Dear Sirs,

We would be interested in receiving one copy of the following Technical Report:

- "NEW ALGORITHM FOR PATTERN MATCHING WITH OR WITHOUT MISMATCHES"
  Ricardo A. Baeza-Yates, Gaston H. Gonnet
  CS-88-37

Could you, please, send the document and invoice to:

- LABORATOIRES DE MARCOUSSIS
  Service DOCUMENTATION CENTRALE
  Route de Nozay
  91480 MARCOUSSIS - FRANCE

We thank you in advance for your cooperation.

Yours faithfully,

[Signature]

S. DESMOUSSEAUX (Mme)
**New Algorithm for Pattern Matching with or without Mismatches**

**CS-88-37**

**DATE REQUISITIONED:** March 27/89  
**DATE REQUIRED:** ASAP  
**ACCOUNT NO.:** 126631741

**REQUISITIONER:** G. Gonnet  
**PHONE:** 4460  
**DEPT.:** C.S.  
**Bldg. Room:** DC 2314

**MAILING INFO:**  
**NAME:** Sue DeAngelis  
**Bldg. Room:** DC 2314

**Copyright:** I hereby agree to assume all responsibility and liability for any infringement of copyrights and/or patents granted as a result of the processing of, and reproduction of, any of the materials herein requested. I further agree to indemnify and hold the University of Waterloo harmless from any liability which may arise from said processing or reproducing. I also acknowledge that any materials processed as a result of this requisition are for educational use only.

**NUMBER OF PAGES:** 14  
**NUMBER OF COPIES:** 50

**TYPE OF PAPER STOCK:** Alpac Ivory  
**PAPER SIZE:** 10 x 14 Glosscoat  
**PAPER COLOUR:** Black  
**PRINTING:** 7 x 10 saddle stitched  
**NUMBERING:**

<table>
<thead>
<tr>
<th>NEGATIVES</th>
<th>QUANTITY</th>
<th>OPER. NO.</th>
<th>MACH. NO.</th>
<th>MACH. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F, L, M</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PMT:**

- |  
- |  
- |  

**PLATES:**

- |  
- |  
- |  

**STOCK:**

- |  
- |  
- |  

**BINDERY:**

- |  
- |  

**OUTSIDE SERVICES:**

- |  

**TAXES:**

- **PROVINCIAL**  
- **FEDERAL**

---

**Special Instructions**

- **Beaver Cove**
  - Both cover and inside in black ink please

---

**COPY CENTRE**

- OPER. NO.:  
- MACH. NO.:  
- MACH. NO.:  
- MACH. NO.:  

**DESIGN & PASTE-UP**

- OPER. NO.:  
- TIME:  
- LABOUR CODE:  

- OPER. NO.:  
- TIME:  
- LABOUR CODE:  

- OPER. NO.:  
- TIME:  
- LABOUR CODE:  

**TYPESetting**

- QUANTITY:  

| PAP00000 | T01  
| PAP00000 | T01  
| PAP00000 | T01  

**PROOF**

- |  
- |  
- |  

---

**$ COST:**
**New Algorithm for Pattern Matching with or without Mismatches**  
CS-88-37

<table>
<thead>
<tr>
<th>DATE REQUISITIONED</th>
<th>DATE REQUIRED</th>
<th>ACCOUNT NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept. 30/88</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**REQUISITION-PRINT**  
G. Gonnet  
4460

**MAILING INFO**  
NAME: Sue DeAngelis  
DEPT: C.S.  
BLDG & ROOM NO.: DC 2314

**Copyright:** I hereby agree to assume all responsibility and liability for any infringement of copyrights and/or patent rights which may arise from the processing of, and reproduction of, any of the materials herein requested. I further agree to indemnify and hold harmless the University of Waterloo from any liability which may arise from said processing or reproducing. I also acknowledge that materials processed as a result of this requisition are for educational use only.

<table>
<thead>
<tr>
<th>NUMBER OF PAGES</th>
<th>NUMBER OF COPIES</th>
<th>TYPE OF PAPER STOCK</th>
<th>QUANTITY</th>
<th>OPER. NO.</th>
<th>TIME</th>
<th>LABOUR CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>50</td>
<td>Alpac Ivory</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PAPER SIZE**  
8½ x 11  
10x14 Glosscoat  
10 pt Holland Tint

**Special Instructions**  
Beaver Cover  
Both cover and inside in black ink please

<table>
<thead>
<tr>
<th>NEGATIVES</th>
<th>OPER. NO.</th>
<th>TIME</th>
<th>LABOUR CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLM</td>
<td></td>
<td></td>
<td>C,0,1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NUMBERS</th>
<th>OPER. NO.</th>
<th>TIME</th>
<th>LABOUR CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLM</td>
<td></td>
<td></td>
<td>C,0,1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PMT</th>
<th>OPER. NO.</th>
<th>TIME</th>
<th>LABOUR CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMLT</td>
<td></td>
<td></td>
<td>C,0,1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PLT</th>
<th>OPER. NO.</th>
<th>TIME</th>
<th>LABOUR CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMLT</td>
<td></td>
<td></td>
<td>C,0,1</td>
</tr>
</tbody>
</table>

**COPY CENTRE**  
OPER. NO.  
BLDG. NO.  
MACH. NO.

**DESIGN & PASTE-UP**  
OPER. NO.  
TIME  
LABOUR CODE  
D,0,1

**TYPESETTING**  
OPER. NO.  
TIME  
LABOUR CODE  
D,0,1

**PROOF**  
OPER. NO.  
TIME  
LABOUR CODE  
D,0,1

**BINDERY**  
RNG  
B,0,1

**OUTSIDE SERVICES**

**TAXES - PROVINCIAL [ ] FEDERAL [ ]**

**GRAPHICSERV. OCT. 85 482-2**
New Algorithm for Pattern Matching with or without Mismatches

Ricardo A. Baeza-Yates
Gaston H. Gonnet

Data Structuring Group
Research Report
CS-88-37

September, 1988
New Algorithm for Pattern Matching with or without Mismatches

Ricardo A. Baeza-Yates
Gaston H. Gonnet

Data Structuring Group
Department of Computer Science
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

Abstract

We introduce a family of simple and fast algorithms for solving the classical string matching problem, string matching with don't care symbols and complement symbols, and multiple patterns. We also solve the same problems allowing up to k mismatches. Among the features of these algorithms is that they are real time algorithms, that they don't need to buffer the input, and that they are suitable to be implemented in hardware.

1 Introduction

The string matching problem consists of finding all occurrences of a pattern of length m in a text of length n. We generalize the problem allowing don't care symbols, the complement of a symbol, and any finite class of symbols. We solve this problem for one or more patterns, and with or without mismatches. For small patterns the worst case time is linear on the size of the text.

For example, if we want to search for references to any CS report we can use the pattern [Cc][S's] - [6 - 8][0 - 9].

The main idea is to represent the state of the search as a number, and each search step costs a small number of arithmetic/logical operations, provided that the numbers are big enough to represent all possible states of

*The work of the first author was supported by a scholarship from the Institute for Computer Research of the University of Waterloo and by the University of Chile and that of the second author by a Natural Sciences and Engineering Research Council of Canada Grant No. A-3353.
the search. Hence, for small patterns, we have a $O(n)$ time algorithm using $O(|\Sigma|)$ extra space and $O(m + |\Sigma|)$ preprocessing time, where $\Sigma$ is the alphabet.

For string matching, empirical results show that the new algorithm compares favourably with the Knuth-Morris-Pratt (KMP) algorithm [11] for any pattern length and the Boyer-Moore (BM) algorithm [4] for short patterns. For patterns with don't care symbols and complement symbols, this is the first practical and efficient algorithm in the literature; generalizing this to any finite class of symbols or their complement.

The main properties of this class of algorithms are:

- Simplicity: the preprocessing and the search are very simple, and only bitwise logical operations, shifts and additions are used.

- Real time: the time delay to process one text character is bounded by a constant.

- No buffering: neither the text nor the pattern need to be stored.

It is worth noting that the KMP algorithm it is not a real time algorithm, and the BM algorithm needs to buffer the text.

All these properties indicates that this class of algorithms is suitable for hardware implementation.

2 A Numerical approach to String Matching

After the discovery of linear time string matching algorithms [11,4] a lot of research has been done in the area. Our algorithm is based in finite automata theory [11] and also exploits the fact that in practical applications the alphabet is finite [4].

Instead of trying to represent the global state of the search as in previous algorithms, we use a vector of $m$ different states, in where state $i$ tell us the state of the search between the positions $1, ..., i$ of the pattern and positions $(j - i + 1), ..., j$ of the text, where $j$ is the current position in the text.

Suppose that we need $b$ bits to represent each individual state. Then we can represent the vector state efficiently as a number in base $2^b$:

$$state = \sum_{i=0}^{m-1} s_i 2^{b-i}$$

where the $s_i$ are the individual states. Note that if $s_m$ corresponds to a final state we have to output a match that ends at the current position.

To update the state after reading a new character on the text, we have to:
New Algorithm for Pattern Matching

- shift the state $b$ bits to the left to reflect that we advanced in the text one position. In practice, this sets the initial state of $s_1$ to be by default 0.

- update the individual states according with the new character. For this we use a table $T$ that is defined by preprocessing the pattern with one entry per alphabet symbol, and an operator $op$ that given the old vector state and the table value, we obtain the new state. Note, that this works only if the effect of the operator in the individual state $s_i$ does not produce a carry that will affect state $s_{i+1}$.

Then each search step is:

$$state = (state \ll b) \ op \ T[curr\ char]$$

where $\ll$ denotes the shift left bitwise operation. The complexity of the search time in the worst case is $O(\lceil \frac{mb}{w} \rceil n)$, where $\lceil \frac{mb}{w} \rceil$ is the time to compute a shift or other simple operation on numbers of $mb$ bits using a word size of $w$ bits. In practice (small patterns, word size 32 or 64 bits) we have $O(n)$ worst case time.

The definition of the table $T$ will be basically the same for all cases. We define

$$T_x = \sum_{i=0}^{m-1} \delta(\text{pat}_{i+1} = x)2^{bi}$$

where $\delta(C)$ is 0 if the condition $C$ is true, or 1 otherwise; for every symbol $x$ of the alphabet. Therefore we need $b \cdot m \cdot |\Sigma|$ bits of extra memory, and if the word size is at least $b \cdot m$, only $|\Sigma|$ extra words are needed. We setup the table preprocessing the pattern before the search. This can be done in $O(\lceil \frac{mb}{w} \rceil (m + |\Sigma|))$ time.

For each kind of patterns or searching problem, we will adequately choose $b$ and $op$. An idea similar to this one was presented by Gonnet [8] applied to searching the signatures of a text.

For string matching we need only 1 bit (that is $b = 1$), where $s_i$ is 0 if the last $i$ characters had matched or 1 if not. After this the choice for $op$ is almost unique: a bitwise logical or. We have to report a match if $s_m$ is 0, or equivalently if $state < 0^{m-1}11...$. Figure 1 shows an efficient implementation.

3 String Matching with Classes

Now we extend our pattern language to allow don’t care symbols, complement symbols and more. Formally, every position in the pattern can be:
New Algorithm for Pattern Matching

- $x$: a character from the alphabet.
- $\Sigma$: a don’t care symbol (matches any symbol).
- $[\text{characters}]$: a class of characters, in where we allow ranges (for example $a - z$).
- $\neg C$: a complement of a character or class of characters ($C$). That is, matches any character that not belongs to this class.

For example, the pattern $[Pp]a-[aeiou]\Sigma-a[p - tu - z]$ matches the word Patter, but not python or Patton.

For this pattern language, we only have to modify the table $T$, such that, for each position, we process every character in the class. To maintain $O((\frac{m^2}{m^2})(m + |\Sigma|))$ time (instead of $O((\frac{m^2}{m^2})m|\Sigma|)$ time) where $m$ is now the size of the description of the pattern (and not its length) we include don’t cares and complements in the initial value of each entry of the table.

String matching with don’t care patterns was addressed before in [6] ($O(n \log^2 m \log \log m \log |\Sigma|)$ asymptotic search time), and also in [13] including complement symbols (same complexity). However, these are theoretical results, and their algorithms are not practical. Pinter also gives a $O(mn)$ algorithm that is faster than a naive algorithm. For small patterns the complexity of our algorithm is much better, and also a lot easier to implement.

Attempts to adapt the KMP algorithm to this case had failed [6,13], and for the same reason the BM algorithm as presented in [11] cannot solve this problem. It is possible to use Horspool version of the BM algorithm [9], but the worst case is $O(mn)$, and on average if we have a don’t care character near the end of the pattern, the whole idea of the shift table is worthless. By mapping a class of characters to a unique character, Karp and Rabin algorithm [10] solves this problem too. However, this a probabilistic algorithm, and if we check each reported match, the search time is $O(n + m + mM)$, where $M$ is the number of matches. Potentially, $M = O(n)$, and, on the other hand, their algorithm is slower in practice (because of the use of multiplications).

4 String Matching with at most $k$ Mismatches

Landau and Vishkin [12] gave the first efficient algorithm to solve this particular problem. Their algorithm uses $O(k(n + m \log m))$ time and $O(k(n+m))$ space. While it is fast, the space required is unacceptable for practical purposes. Galil and Giancarlo [7] improved this algorithm to $O(kn + m \log m)$ time and $O(m)$ space. This algorithm is practical for small $k$. However, if $k = O(m)$ it is not so.
New Algorithm for Pattern Matching

We solve this problem explicitly only for one string, but the solution can be easily extended for patterns with classes and multiple patterns, using the same ideas of the previous and next sections.

In this case one bit is not enough to represent each individual state. Now we have to count matches or mismatches. In both cases at most \( O(\log m) \) bits per individual state are necessary because \( m \) is a bound for both, matches and mismatches. Note, too, that if we count matches, we have to complement the meaning of \( \delta \) in the definition of \( T \). Then, we have a simple algorithm using

\[
B = \lceil \log_2(m + 1) \rceil
\]

and \( op \) being addition. If \( b_m \leq k \) then we have a match. Note that this is independent of the value of \( k \).

Therefore we need \( O(\Sigma m \log m) \) bits of extra space. If we assume that we can always represent the value of \( m \) in a machine word, we need \( O(\Sigma m) \) words and preprocessing time. However for small \( m \), we need only \( O(\Sigma) \) extra space and \( O(\Sigma + m) \) preprocessing time. For a word size of 32 bits, we can fix \( B = 4 \) and we can solve the problem up to \( m = 8 \), as presented in Figure 4, in where we count matches.

Clearly only \( O(\log k) \) bits are necessary to count if we allow at most \( k \) mismatches. The problem is that when adding we have a potential carry in to the next state. We can get around this problem, by having an overflow bit, such that we remember if overflow had happened, but that bit is set to zero at each step of the search. In this case we need

\[
B = \lceil \log_2(k + 1) \rceil + 1
\]

At each step we record the overflow bits in an overflow state, and we reset the overflow bits of all individual states (in fact, we only have to do this each \( k \) steps, but it is not practical to get in all that trouble). Note, that if \( k > m/2 \), then we count matches. The only problem for this case, is that is not possible to tell how many errors there are in a match. Table 1 shows up to what \( m \) we can use for a 32 bits word.

Therefore, with a slightly more complex algorithm we can solve more cases, and using only \( O(\Sigma \log k) \) extra bits.

5 Multiple Patterns

We consider the problem of more than one pattern, for patterns with classes (also we can extend this to mismatches). To denote the union symbol we use "\( \mid \)", for example \( p_1 \mid p_2 \) searches for the pattern \( p_1 \) or the pattern \( p_2 \).
Table 1: Maximum pattern length \( (m) \) for a 32 bits word depending of \( k \).

<table>
<thead>
<tr>
<th>( k, m - k )</th>
<th>Bits per state</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>2-3</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

The KMP algorithm and the BM algorithm had been extended already to this case (see [1] and [5] respectively), achieving a worst case time of \( O(n + m) \), where \( m \) is the total length of the set of patterns.

In our case, if we keep one vector state per pattern, we have an immediate \( O(\lceil \frac{m}{w} \rceil n) \) time algorithm, for a set of \( s \) strings. On the other hand, if we use \( s \) bits for each individual state, we can keep all the information in only one vector state achieving \( O(\lceil \frac{m}{w} \rceil n) \) search time. The disadvantage is that now we need numbers of size \( s \max(|p_i|) \) bits, and then \( O(|\Sigma|s \max(|p_i|)) \) extra space.

The preprocessing is very similar to the one in Figure 3. The main differences are that we have to count the number of patterns to compute \( B \), and that we have to compute the maximum pattern length. The only change in the search phase is the match testing condition:

\[
\text{if( (state & mask) != mask ) /* Match? */}
\]

where mask has a one bit in the adequate position for each pattern. Note that this indicates that a pattern ends at the current position, and it is not possible to say where the pattern starts without wasting \( O(\lceil \frac{m}{w} \rceil sM) \) time, being \( M \) the number of matches.

6 Implementation

In this section we present efficient algorithms to count the number of matches of the different classes of patterns in a text using one word numbers in the C programming language. The programming is independent of the word size as much as possible. We use the following symbolic constants:

- \text{MAXSYM}: size of the alphabet. For example, 128 for ASCII code.
- \text{WORD}: word size in bits (32 in our case).
- \text{B}: number of bits per individual state (1 for string matching).
New Algorithm for Pattern Matching

- EOS: end of string (0 in C).

Figure 1 shows the string matching algorithm.

```
Faststrmat( text, pattern )
register char *text;
char *pattern;
{
    register unsigned int state, lim;
    unsigned int T[MAXSYM];
    int i, j, matches;
    if( strlen(pattern) > WORD )
        Error( "Use pattern size <= word size" );
    /* Preprocessing */
    for( i=0; i<MAXSYM; i++ ) T[i] = "\0";
    for( lim=0, j=1; *pattern != EOS; lim |= j, j <<= B, pattern++ )
        T[*pattern] &= ~j;
    lim = ~(lim >> B);
    /* Search */
    matches = 0; state = "\0"; /* Initial state */
    for( ; *text != EOS; text++ )
    {
        state = (state << B) | T[*text]; /* Next state */
        if( state < lim )
            matches++; /* Match at current position-len(pattern)+1 */
    }
    return( matches );
}
```

Figure 1: Shift-Or algorithm for string matching.

Another implementation is possible using op as a bitwise logical and operation, and complementing the value of $T_x$ for all $x \in \Sigma$.

Experimental results for searching for all possible matches 100 times a text of length 50K are presented in Table 2. For each pattern, a prefix from length 2 to 10 was used. The patterns were chosen such that each first letter has a different frequency in English text (from most to least frequent). The timings are in seconds and they have an absolute error bounded by 0.5 seconds. They include the preprocessing time in all cases.

The algorithms implemented are Boyer-Moore as by Horspool [9] (or BMH) which according to Baeza-Yates [2] is the fastest practical version of this algorithm, Knuth-Morris-Pratt as suggested by their authors [11] (or KMP$_1$) and as given by Sedgewick [14] (or KMP$_2$), and our new algorithm
<table>
<thead>
<tr>
<th>m</th>
<th>BMH</th>
<th>KMP₁</th>
<th>KMP₂</th>
<th>SO₁</th>
<th>SO₂</th>
<th>BMH</th>
<th>KMP₁</th>
<th>KMP₂</th>
<th>SO₁</th>
<th>SO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36.5</td>
<td>24.4</td>
<td>58.7</td>
<td>30.2</td>
<td>15.8</td>
<td>23.6</td>
<td>15.5</td>
<td>49.9</td>
<td>30.2</td>
<td>13.2</td>
</tr>
<tr>
<td>3</td>
<td>25.2</td>
<td>24.3</td>
<td>59.0</td>
<td>30.2</td>
<td>15.7</td>
<td>16.2</td>
<td>15.0</td>
<td>50.2</td>
<td>30.4</td>
<td>13.0</td>
</tr>
<tr>
<td>4</td>
<td>20.5</td>
<td>24.5</td>
<td>58.7</td>
<td>30.2</td>
<td>15.6</td>
<td>12.6</td>
<td>15.0</td>
<td>50.1</td>
<td>30.2</td>
<td>13.1</td>
</tr>
<tr>
<td>5</td>
<td>17.3</td>
<td>24.3</td>
<td>58.8</td>
<td>30.4</td>
<td>15.8</td>
<td>11.0</td>
<td>15.2</td>
<td>50.1</td>
<td>30.4</td>
<td>13.1</td>
</tr>
<tr>
<td>6</td>
<td>15.3</td>
<td>24.4</td>
<td>58.7</td>
<td>30.3</td>
<td>15.9</td>
<td>9.6</td>
<td>15.1</td>
<td>50.9</td>
<td>30.6</td>
<td>13.4</td>
</tr>
<tr>
<td>7</td>
<td>13.2</td>
<td>24.3</td>
<td>58.6</td>
<td>30.1</td>
<td>15.7</td>
<td>9.0</td>
<td>15.3</td>
<td>50.8</td>
<td>30.5</td>
<td>13.1</td>
</tr>
<tr>
<td>8</td>
<td>12.5</td>
<td>24.4</td>
<td>58.6</td>
<td>30.4</td>
<td>15.6</td>
<td>7.9</td>
<td>15.3</td>
<td>50.7</td>
<td>30.6</td>
<td>13.3</td>
</tr>
<tr>
<td>9</td>
<td>11.6</td>
<td>24.4</td>
<td>59.7</td>
<td>30.1</td>
<td>15.8</td>
<td>7.5</td>
<td>15.3</td>
<td>50.5</td>
<td>30.7</td>
<td>13.3</td>
</tr>
<tr>
<td>10</td>
<td>11.2</td>
<td>24.3</td>
<td>58.3</td>
<td>30.1</td>
<td>15.8</td>
<td>7.1</td>
<td>15.4</td>
<td>50.1</td>
<td>30.2</td>
<td>13.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>m</th>
<th>BMH</th>
<th>KMP₁</th>
<th>KMP₂</th>
<th>SO₁</th>
<th>SO₂</th>
<th>BMH</th>
<th>KMP₁</th>
<th>KMP₂</th>
<th>SO₁</th>
<th>SO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>37.7</td>
<td>21.0</td>
<td>58.2</td>
<td>30.6</td>
<td>11.9</td>
<td>35.2</td>
<td>19.3</td>
<td>57.6</td>
<td>30.2</td>
<td>10.4</td>
</tr>
<tr>
<td>3</td>
<td>25.6</td>
<td>21.0</td>
<td>58.6</td>
<td>31.1</td>
<td>12.3</td>
<td>24.9</td>
<td>19.3</td>
<td>57.4</td>
<td>30.1</td>
<td>10.5</td>
</tr>
<tr>
<td>4</td>
<td>19.9</td>
<td>20.9</td>
<td>57.8</td>
<td>30.4</td>
<td>11.8</td>
<td>19.9</td>
<td>18.8</td>
<td>57.4</td>
<td>29.9</td>
<td>10.4</td>
</tr>
<tr>
<td>5</td>
<td>16.5</td>
<td>20.6</td>
<td>57.8</td>
<td>30.1</td>
<td>11.7</td>
<td>16.7</td>
<td>19.3</td>
<td>57.4</td>
<td>30.0</td>
<td>10.4</td>
</tr>
<tr>
<td>6</td>
<td>14.3</td>
<td>20.6</td>
<td>58.0</td>
<td>30.2</td>
<td>11.6</td>
<td>14.3</td>
<td>19.1</td>
<td>57.6</td>
<td>30.1</td>
<td>10.4</td>
</tr>
<tr>
<td>7</td>
<td>12.9</td>
<td>20.5</td>
<td>57.5</td>
<td>30.1</td>
<td>11.8</td>
<td>13.0</td>
<td>19.3</td>
<td>57.5</td>
<td>30.1</td>
<td>10.4</td>
</tr>
<tr>
<td>8</td>
<td>12.0</td>
<td>20.6</td>
<td>57.9</td>
<td>30.3</td>
<td>12.0</td>
<td>12.2</td>
<td>19.3</td>
<td>57.6</td>
<td>30.0</td>
<td>10.4</td>
</tr>
<tr>
<td>9</td>
<td>11.2</td>
<td>20.7</td>
<td>57.7</td>
<td>30.3</td>
<td>12.1</td>
<td>10.8</td>
<td>19.3</td>
<td>57.3</td>
<td>30.1</td>
<td>10.6</td>
</tr>
<tr>
<td>10</td>
<td>10.3</td>
<td>20.9</td>
<td>58.2</td>
<td>30.3</td>
<td>11.8</td>
<td>10.0</td>
<td>19.1</td>
<td>57.5</td>
<td>30.2</td>
<td>10.5</td>
</tr>
</tbody>
</table>

Table 2: Experimental results for prefixes of 4 different patterns.

as presented in Figure 1 (SO₁) and another version using KMP₁ idea (SO₂),
that is, do not use the algorithm until we see the first character of the pat-
tern. The changes needed for the later case (using structured programming!) are shown in Figure 2. Note that SO₁ and KMP₂ will be independent of
the pattern length, that SO₂ and KMP₁ will be dependent of the frequency
of the first letter of the pattern in the text, and that BMH depends on the
pattern length.

From Table 2 we can see that SO₂ outperforms KMP₁ being between a
40% and 50% faster. Also is faster than BMH for patterns of length smaller
than 4 to 9 depending on the pattern.

Figure 3 shows the preprocessing phase for patterns with classes, using
"-" as the complement character and "\" as escape character. The search
phase remains as before.

For string matching with at most k mismatches and word size 32 bits,
we use $B = 4$, and we count matches, solving the problem up to $m = 8$, as
presented in Figure 4.

Figure 5 shows the changes needed for the case in where we use $O(\log k)$
bits per state.
7 Final Remarks

We have presented a simple class of algorithms that can be used for string matching and some other kinds of patterns, with or without mismatches. The time complexity achieved is linear for small patterns, and this is the case in most applications. For longer patterns, we need to implement integer arithmetic of the precision needed using more than a word per number. Still, if the number of words per number is small, our algorithm is a good practical choice. Other solution, is to use VLSI technology to have a chip that uses register of 64 o 128 bits that implements this algorithm for a stream of text.

This class of algorithm can also be used for other matching problems, for example mismatches with different costs (see [3]) or for patterns of the form \((\text{set of patterns})^\Sigma^*(\text{set of patterns})\) (see [13]).

References


/* Compute length and process don’t care symbols and complements */
for( i=0, j=1, len=0, mask=0; *(pattern+i) != EDS; i++, len++, j <<= B )
{
    if( *(pattern+i) == '*' ) /* Complement */
    {
        i++; mask |= j;
    }
    if( *(pattern+i) == '[' ) /* Class of symbols */
    {
        for( ; *(pattern+i) != ']' ; i++ )
            if( *(pattern+i) == '\' ) i++; /* Escape symbol */
        else if( *(pattern+i) == '\' ) i++; /* Escape symbol */
        else if( *(pattern+i) == '.' ) mask |= j; /* Don’t care symbol */
    }
    if( len > WORD ) Error( "Use B=maxlen <= word size" );
} /* Set up T */
for( i=0; i<MAXSYM; i++ ) T[i] = ~mask;
for( j=1, lim=0; *pattern != EDS; lim |= j, j <<= B, pattern++ )
{
    compl = FALSE;
    if( *pattern == '*' ) /* Complement */
    {
        i++; compl = TRUE;
    }
    if( *pattern == '[' ) /* Class of symbols */
    for( pattern++; *pattern != ']' ; pattern++ )
    {
        if( *pattern == '\' ) pattern++; /* Escape symbol */
        if( compl ) T[*pattern] |= j;
        else T[*pattern] &= ~j;
        if( *(pattern+i) == '=' ) /* Range of symbols */
            for( k=(pattern+i)+1; k<=(++pattern); k++ )
                if( compl ) T[k] |= j;
            else T[k] &= ~j;
    }
    else if( *pattern == '.' ) /* Not a don’t care symbol */
    {
        if( *pattern == '\' ) pattern++; /* Escape symbol */
        if( compl ) T[*pattern] |= j;
        else T[*pattern] &= ~j;
    }
} /* lim = ~lim >> B; */

Figure 3: Preprocessing for Patterns with Classes.

New Algorithm for Pattern Matching

```
Fastmist( k, pattern, text ) /* String matching with k mismatches */
int k; /* (B=4, WORD=32, MAXSYM=128, EOS=0) */
char *pattern, *text;
{
    int i, j, m, matches;
    unsigned int T[MAXSYM];
    unsigned int mask, state, lim;
    if( strlen(pattern)+B > WORD )
        Error( "Fastmist only works for pattern size <= WORD/B" );
    /* Preprocessing */
    for( i=0; i<MAXSYM; i++ ) T[i] = 0;
    for( m=0, j=1; *pattern != EOS; m++, pattern++, j <<= B )
        T[*pattern] |= j;
    lim = (m-k) << ((m-1)*B);
    if( m*B == WORD ) mask = ~0;
    else mask = j-1;
    /* Search */
    matches = 0; state = 0; /* Initial state */
    for( i=1; i<m && *text != EOS; i++, text++ )
        state = (state << B) + T[*text];
    for( ; *text != EOS; text++ )
    {
        state = ((state << B) + T[*text]) & mask;
        if( state > lim ) /* Match at current position-m+1 */
            matches++;
        else matches += m-(state>>(m-1)*B) errors */
    }
    return( matches );
}
```

Figure 4: Pattern Matching with at most k mismatches (simpler version).

“Average”, Proceedings of the 6th International Colloquium on Automata,
Languages and Programming, Springer Verlag 1979, 118-132.

in Complexity of Computation (SIAM-AMS Proceedings 7), R. Karp ed.,
American Mathematical Society, Providence, RI, 1974, 113-125.

SIGACT NEWS 17 (1986), 52-54.

Proc. 2nd ACM SIGACT-SIGMOD Symposium on Principles of
New Algorithm for Pattern Matching

m = strlen(pattern); type = MISMATCH; /* count mismatches */
if (2*k > m) /* String matching with at least m-k matches */
{
    type = MATCH; k = m-k; /* count matches */
}
B = clog2(k+1) + 1; /* clog2(n) is the ceiling of log base 2 of n */
if (m > WORD/B ) Error("Fastmist does not work for this case");
/* Preprocessing */
lim = k << ((m-1)*B);
for (i=1, ovmask=0; i<=m; i++) ovmask = (ovmask << B) | (1 << (B-1));
if (type == MATCH )
    for (i=0; i<MAXSYM; i++) T[i] = 0;
else
{
    lim += 1 << ((m-1)*B);
    for (i=0; i<MAXSYM; i++) T[i] = ovmask >> (B-1);
}
for (j=1; *pattern != EOS; pattern++, j <<= B )
    if (type == MATCH )
        T[*pattern] = j;
    else
        T[*pattern] &= ~j;
if (m*B == WORD ) mask = "0;"
else
    mask = j - i;
/* Search */
matches = 0; state = 0; overflow = 0; /* Initial state */
for (i=1; i<=m & & *text != EOS; i++, text++)
{
    state = (state << B) + T[*text];
    overflow = (overflow << B) | (state & ovmask);
    state &= ~ovmask;
}
for( ; *text != EOS; text++)
{
    state = (((state << B) + T[*text]) & mask;
    overflow = ((overflow << B) | (state & ovmask)) & mask;
    state &= ~ovmask;
    if (type == MATCH )
    {
        if (state | overflow) >= lim 
            matches++; /* Match with more than m-k errors */
    } else if (state | overflow) < lim 
        matches++; /* Match with (state>>(m-1)*B) errors */
}

Figure 5: String Matching with at most k mismatches.

New Algorithm for Pattern Matching

and Experience 10 (1980), 501-506.


