on several properties, including whether or not internalization in the system works in its favor or not. In the rest of this section, we analyze the different cases which arise.

(1) **Order volume $v \leq \sum_{j=1}^{n-1} v_j$:** If the volume of the incoming order is small (i.e. $v \leq \sum_{j=1}^{n-1} v_j$) then the transaction does not directly affect broker β since the order will be fully filled by higher priority (with respect to price) orders.

Let i be the smallest index such that $\sum_{j=1}^{i} v_j \geq v$. Then

$$(OB_\beta, o) \to OB_\beta^{(\text{Buy}, v \leq \sum_{j=1}^{n-1} v_j)}$$

where

$$OB_\beta^{(\text{Buy}, v \leq \sum_{j=1}^{n-1} v_j)} = \langle \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \\ \vdots & \vdots \\ p_{i-1} & 0 \\ p_i & \max[0, v_i - \max[0, v - \sum_{k=1}^{i-1} v_k]] \\ p_{i+1} & v_{i+1} \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle$$

Since the transaction did not involve broker β,

$$R((OB_\beta, o)) = 0.$$

The probability is

$$Pr(b = \text{Buy}) Pr(p \geq p_n, v \leq \sum_{j=1}^{n-1} v_j).$$

(2) $v \geq \sum_{j=1}^{n-1} v_j$ **and there is no internalization:** If the incoming order is large enough then it will clear all order-book view entries with price less than p_n. If there is additional volume to be filled then orders at price p_n will be also be executed. Broker β will only be involved in this transaction once all orders are price p_n with higher time priority are cleared. Then

$$(OB_\beta, o) \to OB_\beta^{(\text{Buy}, \text{NoInt}, v \geq \sum_{k=1}^{n-1} v_k)} = OB'_\beta$$

where

$$OB'_\beta = \langle \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \\ \vdots & \vdots \\ p_{n-1} & 0 \\ p_n & \begin{pmatrix} - & \max[0, v_n - \max[0, v - \sum_{k=1}^{n-1} v_k]] \\ \beta & \max[0, v_\beta - \max[0, v - \sum_{k=1}^{n} v_k]] \\ id_1 & \max[0, v_{id_1} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta]] \\ \vdots & \vdots \\ id_m & \max[0, v_{id_m} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - \sum_{k=1}^{m-1} v_{id_k}]] \end{pmatrix} \end{pmatrix}, P^* \rangle$$

The reward is

$$R((OB_\beta^{(\text{Buy,NoInt},v \geq \sum_{j=1}^{n-1} v_k)}, o)) = F(p_n, \min[v_\beta, v - \sum_{k=1}^{n} v_j]).$$

While we currently place no restrictions on what the function F can be, it should depend on the price at which the transaction takes place as well as on the number of shares that were sold. Candidate functions include

$$F((p_n, \min[v_\beta, v - \sum_{k=1}^{n-1} v_j - v_{id_i}]) = p_n \min[v_\beta, v - \sum_{k=1}^{n-1} v_j - v_{id_i}]$$

but other non-negative functions are also suitable.

The transition probability is

$$Pr(b = \text{Buy})(1 - (Pr(id = \beta) + \sum_{k=1}^{m} Pr(id = id_k)))Pr(p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j)$$

(3) $v \geq \sum_{j=1}^{n-1} v_j$ **and broker** β **internalizes:** If the incoming order has broker identifier equal to β then internalization which benefits β occurs. It's order is filled before any other broker at price p_n, irrespective of time prioritization. We observe that

$$(OB_\beta, o) \to OB_\beta^{(\text{Buy},id=\beta,p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j)} = OB''_\beta$$

where

$$OB''_\beta = \langle \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \\ \vdots & \vdots \\ p_{n-1} & 0 \\ p_n & \begin{pmatrix} - & \max[0, v_n - \max[0, v - \sum_{k=1}^{n-1} v_k - v_\beta]] \\ \beta & \max[0, v_\beta - \max[0, v - \sum_{k=1}^{n-1} v_k]] \\ id_1 & \max[0, v_{id_1} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta]] \\ \vdots & \vdots \\ id_m & \max[0, v_{id_m} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - \sum_{k=1}^{m-1} v_{id_k}]] \end{pmatrix} \end{pmatrix}, P^* \rangle$$

The reward for this state is

$$R(OB_\beta^{(\text{Buy}, id=\beta, p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j)}) = F(p_n, \min[v_\beta, v - \sum_{k=1}^{n-1} v_j]).$$

The probability of this transition is

$$Pr(b = \text{Buy})Pr(id = \beta)Pr(p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j).$$

(4) $v \geq \sum_{j=1}^{n-1} v_j$ **internalization occurs but does not involve** β**:**
If internalization is allowed in the system, then it applies to all brokers, not just broker β. Thus, there may be situations where broker β is affected negatively by internalization when another broker is given priority. This occurs when a buy order arrives with the appropriate price and volume, but has a broker $id_i \in \{id_1, id_2, \ldots, id_m\}$. In this situation, the order corresponding to the identifier is filled before β's order, irrespective of time priority. We observe that

$$(OB_\beta, o) \rightarrow OB_\beta^{(\text{Buy}, id_i, p \geq p_n, v > \sum_{j=1}^{n-1} v_j)} = OB_\beta^*$$

where

$$OB^*_\beta = \langle \begin{pmatrix} p_1 & 0 \\ p_2 & 0 \\ \vdots & \vdots \\ p_{n-1} & 0 \\ p_n & \begin{pmatrix} - & \max[0, v_n - \max[0, v - \sum_{k=1}^{n-1} v_k - v_{id_i}]] \\ \beta & \max[0, v_\beta - \max[0, v - \sum_{k=1}^{n} v_k - v_{id_i}]] \\ id_1 & \max[0, v_{id_1} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - v_{id_i}]] \\ id_2 & \max[0, v_{id_2} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - v_{id_1} - v_{id_i}]] \\ \vdots & \vdots \\ id_i & \max[0, v_{id_i} - \max[0, v - \sum_{k=1}^{n-1} v_k]] \\ id_{i+1} & \max[0, v_{id_{i+1}} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - \sum_{k=1}^{i} v_{id_k}]] \\ \vdots & \vdots \\ id_m & \max[0, v_{id_m} - \max[0, v - \sum_{k=1}^{n} v_k - v_\beta - \sum_{k=1}^{m-1} v_{id_k}]] \end{pmatrix} \end{pmatrix}, P^* \rangle$$

The reward is

$$R(OB_\beta^{(\text{Buy}, id=\beta, p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j)}) = F(p_n, \min[v_\beta, v - \sum_{k=1}^{n-1} v_j - v_{id_i}]).$$

The probability which this occurs is

$$Pr(b = \text{Buy}) \sum_{j=1}^{m} Pr(id_j) Pr(p \geq p_n, v \geq \sum_{j=1}^{n-1} v_j).$$

C.3 Change Orders

The final situation which can arise to change an order-book view is the *modification* of an order which is already listed. While this is not technically a situation where a new order arrives, we can still treat this in the same way in our model. We extend the definition of an order to better reflect changes. In particular, a change order takes the form

$$o = \langle b, p \to p', v \to v', id \rangle$$

where

$$p \to p'$$

indicates that the price is changed from p to p', and

$$v \to v'$$

indicates that the price change affects v' of the v shares.

We can break the change orders into four separate cases and study each individually. First, an order might be changed so that it clears immediately upon the change. If the order in question initially had price $p > p_n$ then it would not have been included in the order book view, and so it's execution does not change the order-book view. Thus, our model can ignore this case.

Second, an order can be changed so that *some* of its volume, v', is traded. Assume that a change order takes the form $o = \langle \text{Change}, p_i \to P^*, v_i \to v', id \rangle$

then

$$(OB_\beta, o) \to OB_\beta^{(\text{Change}, p_i, v_i \to v')}$$

where

$$OB_\beta^{(\text{Change}, p_i, v_i \to v')} = \langle \begin{pmatrix} p_1 & v_1 \\ p_2 & v_2 \\ \vdots & \vdots \\ p_i & v_i - v' \\ p_{i+1} & v_{i+1} \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle$$

The reward is

$$R((OB_\beta, o)) = 0$$

and this occurs with probability

$$Pr(b = \text{Change}) Pr(p_i, v \to v').$$

Thirdly, it is possible for an order to be changed and yet not cross the spread. In particular, if there is an order $o = \langle \text{Change}, p_i \to p_j, v \to v', id \rangle$ then

$$(OB_\beta, o) \to OB_\beta^{(\text{Change}, p_i \to p_j, v \to v')}$$

where

$$OB_\beta^{(\text{Change},p_i\to p_j,v\to v')} = \langle \begin{pmatrix} p_1 & v_1 \\ \vdots & \vdots \\ p_j & v_j + v' \\ \vdots & \vdots \\ p_i & v_i - v' \\ p_{i+1} & v_{i+1} \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle$$

with

$$R((OB_\beta, o)) = 0.$$

This occurs with probability

$$Pr(\text{Change})Pr(p_i \to p_j, v \to v').$$

The final case is the situation where an order which initially had a price of $p > p_n$ is changed so that v' of its shares are now priced at p_i. We can treat this as though a new sell-order has arrived at price p_i for v' shares. This means that

$$(OB_\beta, o) \to OB_\beta^{(\text{Sell},p_i,v')}$$

where

$$OB_\beta^{(\text{Sell},p_i,v')} = \langle \begin{pmatrix} p_1 & v_1 \\ \vdots & \vdots \\ p_i & v_i + v' \\ \vdots & \vdots \\ p_{n-1} & v_{n-1} \\ p_n & \begin{pmatrix} - & v_n \\ \beta & v_\beta \\ id_1 & v_{id_1} \\ \vdots & \vdots \\ id_m & v_{id_m} \end{pmatrix} \end{pmatrix}, P^* \rangle.$$

The reward for this situation is

$$R(OB_\beta, o) = 0$$

and it occurs with probability

$$Pr(b = \text{Change}) Pr(p \to p_i, v \to v').$$

D Equity Statistics

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
AAC.B	17205	15180	18185	6649	5441	912	51.39
AAH	44945	52543	86531	28391	4305	248	33.97
ABX	2416467	2345598	5077850	1880305	349684	347	32.99
ABZ	217742	248941	420440	51964	38474	272	42.34
ACE.B	184795	149222	324746	159882	35223	311	30.44
ACM.A	82088	74705	117786	20505	29789	241	42.41
ACO.X	81612	97591	149452	39755	21375	209	49.59
AEM	1989232	1969490	4062215	1366202	198467	245	41.51
AER.UN	71550	63370	81187	28673	46953	663	20.06
AET.UN	106204	92549	107133	69274	76987	390	21.66
AGA	91098	103759	147597	37970	39107	568	51.94
AGF.B	119970	134067	214652	27784	31796	359	35.80
AGI	69227	57732	71147	31617	47537	655	7.35
AGU	1571445	1438165	2889347	751796	187461	232	43.85
ALA.UN	48574	38924	64754	19831	18017	289	25.90
AL	2714333	3001422	5595404	1597745	410298	259	75.32
ANP	420649	450188	927000	263246	53062	361	7.02
APF.UN	28465	27533	24789	16342	24724	482	8.91
A	180772	253987	480842	203222	55026	1701	3.08
ATA	70576	59877	85833	24074	37242	686	8.67
ATD.B	98428	88307	122066	23102	53171	436	23.82
AUR	190407	189562	234520	75713	122709	512	24.99
AVM	75106	73591	117689	18177	23057	478	16.29
AVN.UN	193221	259437	466719	229061	41114	321	12.61
AXC	77118	100326	139276	29728	32254	587	38.62

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
AXP	277440	232496	510810	132113	25339	215	19.26
BAM.A	936493	1032390	1850236	514479	153236	217	62.98
BA.UN	66836	86891	85196	42154	60334	375	30.88
BBD.B	143595	125828	85838	45719	154888	1625	4.74
BCB	497552	514391	1074527	368239	41422	239	17.48
BCE	1324147	1136569	2262003	867207	409058	509	35.81
BEI.UN	171855	151150	287911	25317	29913	314	45.25
BFC.UN	31770	38068	47583	10999	16028	424	27.08
BLD	225934	242706	517541	229262	32136	335	6.10
BLE	164521	184594	207199	64395	123268	837	14.81
BMO	511938	451484	695595	185390	231775	316	69.91
BNP.UN	107891	85080	157810	35225	27526	398	30.73
BNS	450787	460287	595371	174527	282012	336	53.16
BPO	858145	812109	1676670	508456	63577	224	40.98
BTE.UN	200540	225582	402455	154172	49917	325	20.81
BVF	907103	895497	1826930	641991	151164	258	26.13
CAE	151035	156493	244338	79045	62838	534	13.01
CAR.UN	47837	58801	81678	30594	20150	415	20.84
CAS	76267	57551	94638	20301	32969	402	12.19
CBF.UN	25770	33250	40281	9736	12259	397	11.75
CCA	130827	131125	235364	29917	17713	230	42.61
CCO	2719999	2870527	5585745	1824608	395903	242	50.78
CCR.UN	44969	40618	54255	22585	21594	405	37.12
CFP	68929	78038	98985	22425	40485	452	11.79
CFW	145227	159056	274084	16587	21491	297	21.03
CGS	81945	72036	120892	24233	27552	429	10.92
CG	87428	80405	120597	24052	35541	435	11.34

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
CIX.UN	85486	87448	134351	19530	29001	327	28.31
CLC.UN	31671	30304	43443	10095	11750	334	14.91
CLL	57443	51781	44027	20041	48490	1013	3.88
CLS	661787	755884	1631193	719166	85536	469	7.29
CM	416467	440611	671519	107519	148880	270	101.58
CMT	200256	199672	377957	182659	59565	404	11.90
CNE.UN	490352	615899	1243838	561399	76363	406	15.41
CNQ	2673179	2632494	5359085	1721361	345356	246	65.53
CNR	1933653	1898573	3796446	1313988	294795	269	54.76
COM	299242	278302	591219	176589	23197	224	11.25
COS.UN	268666	214968	312647	114088	151844	479	29.24
CPG.UN	66858	77670	87071	31246	47013	463	18.69
CP	1023395	964208	1885599	689324	151355	231	69.21
CRW.UN	39611	58271	73180	17613	18412	472	25.16
CSH.UN	47112	45796	52512	14344	33317	625	15.41
CSN	1131975	1088818	2175938	704196	86496	177	46.43
CTC.A	189150	186946	309698	52002	50723	224	76.19
CTL	16817	13195	16001	5866	10077	635	3.57
CUF.UN	26016	29608	37493	13341	12332	378	24.56
CU	130552	150613	247503	41462	24838	209	45.37
CWB	89685	86574	141422	26829	25338	293	24.85
CWI.UN	85721	61264	117076	12359	23141	405	16.37
CWT.UN	36217	35622	43233	17334	22430	405	27.46
DAY.UN	44226	36737	35469	16230	36828	485	10.02
DDV	142910	167157	262906	32667	36087	300	37.59
DHF.UN	23812	33190	39369	16633	11246	360	17.90
DII.B	52336	87380	117250	32516	16798	305	36.51

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
DML	354600	381325	549339	189647	188872	564	14.29
D.UN	44479	49573	66827	20338	20100	352	40.07
DW	41667	55809	75126	10952	15158	379	16.17
DY	63041	82415	74206	52275	62462	2443	4.04
ECA	2851777	2717118	5510441	1507534	436026	318	60.64
EFX.UN	26194	15777	29718	3878	6859	407	10.93
ELD	403781	375826	740032	249970	137482	889	6.69
ELR	49929	54928	37857	18927	55957	3007	2.25
EMA	54324	58516	75323	17583	29612	324	21.02
EME	36704	34013	43828	11463	21058	429	6.30
EMP.A	47557	34362	69946	14566	5983	266	41.60
ENB	429048	420928	768740	313906	124207	287	37.43
EP.UN	27438	25048	33530	6177	14036	292	26.29
EQN	41576	43710	27087	13443	45430	3400	2.49
ERF.UN	539236	651008	1201532	459712	72618	298	49.84
ESI	151594	161489	220858	55711	80035	381	19.95
EXE.UN	46733	55157	82248	10252	14501	539	16.91
FCE.UN	42162	37630	61026	13684	13411	542	10.80
FCP	58054	58611	56614	21863	50081	946	4.96
FDG.UN	903359	962376	1936229	752336	92745	238	27.72
FEL.UN	51361	43262	60825	20564	28598	492	8.86
FET.UN	49720	52034	71026	16089	22575	385	18.19
FFH	186855	296278	476515	260665	14400	163	248.98
FGL	59861	65198	105497	16186	12747	336	21.10
FLY.A	91114	91880	153478	35516	23485	290	23.72
FM	176675	193938	269001	83768	85423	343	77.13
FNX	126843	144549	173188	58234	86387	619	26.89

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
FRG	440419	486673	932317	325429	80396	329	14.46
FRU.UN	30406	32478	43482	11904	13004	349	14.71
FSV	175422	203481	362966	48252	9581	187	32.60
FTS	109258	104324	143542	43246	64371	398	27.93
FTT	147778	181553	269599	41830	46749	287	54.57
GAM	496021	503185	922139	300971	126748	387	17.87
GBU	49023	44076	46795	18048	37926	1133	4.32
GC	15478	24402	26992	5977	9776	443	13.58
GIB.A	172931	160529	283694	115338	73225	692	10.44
GIL	520640	631455	1094421	210817	63058	224	63.14
GMP.UN	50870	57778	91214	9696	10898	422	22.66
GNA	571104	618085	1241933	470590	68846	280	14.67
GO.A	66562	59655	94242	19194	26411	565	16.58
GSC	460487	423312	1089901	450878	58420	870	4.84
G	2993937	3255941	6875682	2847471	525841	419	28.10
GWO	174095	146447	230118	58527	74974	298	35.26
HCG	49303	81854	111093	20132	12992	301	37.01
HPX	63968	81498	100115	19791	35633	528	13.75
HR.UN	47531	41997	46884	19352	33383	385	24.98
HSE	255267	304750	437174	116999	102025	249	82.14
HTE.UN	323899	428629	742119	340897	89752	358	29.76
IAG	131467	141193	230355	31137	31201	249	36.50
IGM	158151	142815	236935	41089	52245	266	52.08
IIC	85005	57308	99366	19275	30618	263	50.08
IMG	583190	547981	1293888	658929	119006	662	8.64
IMN	143160	147668	211666	64035	61675	307	65.84
IMO	629586	631226	1069311	336312	171749	250	44.64

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
IOL	236688	352600	588815	179551	10906	165	32.17
IPL.UN	26637	27548	22114	11626	26021	693	9.41
IPS	1888928	2021099	3798691	1213563	120807	173	152.37
IQW	223153	229008	417771	130524	52699	334	14.85
IVN	752804	742109	1548896	490630	119945	441	13.82
KEY.UN	28140	30512	40690	11521	12606	414	17.90
KFS	215331	209317	396975	123238	43415	293	21.64
KRY	366000	416232	807900	277668	120083	990	4.55
K	1556708	1654896	3570931	1571716	292717	608	15.33
LB	46730	42014	60674	14516	19756	260	33.15
LGY.UN	31310	36901	36647	14658	25665	799	13.67
LIF.UN	35667	38002	53547	12097	13151	349	31.20
LIM	140621	129027	121527	73201	132872	1626	19.78
LNR	55956	61234	92233	22105	16403	278	17.21
L	171044	148811	221638	44892	79846	267	48.53
LUN	905687	921817	1785909	937092	197285	519	13.66
MAE	236097	215499	449932	166567	77657	725	4.99
MB	121177	115786	206036	27181	23265	433	23.14
MBT	99252	112436	153447	31589	46469	279	48.06
MDA	93687	139242	200085	29841	23708	307	48.01
MDS	226588	225251	386848	129710	87578	350	21.64
MFC	1108574	955251	1925499	713280	290504	396	39.83
MFI	71917	112467	150648	21963	26139	317	15.11
MG.A	906636	928336	1718455	417238	105823	177	89.28
MNG	984131	977285	1979077	566543	84170	237	29.32
MRU.A	102083	116887	174677	32008	34822	327	38.11
MTL.UN	85846	97754	151954	15745	24112	373	19.96

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
MX	684765	636813	1225230	360941	132727	250	27.58
NAE.UN	53554	41740	52533	25131	34343	358	12.32
NAL.UN	57119	56775	84664	28686	23632	388	25.78
NA	167237	177388	198636	60817	118020	293	63.84
NBD	68787	65923	66654	27175	58401	563	8.55
NB	60737	66829	103372	16603	15844	216	32.40
NCX	815463	824360	1556574	446084	121518	222	36.95
NG	386909	396189	811752	316173	58760	293	17.36
NGX	663636	546146	1323016	278135	46796	908	3.89
NKO	164737	150391	275611	51072	26969	262	87.00
NNO	478237	441137	951942	249652	87758	917	5.20
NPI.UN	43047	39603	65697	28103	10954	489	13.20
NRM	508182	619617	1086750	302546	67179	312	12.75
NT	1974570	1964405	4176455	1453631	232737	340	28.99
NVA	76477	90993	139714	22132	19435	428	15.01
NWF.UN	27032	25821	33448	12622	12818	367	17.94
NXY	1895120	1879248	3621636	1253182	332415	299	55.82
OCX	96153	108610	130280	37699	61319	328	34.99
OIL	95944	89788	84940	39845	87098	696	9.01
OPC	193663	205758	297322	73733	87653	457	21.64
OTC	615743	598483	1209118	281472	38774	194	25.48
PAA	1255991	1234875	2570414	591404	55957	234	32.36
PBG	154673	162931	236367	64326	68151	476	23.74
PCA	1448765	1417021	2683139	953152	371488	334	47.46
PD.UN	860634	858735	1770497	735053	134591	306	27.82
PEY.UN	66705	88493	103304	42712	45479	365	18.29
PGF.UN	388152	476623	971518	547291	75395	399	19.55

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
PGX.UN	57462	65144	76055	19337	37582	516	13.65
PIF.UN	35684	49069	55203	36251	25172	382	16.11
PJC.A	58998	62853	60298	28591	50789	360	14.94
PMT.UN	71012	60835	84091	41416	40267	453	10.51
PMZ.UN	38864	51208	67272	15786	16063	400	20.54
PNP	154613	165403	247014	159741	67052	496	16.30
POT	1973644	2007009	3812164	1254117	143734	157	165.87
POU	127233	122034	197063	32953	41675	429	21.55
POW	223103	215977	341372	78959	87477	324	38.34
PSI	29062	27415	37612	12274	12893	316	15.32
PTI	26564	25026	27310	11068	17880	435	4.75
PVE.UN	249694	356556	656860	279637	47437	427	12.85
PWF	227848	217086	334740	76228	94188	250	39.85
PWI.UN	206896	280015	455830	130698	48320	288	22.90
PWT.UN	598320	835921	1451833	565128	134362	287	34.76
QBR.B	90333	103105	160678	30826	21437	285	39.66
QLT	363838	355589	740270	146574	30002	270	8.60
RCI.B	804159	817455	1386612	450065	256638	312	41.09
REF.UN	70298	63906	102334	41145	24624	336	31.25
REI.UN	57267	67399	64468	31415	52060	418	25.73
RER	65343	62752	87737	18707	32497	586	10.03
RET.A	65662	91936	134431	21103	15537	271	23.12
RIM	5300654	5249170	10508965	3380361	248700	134	161.11
ROC	139029	150857	249010	26730	29860	263	21.27
RON	99242	94050	131765	26590	48467	362	23.74
RUS	145815	120972	212528	37930	42927	363	30.54
RY	685343	648685	972404	327426	332436	363	58.06

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
SAP	183452	160242	281814	42560	45418	249	44.15
SBY	53393	33094	61903	14477	17376	442	45.43
SCC	10427	15004	20761	8092	1265	460	28.04
SCL.A	137471	164580	260252	43627	30551	279	28.47
SC	121777	128288	138692	46932	91784	308	50.91
SGF	73536	69248	80499	26851	51788	917	6.50
SHN.UN	74882	82752	110350	66713	45311	464	13.51
SIF.UN	42707	54411	72905	8731	18128	535	13.81
SJR.B	392310	348986	644468	252328	101162	274	43.04
SLF	720341	611364	1183748	520105	191570	310	51.61
SLW	686869	854148	1716629	659993	129836	516	12.06
SNC	174269	156540	245872	46744	69422	343	34.77
SSO	946597	1080840	2089559	612710	39048	178	40.27
S	110628	126097	126061	64404	98149	692	15.93
ST.A	3880	4886	5349	1694	1009	862	40.16
STN	213911	210866	397275	87645	25856	277	32.61
SU	3884135	3833051	7747457	1994819	422670	225	88.65
SVM	104610	108654	178028	34154	27014	322	18.81
SVY	111022	122487	191280	30465	39076	411	20.91
SWP	56853	66481	76974	22968	39633	744	8.71
SXR	267727	280382	366765	273398	165405	876	16.54
SYN	105544	110029	171412	21879	35806	531	13.61
TA	218832	208191	321967	123476	93677	363	26.03
TCK.B	1071441	1121864	1861142	627184	311592	318	67.59
TCL.A	69857	90394	136730	26829	15000	284	21.47
TCM	26602	26196	29979	7587	19459	616	16.81
TCW	136508	134522	199909	41232	58510	382	24.24

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
TDG.UN	39460	42108	42845	19332	32046	443	15.65
TD	597731	583711	882792	379841	263633	290	69.53
TEO	276537	310280	566592	156713	24793	183	31.80
TET.UN	55546	49716	67912	16952	28765	435	10.84
THI	600640	569213	1180788	456158	100613	266	35.10
TIF.UN	34713	31163	46191	20349	15435	407	14.27
TIH	42366	46062	64449	12420	16167	324	27.30
TLM	1979097	1872908	4244927	1686765	358522	593	20.71
TNK	129207	159476	271830	90950	37352	822	22.83
TNX	138059	135209	271923	86744	17556	303	6.10
TOC	324300	311456	501488	197825	133753	417	47.47
TPW.UN	32809	22665	35667	9946	14785	571	7.48
TRE	77381	84034	102722	27426	50430	715	12.63
TRP	592249	565562	1088459	493615	166441	342	38.96
TRZ.B	63881	98393	148029	52853	12660	284	35.73
TS.B	45602	56293	79910	15051	14532	278	20.73
T	405535	507515	735789	177681	151093	297	60.88
TUI.UN	37866	32793	31942	15169	30644	746	5.89
TWF.UN	77732	94983	155927	62801	18990	348	17.62
UEX	72710	76626	77870	37146	56453	885	6.72
UTS	109907	109890	77225	39744	117347	1201	4.63
UWH.UN	24288	18828	23613	5964	13029	874	20.22
VET.UN	127358	149690	245894	76221	31356	307	33.15
WFT	100733	111182	193684	31950	9743	234	42.18
WJA	91924	90601	116277	53558	56954	514	15.85
WN	144965	136007	234526	41722	33997	199	73.84
WTE.UN	33549	40691	46589	30135	23644	353	12.27

Equity	Number Sell Orders	Number Buy Orders	Number Cancels	Number Change Requests	Number Trades	Average Trade Volume	Average Price
WTO	207653	207656	298531	77205	102499	558	35.67
X	114441	114053	147673	44619	64906	356	46.18
YLO.UN	89372	96744	69532	45620	100308	536	13.97
YRI	1418636	1267050	2943480	1167991	259845	506	15.94

E Brokerage Data

Brokerage	Number of Trades	Average Trade Volume	Proportion of Trades: Internal	Percentage Gain from Active Internalization
Polar Securities Inc.	5	316.00	-	-
J.C. Clark Ltd.	10	200.20	-	-
Timber Hill Canada Company	11	2281.82	-	-
MCA Securities Inc.	288	289.38	-	-
Trapeze Capital Corp.	197	692.41	-	-
Evergreen Capital Partners Inc.	263	845.25	-	-
CTI Capital Inc.	697	313.82	-	-
Industrial Alliance Securities Inc	684	399.44	-	-
Le Groupe Option Retraite Inc	1,279	282.90	-	-
Norstar Securities Limited Partnership	369	1156.00	-	-
e3m Investments Inc.	588	749.46	-	-
Acker Finley Inc.	1,037	567.70	-	-
Integral Wealth Management	993	620.35	-	-
Interactive Brokers Canada Inc.	2,950	316.88	-	-
Woodstone Capital Inc.	872	1118.49	-	-
Brant Securities Ltd.	1,669	612.41	-	-
Byron Securities Ltd.	3,473	365.72	1.35%	107.57%
Qtrade Securities Inc.	3,395	414.85	0.02%	-1.40%

Brokerage	Number of Trades	Average Trade Volume	Proportion of Trades: Internal	Percentage Gain from Active Internalization
M Partners Inc.	1,400	1068.74	2.85%	20.35%
Maple Securities Canada Ltd.	14,071	101.83	-	-
Man Financial Canada Co.	2,167	676.35	0.23%	-0.98%
Caldwell Securities Ltd.	3,577	433.83	0.12%	-1.32%
Standard Securities Capital Corp.	2,526	685.58	-	-
Berkshire Securities Inc.	5,392	322.96	0.41%	-2.81%
Global Securities Corp.	2,962	645.33	0.01%	-
Gateway Securities Inc.	2,398	873.25	-	-
Fraser Mackenzie Ltd	1,838	1240.28	1.19%	0.93%
Standard Securities Capital Corp.	5,107	445.95	0.19%	34.20%
The Jitney Group Inc.	1,720	1378.60	-	-
Loewen Ondaatje McCutcheon Ltd.	2,231	1196.46	0.96%	9.05%
Toll Cross Securities	2,421	1322.59	0.32%	-18.42%
Northern Securities Inc.	3,918	836.19	0.01%	-0.07%
Leede Financial Markets Inc.	7,521	481.54	0.30%	-
Pictet Canada L.P.	7,321	625.36	-	-
Pope & Company	8,622	553.88	0.05%	85.61%
Laurentian Bank Securities Inc.	12,127	436.18	0.29%	57.04%
Versant Partners Inc	6,075	909.98	0.16%	-1.70%
Pollitt & Co. Inc.	10,028	682.52	0.01%	100.44%

Brokerage	Number of Trades	Average Trade Volume	Proportion of Trades: Internal	Percentage Gain from Active Internalization
Infinium Capital Corp.	23,981	246.82	0.33%	128.64%
Clarus Securities Inc.	8,322	932.63	0.56%	-19.65%
Bolder Investment Partners Ltd.	9,227	797.88	-	-
Wolverton Securities Ltd.	10,388	692.25	0.29%	443.49%
Octagon Capital Corporation	11,958	705.21	0.36%	88.27%
MGI Securities Inc.	10,862	797.73	0.55%	13.89%
MacDougall MacDougal & MacTier Inc.	21,983	398.67	0.04%	105.01%
Jennings Capital Inc.	13,034	861.47	1.47%	-0.70%
Salman Partners Inc.	12,360	1259.32	0.02%	-66.10%
Odlum Brown Ltd.	38,436	334.65	0.24%	39.91%
Pacific Int'l Securities Inc.	14,185	958.97	0.11%	29.44%
Quest Capital Group Ltd.	23,609	588.59	0.29%	187.21%
Dominick & Dominick Securities Inc.	23,495	653.54	0.23%	129.60%
Maison Placements Canada Inc.	17,758	860.41	0.29%	50.76%
Jones Gable & Company Ltd.	38,404	398.55	1.31%	74.64%
Brockhouse & Cooper Inc.	49,718	315.93	0.38%	29.86%
Union Securities Ltd.	29,275	716.77	0.17%	21.56%
Westwind Partners Inc.	21,959	1113.70	0.78%	58.59%
Research Capital Corp.	49,592	564.23	0.88%	63.39%
The Jitney Group Inc.	75,027	420.06	0.65%	35.08%

Brokerage	Number of Trades	Average Trade Volume	Proportion of Trades: Internal	Percentage Gain from Active Internalization
Wellington West Capital Markets	30,863	1210.28	0.69%	-4.72%
Tristone Capital Inc.	48,913	801.54	1.27%	17.26%
Instinet Canada Ltd.	86,857	484.72	0.06%	57.92%
Independent Trading Group	209,321	197.10	2.70%	87.82%
State Street Global Markets Canada Inc.	84,005	553.93	0.36%	4.87%
FirstEnergy Capital Corp.	75,320	662.60	1.20%	15.94%
Citigroup Global Markets Canada	106,630	472.75	1.67%	14.21%
Haywood Securities Inc.	47,576	1097.32	1.47%	10.67%
Peters & Co. Ltd.	66,167	844.46	1.44%	22.52%
Paradigm Capital Inc.	58,058	1099.68	1.14%	9.09%
HSBC Securities (Canada) Inc.	138,290	483.35	0.35%	85.81%
Commission Direct Inc.	137,051	592.89	0.85%	13.70%
Penson Financial Services Canada, Inc.	195,738	422.53	0.50%	90.82%
Hampton Securities Ltd.	235,463	418.27	2.53%	93.55%
Morgan Stanley Canada Ltd.	263,989	380.89	1.25%	21.04%
Blackmont Capital Inc.	139,422	819.22	0.98%	12.76%
W.D. Latimer Co. Ltd.	370,428	306.03	2.31%	72.71%
Goldman Sachs Canada Inc.	439,685	340.88	1.01%	40.72%
Cormark Securities Inc./ Valeurs Mobilieres Cormark Inc.	147,870	1055.00	0.93%	10.31%
Genuity Capital Markets	353,307	446.71	1.24%	15.59%

Brokerage	Number of Trades	Average Trade Volume	Proportion of Trades: Internal	Percentage Gain from Active Internalization
Dundee Securities Corp.	187,248	881.42	1.02%	20.23%
Orion Securities Inc. Valeurs mobilires Orion Inc.	320,782	546.55	1.47%	39.30%
Raymond James Ltd.	282,647	754.51	2.15%	13.80%
Credit Suisse Securities (Canada), Inc.	791,908	320.97	2.10%	21.07%
FIMAT Derivatives Canada Inc.	1,105,552	235.97	0.80%	53.50%
E*TRADE Canada Securities Corp.	870,076	347.15	2.65%	110.22%
GMP Securities Limited	216,158	1566.93	1.15%	11.78%
Desjardins Securities Inc.	825,314	439.75	2.82%	49.85%
Canaccord Capital Corp.	508,475	735.22	2.72%	31.23%
UBS Securities Inc./ UBS Valeurs Mobilires Canada Inc.	629,062	708.95	1.56%	17.19%
Merrill Lynch Canada Inc.	1,277,961	453.34	2.17%	24.85%
Scotia Capital Inc.	921,014	695.48	2.36%	21.71%
ITG Canada Corp.	2,689,043	301.40	4.60%	62.75%
National Bank Financial Inc.	1,473,160	574.20	3.95%	57.27%
BMO Nesbitt Burns Inc.	2,805,509	493.11	4.09%	33.11%
CIBC World Markets Inc.	3,051,158	454.48	4.69%	31.35%
RBC Capital Markets	4,051,635	468.65	6.45%	29.04%
TD Securities Inc.	3,847,824	513.67	7.04%	39.54%
Anonymous	13,367,108	325.94	-	-

F Injection Statistics

In this appendix we list results from our studies on quantifying internalization, over a large number of brokers. Among this enlarged set of brokers, no brokerage had a benefit that was more than 0.1426%, while no broker was penalized by more than 0.023%.

ID	Broker	Ave	Max	Min	Volume
1	Anonymous	0	0	0	NA
2	RBC Captial Markets	-0.00378	-0.03262	0.01431	2.15E+09
3	Tristone Capital Inc.	-0.00001	-0.00065	0.00011	2.90E+07
4	Versant Partners Inc.	-0.00001	-0.00291	0.00005	6.45E+06
5	Penson Financial Services Canada Inc. Services Financiers Penson Cda Inc.	-0.00019	-0.01482	-.00168	6.25E+07
7	TD Securities Inc.	-0.00393	-0.14254	0.01354	2.7E+09
8	Maple Securities Canada Ltd.	-0.00004	-0.00287	0.0000	1.32E+06
9	BMO Nesbitt Burns	-0.00281	-0.05386	0.02241	2.39E+09
14	ITG Canada Corp.	-0.00671	-0.10040	0.00407	1.73E+09
15	UBS Securities Inc. UBS Valeurs Mobilieres Canada Inc.	-0.00042	-0.00943	0.00880	5.8E+08
19	Desjardins Securities Inc.	-0.00066	-0.2497	0.00539	3.49E+08
33	Canaccord Capital Corp.	-0.00066	-0.01789	0.00629	4.4E+08
36	W. D. Latimer Co. Ltd.	-0.00008	-0.00412	0.00159	7.76E+07
39	Merrill Lynch Canada Inc.	-0.00150	-0.02237	0.00901	6.11E+08
53	Morgan Stanley Canada Ltd.	-0.00016	-0.00376	0.00315	1.04E+08
65	Goldman Sachs Canada Inc.	-0.00076	-0.02033	0.01836	3.19E+08
69	Citigroup Global Markets Canada	-0.00005	-0.00289	0.00138	3.63E+07
73	Cormark Securities Inc. Valeurs Mobilieres Cormark Inc.	-0.00021	-0.00876	0.00103	3.01E+08

ID	Broker	Ave	Max	Min	Volume
74	GMP Securities Ltd.	-0.00025	-0.01470	0.00182	1.2E+09
79	CIBC World Markets Inc.	-0.00358	-0.03237	0.01308	3.98E+09
80	National Bank Financial Inc.	-0.00155	-0.04125	0.01595	9.92E+08
85	Scotia Captial Inc.	-0.00154	-0.02728	0.00682	7.78E+08
88	ETRADE Canada Securities Inc.	-0.00051	-0.00594	0.00660	5.63E+08
89	Raymond James Ltd.	-0.00048	-0.01172	0.00202	2.21E+08
90	Lehman Brothers Canada Inc.	0.0000	0.0000	0.0000	2.21E+08
101	FIMAT Derivatives Canada Inc.	-0.00068	-0.01599	0.00211	5.28E+08