

## **An Investigation on Sieve and Detour Effects Affecting the Interaction of Infrared Radiation with Plant Leaves**

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### **Abstract:**

The retrieval of plant biophysical and biochemical properties from high spectral resolution data represents an active area of research within the remote sensing field. Scientific studies in this area are usually supported by computational simulations of light attenuation processes within foliar tissues. In heterogeneous organic materials, like plant leaves, sieve and detour effects can affect these processes, and ultimately change the light gradients within these tissues and their spectral signatures. Although these effects have been extensively examined for applications involving the interactions of visible radiation with plant leaves, little is known about their role in the infrared domain. In this paper, we describe the procedural basis for their incorporation in the modeling of infrared radiation transport (in the range of 750 to 2500 $nm$ ) within plant leaves. We also assess their impact on the predictability of simulation solutions relating the directionality of the incident radiation and the internal arrangement of the tissues to changes on foliar spectral signatures in this domain. Our investigation is grounded by observations involving modeled results and quantitative and qualitative data reported in the literature.

### I. INTRODUCTION

The measurement and modeling of foliar optical properties are object of extensive research in remote sensing and related fields. As a result, a substantial number of models have been developed to simulate light interactions with plant leaves [1], with potential target applications ranging from the estimation of foliar biophysical parameters through inversion procedures [2] to the analysis of plant responses to environmental changes [3]. The key task performed by these models involves the simulation of light attenuation within the foliar tissues, which causes a reduction in the internal photon fluence rate. This reduction, known as light gradient [4], is associated with variations in the penetration depth [5] and trapping of the incident light [6]. Accordingly, its study and modeling can provide valuable tools for the analysis of changes in the spectral signature of plants due to nutrient and water stress [7], and enhance the current understanding of photobiological phenomena such as photosynthesis, phototropism and photomorphogenesis [8], [9].

The foliar light gradient is affected by both the directional quality of the incident light [9], [10], [11], which enters the leaf either through collimated radiation (*e.g.*, direct sunlight

[12]) or diffuse radiation (*e.g.*, skylight [12]), and the foliar structural and biophysical characteristics, which determine the degree of attenuation of light being propagated within the leaf. Although it is possible to estimate the probability of a photon being attenuated within a foliar tissue either due to scattering or absorption, it is important to note that these two processes are intertwined [13]. Hence, for modeling purposes, the computation of the absorption probability needs to account for structural factors affecting the propagation of light under *in vivo* conditions. Due to the nonhomogeneous distribution of absorbers within the foliar tissues light can be propagated without encountering these foliar constituents, a phenomenon known as sieve effect [14], [15], [4]. Alternatively, when the propagated light encounters an absorber, its direction of propagation can be changed, a phenomenon known as detour effect [14], [15], [4]. These phenomena have opposite influences on the absorption of light within the foliar tissues: sieve effect reduces the probability of light absorption within these tissues, while detour effect enhances it [14], [13], [16].

Detour and sieve effects have been extensively studied and measured with respect to the propagation of visible light in foliar tissues and its interaction with photosynthetic pigments [17], [15], [18], [19], [20]. However, to the best of our knowledge, quantitative data regarding the role of these effects in interactions of infrared radiation with these tissues are not readily available in the scientific literature. Although this aspect may represent a temporary hurdle, we believe that it must not prevent the advance of research in this area. We remark that investigations involving foliar light regime in the near (700 to 1300 $nm$ ) and middle (1300 to 2500 $nm$ ) infrared regions [21] are central to the characterization of leaf constituents of high value from an industrial and agricultural standpoint, such as cellulose, starch, lignin and protein [7], [21], and to the assessment of water stress in regions of vegetation [22], [23]. In this paper, we examine the algorithmic simulation of these effects in the infrared domain from 750 to 2500 $nm$ , henceforth referred to as the investigated spectral domain. More specifically, we target modeling efforts relating the directionality of incident infrared radiation in this domain to variations of foliar photon transport patterns. We also address aspects affecting the quantitative accuracy of such simulations and discuss future prospects in this area.

## II. PROCEDURAL SIMULATION OF SIEVE AND DETOUR EFFECTS

Recently, Baranoski [24] has proposed two models, ABM-B and ABM-U, to algorithmically simulate the interaction of infrared radiation (in the range of 750 to 2500 $nm$ ) with bifacial and unifacial plants leaves respectively. Bifacial leaves are characterized by a differentiated mesophyll tissue. In these leaves, this tissue is usually composed of one or more layers of palisade cells, and a loosely packed layer of spongy cells [25]. Unifacial leaves, on the other hand, are characterized by an undifferentiated mesophyll tissue. In monocotyledon species with unifacial leaves, this tissue visually resembles the spongy layer of bifacial leaves, but with a smaller portion of its volume occupied by air [26], [27].

In this section, for the sake of completeness, we describe how sieve and detour effects were incorporated in the absorption testing performed by the ABM-B and ABM-U. However, the

following description addresses mainly unpublished issues related to the procedural simulation of these effects. For general information about these models, the reader is referred to the publication by Baranoski [24].

The infrared absorption testing performed by the ABM-B and ABM-U is based on a modified Beer-Lambert law which geometrically accounts for changes on the optical pathlength due to sieve and detour effects. These changes are often quantified in terms of a factor of intensification [17] or apparent scattering pathlength [19] in remote sensing related applications, and differential pathlength factor in biomedical applications [28]. As mentioned earlier, although these changes on the optical path have been investigated with respect to the presence of photosynthetic pigments, such as chlorophyll and carotenoids, which are relevant in the visible domain, to the best of our knowledge neither its quantification with respect to foliar absorbers acting in the investigated spectral domain nor a unified theory of sieve and detour effects is currently available in the literature. For this reason, the sieve and detour effects were incorporated in the ABM-B and ABM-U using a nondeterministic approach based on qualitative information inferred from available data.

In the ABM-B and ABM-U, light transport is simulated as a random walk process whose states correspond to the main tissue interfaces found in bifacial and unifacial leaves respectively. The transition probabilities of this random walk are associated with the Fresnel coefficients computed at each interface, and the termination probabilities are associated with the free path length of the ray (photon) traveling in the mesophyll tissue. Once a slant ray at a certain wavelength  $\lambda$  enters the mesophyll tissue, it may be propagated or absorbed. The absorption probability is given by:

$$\mathcal{P}(\lambda) = 1 - \epsilon^{-\mu_a(\lambda) \frac{h}{\cos \theta}}, \quad (1)$$

where  $\mu_a(\lambda)$  represents the effective absorption coefficient of the medium,  $h$  corresponds to the medium thickness and  $\theta$  represents the angle between the ray direction, after being adjusted to account for the detour effect as described below, and the medium's normal direction. If  $\mathcal{P}(\lambda) < \xi$ , where  $\xi$  is a random number uniformly distributed in the interval  $[0..1]$ , then the ray is considered to be absorbed. Otherwise, it is assumed that the ray continues its run through the mesophyll tissue without triggering absorption events.

We remark that the direction of propagation of a ray traveling in the mesophyll tissue can be altered due to geometrical scattering caused by tissue structures larger than the wavelength of the light. In the ABM-B and ABM-U, this deviation is represented by the angle  $\theta_s$  between the medium's normal direction and the ray's altered trajectory, which in turn is obtained by perturbing its direction of propagation using a warping function that takes into account the aspect ratio of the medium's cells [24]. Note that regardless of the magnitude of  $\theta_s$ , it is assumed that is probabilistically possible that the ray goes through the mesophyll tissue without triggering absorption events, *i.e.*, the sieve effect is independent of the geometrical scattering. This assumption is consistent with the general theory of sieve effect [14], [29].

From photometric experiments on bifacial and unifacial plant leaves reported in the literature [10], [13], it can be observed that as the angle between the medium's normal and the direction of propagation of the light traveling in the mesophyll tissue increases, the photon

flux decreases, *i.e.*, there is a noticeable increase in the medium's absorption efficiency (detour effect). Conversely, as the angle decreases, the photon flux increases, *i.e.*, there is a noticeable decrease in the absorption efficiency (sieve effect). This aspect illustrates not only that detour and sieve effects may be inversely related as suggested by their opposite dependence on the distribution of the absorbers, but also that this relationship may be correlated with the angular distribution of the slant rays traveling in the mesophyll tissue.

It is important to note that the above cited measurements were performed for a selected number of wavelengths (below  $750nm$ ). It is assumed, however, that the qualitative observations related to the spatial distribution of the propagated photons can be extended to the investigated spectral domain. This assumption is supported by studies performed by Seyfried et al. [30] involving plant tissue spectrophotometric measurements from  $350$  to  $1000nm$ . These studies indicated that incident collimated light quickly becomes diffuse when it penetrates the foliar tissues, and although the geometrical pathlength is small for small angles of penetration, the optical pathlength is large. Furthermore, experiments by Okada et al. [31] on the estimation of the optical pathlength of collimated light propagating at  $800nm$  in inhomogeneous tissues provide an upper bound for the angular deviation due to light interaction with absorbers. This bound was also incorporated in the procedural simulation of sieve and detour effects. It corresponds to approximately one radian for phantoms (objects resembling organic materials in mass, composition, and dimensions, which are used in biomedical investigations on the absorption of radiation within living tissues) with absorptive and scattering characteristics similar to plant tissue compounds.

In order to account for the sieve and detour effects inverse angular relationship described above and avoid undue complexity, the ABM-B and ABM-U make use of a simple formula to obtain  $\theta$ . According to this formula, the value of  $\theta$  in radians corresponds to the cosine of  $\theta_s$ , *i.e.*, the higher the angle  $\theta_s$ , the lower the angle  $\theta$  (between zero and one radian), and consequently the lower the probability of absorption (Equation 1).

### III. EXPERIMENTS AND DISCUSSION

Quantitative comparisons performed by Baranoski [24] showed good quantitative agreement between measured data and modeled results provided by the ABM-B and ABM-U. This aspect suggests that the accuracy of the procedural simulation of foliar sieve and detour is within acceptable limits since the modeled curves presented root mean square errors smaller than 0.03, which indicates good spectrum reconstruction according to Jacquemoud et al [32]. Although data scarcity prevents a more comprehensive quantitative analysis of these simulations, their effects on the predictability of the modeled results are further examined in this section with respect to the relationship between the directional quality of the incident light (collimated or diffuse) and differences in leaf anatomy (bifacial or unifacial). For these experiments we selected a bifacial dicotyledon soybean leaf and a monocotyledon unifacial corn leaf, and used the the same characterization data provided by Baranoski [24].

In order to facilitate the comparison of modeled results with observations reported in the literature, the corn specimen's thickness was changed to have the same value used for the

soybean specimen, namely  $0.0166\text{cm}$ . This value represents a 20% reduction in its original thickness which is accompanied by a 20% increase in the aspect ratio of its epidermal and mesophyll cell caps [25], [24]. We also remark that both models take into account the same number of optical interfaces (six), and we used the same refractive indices for the cutinized epidermal cell wall of both specimens [24].

Spectrophotometric measurements performed by Walter-Shea et al. [33] on corn and soybean leaves (considering directional light incident on the adaxial surface of these specimens) show that as the angle incidence with respect to the leaf's normal increases, the reflectance of light by monocotyledon (corn) and dicotyledon (soybean) specimens increases and the transmittance decreases. Since diffuse incident light includes photons from all directions, *i.e.*, small and large angles of incidence, one can expect for both specimens higher reflectance values when diffuse incident light is employed, and higher transmittance values when the specimens are illuminated by collimated light. As can be observed in the graphs presented in Figures 1 and 2, which were obtained considering diffuse and collimated ( $2.5^\circ$ ) incident light, the simulations succeeded in predicting these spectral variations with respect to the directionality of the incident light. Furthermore, according to Verdebout et al. [21], for the same thickness, monocotyledons whose mesophyll cells are more uniformly distributed have a lower near-infrared reflectance and a higher near-infrared transmittance than dicotyledons which have a palisade and spongy mesophyll. This statement was also confirmed by the spectrophotometric reflectance and transmittance measurements performed by Walter-Shea et al. [33]. The stochastic simulations described above can also predict this behavior as illustrated by the modeled reflectance and transmittance curves for directional (collimated) incident light presented in Figures 1 and 2 respectively.

These spectral variations have been explained in terms of the more lacunous structure of the dicotyledon leaf, which allows for greater amount of geometrical scattering [34], [21]. According to studies by Vogelmann and Björn [6] on the light trapping role of these cavities, the various optical boundaries within the foliar tissues control the penetration and distribution of light within these tissues, and can cause photons to bounce back and forth between different tissue layers before being either absorbed or propagated back to the environment. Accordingly, in the ABM-B and ABM-U, a ray may also interact few times with the different foliar optical interfaces. The graphs presented in Figure 3 illustrate the pattern of interactions that collimated rays at  $1000\text{nm}$  have to go through before being absorbed within the bifacial (soybean) and unifacial (corn) specimens' mesophyll tissue. As can be observed in these graphs, although a large proportion of the absorbed rays stop after two interactions, *i.e.*, after starting their first run in the mesophyll tissue, the number of interactions decreases asymptotically as expected from an optical system prone to a certain degree of light trapping.

As observed by several researchers [14], [11], [18], [13], [4], sieve and detour effects have a direct impact on the visible light attenuation processes that take place within the plant leaves and affect their spectral signatures in this domain. The modeled curves presented in Figures 4 to 7 suggest that a similar premise may be valid for the investigated spectral domain since without properly accounting for the relationship between sieve and detour effects (*e.g.*, using  $\theta$  equal  $\theta_s$  in Equation 1) there is a noticeable quantitative difference in

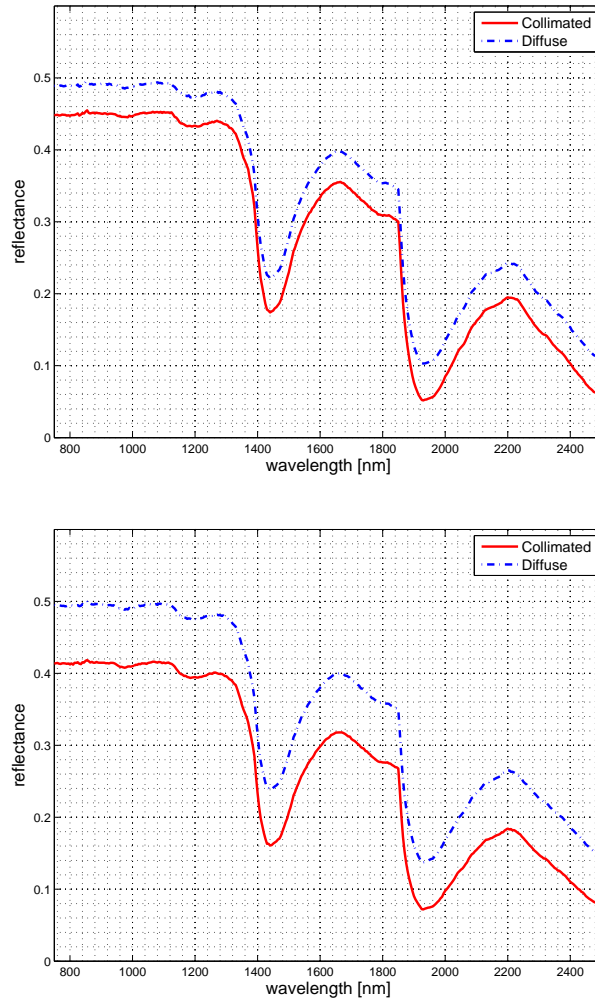


Fig. 1. Modeled *reflectance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained considering diffuse and collimated ( $2.5^\circ$ ) light incident on the specimens' adaxial surface.

the results provided by the ABM-B and ABM-U. In addition, the modeled curves presented in Figures 4 to 7 also indicate that the net result of the procedural simulation of sieve and detour effects is a reduction of absorption efficiency for most part of the investigated spectral domain. This aspect may be explained by the highly heterogeneous distribution of absorbers interacting with radiation in the investigated spectral domain [14].

#### IV. CONCLUDING REMARKS

The relationship between attenuation processes, the directionality of the incident light, the internal arrangement of the tissues and sieve and detour effects has been fairly documented

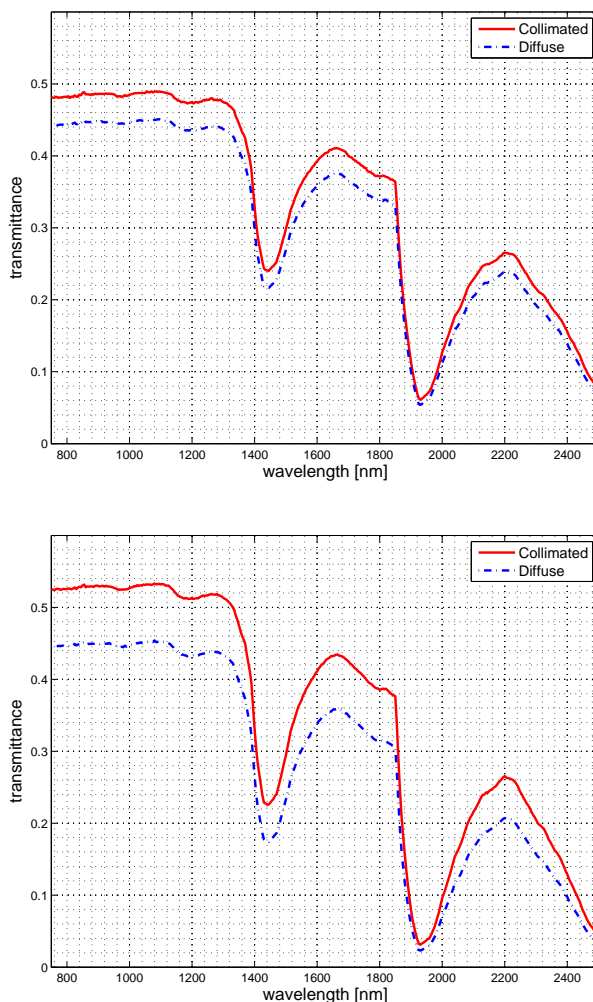


Fig. 2. Modeled *transmittance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained considering diffuse and collimated ( $2.5^\circ$ ) light incident on the specimens' adaxial surface.

for applications involving the visible domain. However, as mentioned earlier, reports addressing this relationship in the infrared domain are scarce in the literature. For this reason, certain aspects observed in the visible domain could not be incorporated in the procedural simulation of sieve and detour effects in the infrared domain (in the range of  $750$  to  $2500\text{nm}$ ). For example, in studies involving the visible domain, it was observed that variations on the optical pathlength have a wavelength dependence and they are also correlated to the absorbers' concentration [18]. Clearly, there is a lack of quantitative evidence to confirm these dependences with respect to compounds that attenuate light in the investigated spectral domain. Viewed in this context, this work also aims to highlight the need for actual measured data to support further research on the interaction of infrared

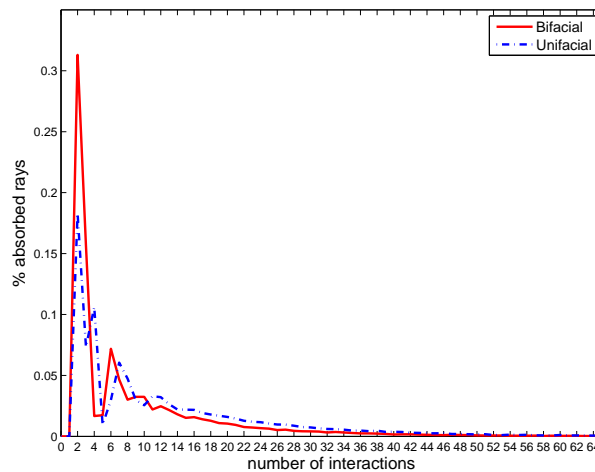


Fig. 3. Number of interactions that rays traveling at  $1000\text{nm}$  have to go through before being absorbed within the bifacial (soybean) and unifacial (corn) specimens' mesophyll tissue. Measurements were performed considering an angle of incidence of  $2.5^\circ$ .

radiation with plant leaves. However, we believe that this circumstantial scarcity of data should not prevent investigations in this area. After all, oftentimes theories and hypotheses are the required catalysts to fundamental experimental research [35].

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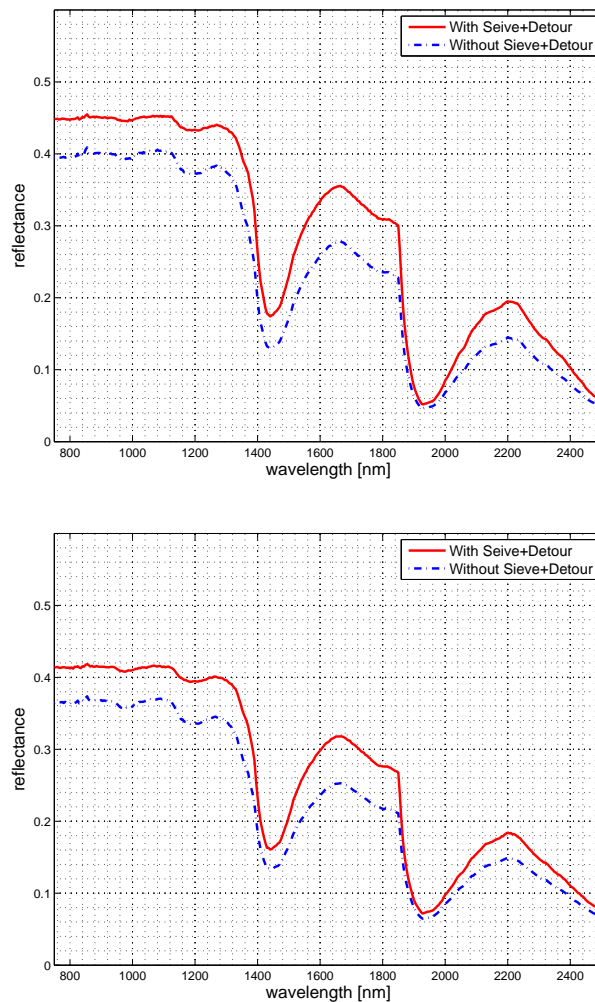


Fig. 4. Modeled *reflectance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained with and without the procedural simulation of sieve and detour effects. Measurements were performed considering *collimated* light ( $2.5^\circ$ ) incident on the specimens' adaxial surface.

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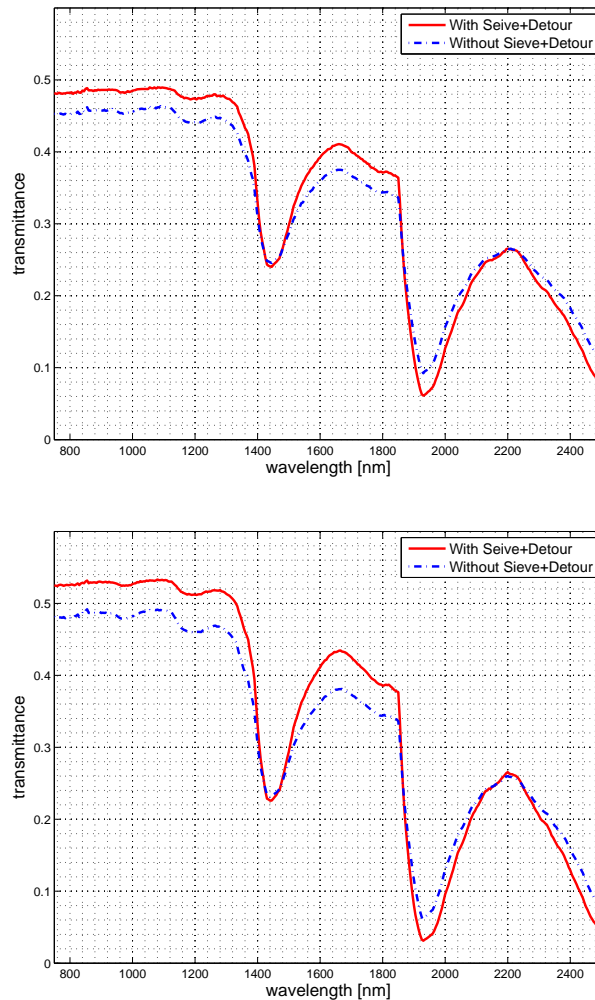


Fig. 5. Modeled *transmittance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained with and without the procedural simulation of sieve and detour effects. Measurements were performed considering *collimated* light ( $2.5^\circ$ ) incident on the specimens' adaxial surface.

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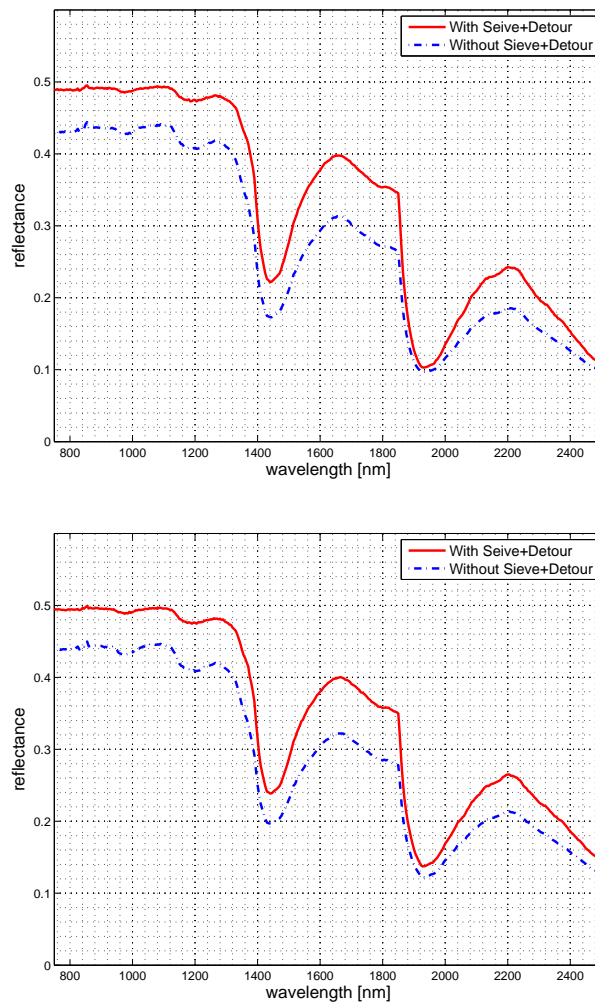


Fig. 6. Modeled *reflectance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained with and without the procedural simulation of sieve and detour effects. Measurements were performed considering *diffuse* light incident on the specimens' adaxial surface.

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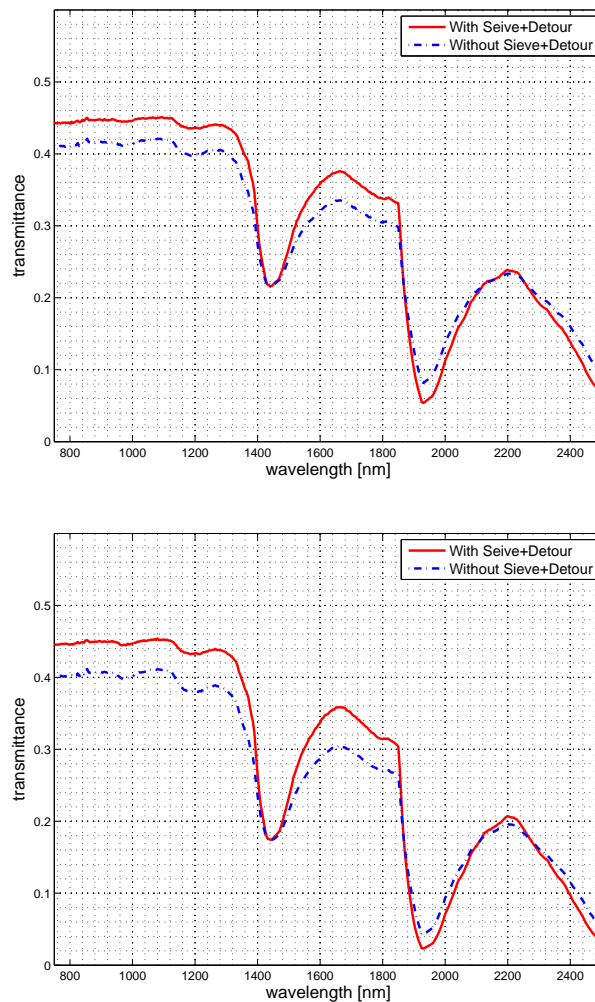


Fig. 7. Modeled *transmittance* curves for a soybean leaf (top) and a corn leaf (bottom) obtained with and without the procedural simulation of sieve and detour effects. Measurements were performed considering *diffuse* light incident on the specimens' adaxial surface.

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