Abstract.
The turning point problem
\[
\begin{cases}
-\varepsilon \Delta u + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0 \quad (x, y) \in [(-1, 1) \times (-1, 1)] \\
u(-1, y) = V_a, \quad u(1, y) = V_b, \\
u(x, -1) = V_c, \quad u(x, 1) = V_d,
\end{cases}
\]
is known to have some extremely small eigenvalues. No successful numerical solution to this problem has been reported. In this paper, a numerical procedure is proposed. All four boundary layers are well defined and the numerical singularity is successfully removed.

Key words. boundary layer, domain decomposition, overlap, Schwarz Alternating Method (SAM), turning point problem

AMS(MOS) subject classifications. 65F10, 65N20

1. Introduction. The singularly perturbed elliptic partial differential equation
\[
-\varepsilon \Delta u + a(x, y) \frac{\partial u}{\partial x} + b(x, y) \frac{\partial u}{\partial y} + k(x, y)u = 0
\]
has been extensively studied by many researchers. When \( \varepsilon \to 0 \), the solution to this problem becomes difficult. A typical property of the solution is the existence of boundary layers. This problem has many important applications including the solution of the Navier-Stokes equations and stochastic differential equations [13].

In (1), the lower order operator represents the deterministic flow field while the second order part represents a slow diffusion of particles. Therefore, the results will depend on the nature of the underlying flow. In [13], Matkowsky classifies the singularly perturbed elliptic boundary value problem into three cases according to how the particles are diffusing (see Fig. 1):

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* This research was supported by the Natural Sciences and Engineering Research Council of Canada.
† Department of Computer Science, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.