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Dear Dr. Mavaddat,

I should be very obliged to you for sending me a copy of your paper entitled:

A Functional Model of RT Design
CS-88-16

Please, if possible.

Thanking you in advance, yours sincerely,

R. Karl-Adolf Zech

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A Functional Model of Register-Transfer Designs

Farhad Mavaddat

Dept. of Computer Science
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A Functional Model of Register-Transfer Designs

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ABSTRACT

We propose a strongly-typed functional model of register-transfer-level design specifications. The model is influenced by Gordon's register-transfer model of digital design and, compared to it, is presented from a more intuitive point of view, which in a way is closer to the reality of the RT design. We use the typed nature of the design environment to develop a semantic model for our SDC design notation, reported earlier,\(^1\) and to enforce correct composition of SDC-based designs.

The model can be used for design specification purposes as well as for analysing and reasoning about designs.
A Functional Model of Register-Transfer Designs

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1. Motivation

Register-Transfer (RT) based designs enjoy a high degree of structural regularity, which contributes to their acceptance as suitable models of VLSI design. This regularity is manifested by the explicit separation of a design into a PLA-type control-part and a slice-based data-path-part, and leads to an efficient placement and routing scheme. Layout efficiency and regularity has been the main motivation behind a number of silicon compilation activities in the past,\textsuperscript{2-4} and will likely continue to contribute to future developments.

Theoretical interest in RT-level modeling has also gained momentum over the last few years\textsuperscript{5, 6} and, among others, Gordon’s functional model\textsuperscript{5} has received special attention. Unfortunately, that model (among others) does not capture the above-mentioned regularities in an explicit form and so, fails to act as a direct representation of the corresponding RT design.

It is our belief that for a design abstraction to gain acceptance, there should be a one-to-one link between the objects of the design and the elements of the model, similar to the relationship between the logic-gate-based designs and their corresponding Boolean algebra model.

It is the purpose of this report to adapt a modified and extended version of Gordon’s model to the direct capture of digital designs. To achieve this, we apply the model to the SDC design primitives\textsuperscript{1, 7} and show that SDC-based designs can be modeled in a direct, one-to-one form, using the proposed functional model.
2. Defining Combinational Modules

Let $T$ be the set of basic signal types used in communication with a device. We define an $m$-input, $n$-output ($m \times n$-put) combinatorial device $D$, shown in Figure 1, to be of type:

$$D : (t_1 \times t_2 \times \cdots \times t_m) \rightarrow (t_{m+1} \times t_{m+2} \times \cdots \times t_{m+n})$$

if $t_1, t_2, \cdots, t_{m+n} \in T$ represent the types of values appearing at the $m$ input and the $n$ output ports of device $D$ respectively.

We define the behavior of $D$ by:

$$D = \lambda(\eta_1, \eta_2, \cdots, \eta_m)(E_1, E_2, \cdots, E_n);$$

where the right side of (2) is a short form for:

$$\lambda(\eta_1, \eta_2, \cdots, \eta_m). E_i, \quad 1 \leq i \leq n$$

$\eta_j : t_j \in T, \ 1 \leq j \leq m$ is the $j$th input port's value, and $g_k : (t_1 \times t_2 \times \cdots \times t_m) \rightarrow t_{m+k}$, defined by $g_k = \lambda(\eta_1, \eta_2, \cdots, \eta_m). E_k$, $1 \leq k \leq n$, defines the $k$th output-port's value.

![Figure 1](image)

3. Defining Sequential Circuits

At every state, the behavior of a Mealy-type sequential machine $B$, shown in Figure 2, has two components. First, its combinational behavior, $B_{cmb}$, under the influence of the current state and port inputs, and second, its next state behavior, $B_{seq}$, under the influence of the state and the port inputs at the time of transition to the next state. Therefore, the behavior of an $m \times n$-put, $q$-state sequential machine $B$, at state $(s_1, s_2, \cdots, s_q)$, $s_j : t_j \in T, \ 1 \leq j \leq q$, is modeled by two combinational circuits of types:
$$B_{cmb} : (t_1 \times t_2 \times \cdots \times t_q \times t_{q+1} \times t_{q+2} \times \cdots \times t_{q+m}) \rightarrow \quad (t_{q+m+1} \times t_{q+m+2} \times \cdots \times t_{q+m+n})$$

(3)

and,

$$B_{seq} : (t_1 \times t_2 \times \cdots \times t_q \times t_{q+1} \times t_{q+2} \times \cdots \times t_{q+m}) \rightarrow \quad (t_1 \times t_2 \times \cdots \times t_q)$$

(4)

and is defined by:

$$\begin{cases} 
B_{cmb} = \lambda (\eta_1, \eta_2, \cdots, \eta_q, \eta_{q+1}, \cdots, \eta_{q+m}) \cdot \left( \begin{array}{c} E_1, E_2, \cdots, E_n \\ F_1, F_2, \cdots, F_q \end{array} \right) \\
B_{seq} \end{cases}$$

(5)

A few observations are appropriate at this point.

- $E_1, E_2, \cdots, E_n$ are the $n$ output port values produced in response to the corresponding input-port and input-state values at all times.

- $F_1, F_2, \cdots, F_q$ are the $q$ next-state values produced in response to the corresponding input-port and input-state values at every step. They are evaluated at the time of the transition to the next state.

- The $q+m$ inputs represent the $m$ input-port (environment) and $q$ input-state values. To distinguish between the state and the port inputs we (sometimes) move the input-state bound variables to the left of the equality symbol, while keeping the environment inputs on the right side of the definition.

- We write $B(s_1, s_2, \cdots, s_q)$ to represent module $B$ at state $(s_1, s_2, \cdots, s_q)$, and $B(F_1, F_2, \cdots, F_q)$ to define a next state $(F_1, F_2, \cdots, F_q)$ for $B$, where $F_j : t_j \in T$ for $1 \leq j \leq q$ is the new value for the $j$th state variable.
Combining the two components of (5) into a single definition, and following the new practice of separating the bound variables, we write

\[ B(s_1, s_2, \ldots, s_q) = \lambda(\eta_1, \eta_2, \ldots, \eta_m). ((E_1, E_2, \ldots, E_n), B(F_1, F_2, \ldots, F_q)) \]  

(6)

to represent the behavior of \( B \) and re-write (3) and (4) as

\[ B_{cmb}(s_1, s_2, \ldots, s_q) = \lambda(\eta_1, \eta_2, \ldots, \eta_m). (E_1, E_2, \ldots, E_n) \]  

(7)

and

\[ B_{seq}(s_1, s_2, \ldots, s_q) = \lambda(\eta_1, \eta_2, \ldots, \eta_m). (F_1, F_2, \ldots, F_q) \]  

(8)

to represent \( B \)'s combinational and sequential behaviors, respectively.

4. Composite Modules

So far we have concentrated on the definition of primitive modules, or the leaf cells.\(^8\) It is the purpose of this section to propose a formalism for the definition of composite modules in terms of the instances of primitive and/or other (possibly predefined) composite modules. In the context of a composite module, we refer to the lower level instances as its sub-modules.

An \( m \times n - put \) composite module \( f^c \) is defined as the interconnection of \( s \) submodules \( f^0, f^1, \ldots, f^{s-1} \), and a (hypothetical) \( n \times m - put \) environment module \( f^s \), where the input and output ports of \( f^s \) respectively define the output and the input ports of \( f^c \), as shown in Figure 3. Furthermore, we define:

- \( I = \bigcup_{i=0}^{s} I_i \), \( O = \bigcup_{i=0}^{s} O_i \), as the set of internal input and the output ports respectively, where \( I_i \), \( 0 \leq i \leq s \), and \( O_i \), \( 0 \leq i \leq s \), are the respective sets of input and output ports of the \( i \)th module, and

- \( P = \{ p_1, p_2, \ldots, p_t \} \) as the set of nets used in connecting the submodules, such that \( h: O \cup I \rightarrow P \) is a total function assigning a single net to every port, \( h: O \rightarrow P \) is one-to-one, and \( h: I \rightarrow P \) is onto.

To capture the net connections of a module, say \( f^i \) (\( m^i \times n^i - put \), \( q^i - state \)), in a functional form, we write

\[(y_1, y_2, \ldots, y_{n^i}) = f_{cmb}(s_1, s_2, \ldots, s_{q^i})(x_1, x_2, \ldots, x_{m^i})\]  

(9)
as a short form for

\[ y_j = \left( \lambda(\eta_1, \eta_2, \ldots, \eta_{q^i}, \eta_{q^i+1}, \eta_{q^i+2}, \ldots, \eta_{q^i+m^i}) \cdot E_j \right) \]

\[ (s_1, s_2, \ldots, s_{q^i}, x_1, x_2, \ldots, x_{m^i}) \quad 1 \leq j \leq n^i, \]

where

\[ f_{\text{emb}}^i = \lambda(\eta_1, \eta_2, \ldots, \eta_{q^i}, \eta_{q^i+1}, \eta_{q^i+2}, \ldots, \eta_{q^i+m^i}) \cdot (E_1, E_2, \ldots, E_{n^i}), \]

and \( y_j \in h(O^i), \ 0 \leq j \leq n^i \) and \( x_j \in h(I^i), \ 0 \leq j \leq m^i \) are the values of the nets connected to the corresponding ports. Thus, the behavior of the module \( f^c \), composed of the interconnection of submodules: \( f^0, f^1, \ldots, f^s \), using the connection nets \( P \), can be defined as:

\[ f^c(S^1, S^2, \ldots, S^s) = \lambda(h(O^s)) \cdot (\text{rec} \]

\[ (Y^i = f_{\text{emb}}^i(S^i)(X^i) \quad 1 \leq i \leq s-1) \]

\[ \text{in} \quad (h(I^s), f^c(f_{\text{seq}}^i(S^i)(X^i) \quad 1 \leq i \leq s-1)) \]

where

\[ Y^i = (y_1^i, y_2^i, \ldots, y_{n^i}^i), y_j^i \in P - h(O^s), \ 1 \leq j \leq n^i, 1 \leq i \leq s-1, \]

and

\[ X^i = (x_1^i, x_2^i, \ldots, x_{m^i}^i), x_j^i \in P, \ 1 \leq j \leq m^i, 1 \leq i \leq s, \]

are the net values, \( S^i = (s_1^i, s_2^i, \ldots, s_{q^i}^i) \) is the set of states of \( f^i \), \( q^i \) is the number of state variables in \( f^i \), \( 1 \leq i \leq s \), and rec and in are defined as in.\(^9\)
5. The SDC Model of Register-Transfer Design

Let $T = \{ S, C, D \}$ be the set of signal types used in the design of a register-transfer-based design, where:

- $S$ is the type of signal indicating the truth values of the assertions made about the status of the data-path.
- $C$ is the type of signal selecting among the path alternatives inside the data-path.
- $D$ is the type of signal carrying data values among the path slice components. Such data values depend on the width of slice being defined. For example, for a binary slices, we have $D_b = \{ 0, 1 \}$, and in case of a decimal slice, we have $D_d = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$.

In the remainder of this report we will use small letter identifiers as variables ranging over the above sets of values and $\langle \rangle$, $\langle \rangle$, and $\langle \rangle$ to enclose $S$, $C$, and $D$ type variables, respectively.

We now introduce four design primitives which constitute the building blocks of our SDC model.

5.1. The Selector-Slice Primitive

A selector—slice $sel$, Figure 4, is a combinational device of type $sel : D \times D \times C \rightarrow D \times C$, defined by:

$$sel = \lambda\{d_1, d_2\}[c] \cdot \{c \rightarrow d_2, d_1\}[c], \quad \text{(12)}$$

where $\rightarrow$, in the context of an expression, stands for the if_then_else operator. Definition (12) indicates that the output of a selector is equal to one of its two data inputs and the selection is made according to the value of input $c : C$.

![Figure 4](image-url)
5.2. Functional Primitives

Functional -slices are a family of \((m + k) \times (n + k)\)-put combinational devices of type \(D^m \times S^k \rightarrow (D^n \times S^k)\), as shown in Figure 5, where: \(m \geq 1\), \(1 \geq k \geq 0\), \(1 \geq n \geq 0\), and \(n + k \geq 1\).

![Figure 5](image)

Thus, the behavior of a functional primitive can be defined by one of the following three definition schemes:

\[
\lambda\{d_1, d_2, \ldots, d_m\} \langle s \rangle . \{E\} \langle F \rangle
\]

\[
\lambda\{d_1, d_2, \ldots, d_m\} \langle s \rangle \langle F \rangle
\]

\[
\lambda\{d_1, d_2, \ldots, d_m\} . \{E\}.
\]

Depending on the nature of application, the number and type of operators used in \(E\) and \(F\) may vary. For example, multiplication might not be allowed when the model is used as the input to a silicon compiler, while its use in simulation applications might be allowed. In a similar way, addition might prove to be un-acceptable when the model is used for reasoning about hardware, but acceptable when the model is intended for silicon implementation. We now present a few typical functional-slices specified according to the techniques discussed in (13)-(15).

Ex.1- Binary 'and' Slice:
A binary and slice is a \(D_b^2 \rightarrow D_b\) type device defined by:

\[
\text{and} = \lambda\{a, b\} . \{a \land b\}.
\]

Ex.2- Binary 'equal' Slice:
A binary equal slice is a \(D_b^2 \times S \rightarrow S\) type device defined by:
\textbf{equal} = \lambda\{a, b\} \langle s \rangle . \langle s \land (a \oplus b) \rangle, \quad (17)

where \(a\) and \(b\) are the slice's data inputs, \(s\) is the status input indicating the result of comparisons at more significant slices, and \(s \land (a \oplus b)\) is the status output to the less significant neighbouring slice.

**Ex.3- Decimal ‘add’ Slice:**

A decimal \textbf{add} slice is a \(D_d^2 \times S \rightarrow D_d \times S\) type device defined by:

\[
\text{add} = \lambda\{a, b\} \langle s \rangle . \langle (a + b + \text{num}(s)) \mod 10 \rangle [\text{num}(s) + a + b > 9 \rangle \quad (18)
\]

Where \(a\) and \(b\) are the slice’s data inputs, \(s\) is the carry input from the less significant neighbouring slice, \((a + b + \text{num}(s)) \mod 10 \in D\) is the data output, \((\text{num}(s) + a + b > 9) \in S\) is the carry to the more significant neighbouring slice, and \(\text{num} : S \rightarrow D\) is a function that produces the numerical equivalent of the status input signal.

**5.3. The Controller Primitives**

Controllers are a family of \(m \times n\) devices of type \(C^p \times S^q \rightarrow C^s \times S^t\), where \(m = p + q\) and \(n = s + t\), \(p \geq 0\), \(q \geq 0\), \(s \geq 1\), and \(t \geq 0\); a typical way of defining this might be as:

\[
\lambda[\eta_1, \eta_2, \ldots, \eta_p \rangle \langle \eta_{p+1}, \eta_{p+2}, \ldots, \eta_{p+q} \rangle . [B_1, B_2, \ldots, B_s] \langle S_1, S_2, \ldots, S_t \rangle, \quad (19)
\]

where \(B_i\), \(1 \leq i \leq s\), and \(S_i\), \(1 \leq i \leq t\), are the sum of the products of the bound variables and their complements.

**5.4. The Unit Delay Primitive**

A unit-delay \textbf{del} is a \(1\times1\) device, single state, polymorphic sequential device of type \(\text{del} : (\ast \times \ast) \rightarrow (\ast \times \ast)\), shown in Figure 6 and defined by:

\[
\text{del}(n) = \lambda(i) . (n, \text{del}(i)) \quad (20)
\]
6. Typed Connections

We extend the concept of typed ports to that of typed nets. This assumes that the nets of a particular type connect the ports of the same type. We continue to use the parenthesis pairs [], and { } and ⟨ ⟩, to enclose control, data, and status nets, respectively.

In this spirit we partially re-write (9) as:

\[ \{ y_1, y_2, \ldots, y_{n_d} \} = f \text{ cmb}(s_1, s_2, \ldots, s_q)(x_1, x_2, \ldots, x_m) \]  

(21)

to emphasize the output data connections of \( f \), where \( n_d \) is the number of data outputs of \( f \).

In a similar way, one may specify the control and status type connections, or extend the definition into a single statement of the form:

\[ \{ Y_D \} [ Y_C ] \langle Y_S \rangle = f \text{ cmb}(S) \{ X_D \} [ X_C ] \langle X_S \rangle \]

(22)

so as to re-write (9) in a completely-typed form.

We also write:

\[ \{ f \text{ cmb}(S) \{ X_D \} [ X_C ] \langle X_S \rangle \} \quad [ f \text{ cmb}(S) \{ X_D \} [ X_C ] \langle X_S \rangle ], \]

and

\[ \langle f \text{ cmb}(S) \{ X_D \} [ X_C ] \langle X_S \rangle \]

in order to refer to the net sets: \( Y_D \), \( Y_C \), and \( Y_S \), respectively.

Extending this convention to the sequential behavior, we write:

\[ f \text{ seq}(S) \{ X_D \} [ X_C ] \langle X_S \rangle, \]

(23)

to refer to the next-state values of \( f \).

Finally, we call a composition a data- (or control-, or status- ) composition, if only data (or control, or status ) type nets are used in making the connections.
7. Register-Transfer Slices

A Register-Transfer (RT) slice is defined as the data-composition of primitives and/or other data-composed sub-modules. This guarantees that the control and status ports of an RT-slice remain unconnected. We now present a number of RT slice examples.

- The \(m\)-bit "register-counter" slice:

Following are the behavioral definitions of three submodules: \texttt{sel}, \texttt{del}, and \texttt{inc} used in the composition of the \(m\)-bit "register-counter" slice shown in Figure 7.

\[
\begin{align*}
\text{sel} &= \lambda\{ \text{in}_1, \text{in}_2 \}[ c ] \cdot \{ c \rightarrow \text{in}_2, \text{in}_1 \}[ c ] \\
\text{del}(n) &= \lambda\{ \text{in} \} \cdot (\{ n \}, \text{del}(\text{in})) \\
\text{inc} &= \lambda\{ \text{in} \} \cdot ((\text{in}+1) \mod 2^m).
\end{align*}
\]

Next we write the composition rule for the \(m\)-bit "register-counter" using the above sub-modules, the nets \(y_1, y_2, y_3, \text{in}, \text{c}_\text{in},\) and \(c_\text{ot}\). According to the (11),

\[
\text{count}(n) = \lambda\{ \text{in} \}[ c_\text{in} ] \cdot (\text{rec}( \\
\{ y_1 \}[ c_\text{ot} ] = \text{sel}_\text{cmb}(y_3, \text{in}))[ c_\text{in} ]; \\
\{ y_2 \} = \text{del}_\text{cmb}(n)\{ y_1 \}; \\
\{ y_3 \} = \text{inc}_\text{cmb}(y_2)\text{ in }((y_2)[c_\text{ot}], \text{count}(\text{del}_\text{seq}(n)(y_1))).
\]
This can be expanded to
\[
\text{count } (n) = \lambda \{ \text{in} \} [c_{in}] . \ (\text{rec} ( \\
y_1 = c_{in} \rightarrow \text{in} , y_3; \\
c_{ot} = c_{in} ; \\
y_2 = n ; \\
y_3 = (y_2 + 1) \mod 2^m ) \ \text{in} \ ( \{y_2\} [c_{ot}] , \ \text{count} (y_1))) ,
\]
and reduced to
\[
\text{count } (n) = \lambda \{ \text{in} \} [c_{in}] . \ (\{n\} [c_{in}] , \ \text{count} (c_{in} \rightarrow \text{in} , (n + 1) \mod 2^m ).
\]
Verbally, \text{count} is a single state \((n)\) sequential device with one data input \((\text{in})\), one data output, one \text{control} input \((c_{in})\) and one \text{control} output. The \text{counter}'s next state value is controlled by the value of the control input which selects between \((n + 1) \mod 2^m\) and \text{in} as the next state value.

• The "shift-register" slice:

The following is the behavioral definition of one slice of a shift register shown in Figure 8. The sub-modules \text{sel}, and \text{del} used in the composition of the "shift-register" slice are defined as usual.

![Figure 8](image-url)
The \textit{shift} slice is a new \textit{functional} primitive defined as

\[
\text{shift} = \lambda\{\text{in}_1\}\langle\text{in}_2\rangle \cdot \{\text{in}_2\} \langle\text{in}_1\rangle.
\]

Next, we write the composition rule for forming the "shift register" slice using the above sub-modules and the nets \(y_1, y_2, y_3, y_4, \text{in}, c_{in_1}, c_{in_2}, s_{in}, c_{ot_1}, c_{ot_2},\) and \(s_{ot}\). According to the (11),

\[
\text{shift} -\text{register} (n) = \lambda\{\text{in}\}\langle[c_{in_1}, c_{in_2}]\langle s_{in} \rangle \cdot \text{rec}
\]

\[
\{y_1\}[c_{ot_1}] = \text{sel}_{cmb} \{y_4, \text{in}\}[c_{in_1}];
\]

\[
\{y_2\}[c_{ot_2}] = \text{sel}_{cmb} \{y_3, y_1\}[c_{in_2}];
\]

\[
\{y_3\} = \text{del}_{cmb} (n)\{y_2\};
\]

\[
\{y_4\}[s_{ot}] = \text{shift}_{cmb} \{y_3\}[s_{in}]
\]

\[
\text{in} (\{y_3\}[c_{ot_1}, c_{ot_2}][s_{ot}] ,
\]

\[
\text{shift} -\text{register} (\text{del}_{seq} (n) (y_2))).
\]

This is expanded to

\[
\text{shift} -\text{register} (n) = \lambda\{\text{in}\}\langle[c_{in_1}, c_{in_2}]\langle s_{in} \rangle \cdot \text{rec}
\]

\[
y_1 = c_{in_1} \rightarrow \text{in}, y_4;
\]

\[
c_{ot_1} = c_{in_1};
\]

\[
y_2 = c_{in_2} \rightarrow y_1, y_3;
\]

\[
c_{ot_2} = c_{in_2};
\]

\[
y_3 = n;
\]

\[
y_4 = s_{in};
\]

\[
s_{ot} = y_3
\]

\[
\text{in} (\{y_3\}[c_{ot_1}, c_{ot_2}][s_{ot}] , \text{shift} -\text{register} (y_2))).
\]

and reduced to

\[
\text{shift} -\text{register} (n) = \lambda\{\text{in}\}\langle[c_{in_1}, c_{in_2}]\langle s_{in} \rangle \cdot (\{n\}[c_{in_1}, c_{in_2}](n) ,
\]

\[
\text{shift} -\text{register} (c_{in_2} \rightarrow (c_{in_1} \rightarrow \text{in}, s_{in} , n)).
\]
Hierarchical Design Example:

In the following example we first give the behavioral definition of one slice of a register with add capability, called \texttt{radd}, shown in Figure 9.

![Figure 9]

We then show the use of this unit in a hierarchical design using two such modules. The sub-modules used in composition of the \texttt{radd} are \texttt{sel}, \texttt{del}, and a binary adder called \texttt{badd} defined by

\[
\texttt{badd} = \lambda \{ \text{in}_1, \text{in}_2 \} \langle s_{\text{in}} \rangle \cdot \{ \text{in}_1 \oplus \text{in}_2 \oplus s_{\text{in}} \} \langle \text{in}_1 \wedge \text{in}_2 + \text{in}_1 \wedge s_{\text{in}} + \text{in}_2 \wedge s_{\text{in}} \rangle.
\]

Next, we write the composition rule for forming the \texttt{radd} slice, using the above sub-modules, the nets \( y_1, y_2, y_3, \text{in}_1, \text{in}_2, c_{\text{in}}, s_{\text{in}}, c_{\text{ot}}, \) and \( s_{\text{ot}} \). According to the (11),

\[
\texttt{radd} (n) = \lambda \{ \text{in}_1, \text{in}_2 \} [ c_{\text{in}} ] \langle s_{\text{in}} \rangle \cdot \text{(rec} \left( \begin{array}{l}
\{ y_1 \}[c_{\text{ot}}] = \texttt{sel}_{\text{cmb}} \{ y_3, \text{in}_1 \}[c_{\text{in}}]; \\
\{ y_2 \}[s_{\text{ot}}] = \texttt{badd}_{\text{cmb}} \{ y_1, \text{in}_2 \}[s_{\text{in}}]; \\
\{ y_3 \} = \texttt{del}_{\text{cmb}} (n) \{ y_2 \} \text{ in} \\
\langle \{ y_3, y_2 \}[c_{\text{ot}}] [s_{\text{ot}}] \rangle, \texttt{radd}_{\text{seq}} (n) (y_2)) \text{)).}
\end{array} \right) \text{(rec} \left( \begin{array}{l}
y_1 = c_{\text{in}} \rightarrow \text{in}_1, y_3;
\end{array} \right).
\]

This is expanded to

\[
\texttt{radd} (n) = \lambda \{ \text{in}_1, \text{in}_2 \} [ c_{\text{in}} ] \langle s_{\text{in}} \rangle \cdot \text{(rec} \left( \begin{array}{l}
y_1 = c_{\text{in}} \rightarrow \text{in}_1, y_3;
\end{array} \right).
\]

\[ c_{ot} = c_{in} ; \]
\[ y_2 = y_1 \oplus in_2 \oplus s_{in} ; \]
\[ s_{ot} = y_1 \wedge in_2 + y_1 \wedge s_{in} + in_2 \wedge s_{in} ; \]
\[ y_3 = n \]
\[ \text{in} \ (\{y_3, y_2\} \langle c_{ot} \rangle \langle s_{ot} \rangle, \ \text{radd} \ (y_2)) \]

and reduced to
\[ \text{radd} \ (n) = \lambda \{ \text{in}_1, \text{in}_2 \} \langle c_{in} \rangle \langle s_{in} \rangle . \]
\[ \langle (c_{in} \rightarrow \text{in}_1, n) \oplus \text{in}_2 \oplus s_{in} \rangle \langle c_{in} \rangle \]
\[ \langle (c_{in} \rightarrow \text{in}_1, n) \wedge \text{in}_2 + (c_{in} \rightarrow \text{in}_1, n) \wedge s_{in} + \text{in}_2 \wedge s_{in} \rangle , \]
\[ \text{radd} \ ((c_{in} \rightarrow \text{in}_1, n) \oplus \text{in}_2 \oplus s_{in})). \]

A further reduction yields
\[ \text{radd} \ (n) = \lambda \{ \text{in}_1, \text{in}_2 \} \langle c_{in} \rangle \langle s_{in} \rangle . \]
\[ \langle (n, (c_{in} \rightarrow \text{in}_1, n) \oplus \text{in}_2 \oplus s_{in}) \rangle \langle c_{in} \rangle \]
\[ \langle \text{in}_2 + s_{in} \rangle \wedge (c_{in} \rightarrow \text{in}_1, n) + \text{in}_2 \wedge s_{in} \rangle , \]
\[ \text{radd} \ ((c_{in} \rightarrow \text{in}_1, n) \oplus \text{in}_2 \oplus s_{in})). \]

We are now interested in deriving the behavior of the serial connection of two radds, as shown in Figure 10.

Figure 10
The following derivations lead to the definition of combined behavior. First,

\[\text{Dradd} \ (n, m) = \lambda \{in_1, in_2\}[c_{in_1}, c_{in_2}][s_{in_1}, s_{in_2}] \cdot (\text{rec} (\)
\{y_1, y_2\}[c_{ot_1}, c_{ot_2}][s_{ot_1}, s_{ot_2}] = \text{radd cmb} \ (n) \{in_1, in_2\}[c_{in_1}][s_{in_1}]
\{ot_1, ot_2\}[c_{ot_1}, c_{ot_2}][s_{ot_1}, s_{ot_2}] = \text{radd cmb} \ (m) \{y_1, y_2\}[c_{in_2}][s_{in_2}])
in (\{ot_1, ot_2\}[c_{ot_1}, c_{ot_2}][s_{ot_1}, s_{ot_2}],
\text{Dradd} \ (\text{radd seq} \ (n) \{in_1, in_2\}[c_{in_1}][s_{in_1}],
\text{radd seq} \ (m) \{y_1, y_2\}[c_{in_2}][s_{in_2}])).\]

This is expanded to

\[\text{Dradd} \ (n, m) = \lambda \{in_1, in_2\}[c_{in_1}, c_{in_2}][s_{in_1}, s_{in_2}] \cdot (\text{rec} (\)
y_1 = n ;
y_2 = (c_{in_1} \rightarrow in_1, n) \oplus in_2 \oplus s_{in_1};
c_{ot_1} = c_{in_1};
s_{ot_1} = (in_2, s_{in_1}) \wedge (c_{in_1} \rightarrow in_1, n) + in_2 \wedge s_{in_1};
\text{radd cmb} \ (m) \{y_1, y_2\}[c_{in_2}][s_{in_2}]\}
in (\{ot_1, ot_2\}[c_{ot_1}, c_{ot_2}][s_{ot_1}, s_{ot_2}],
\text{Dradd} \ (\)
(c_{in_1} \rightarrow in_1, n) \oplus in_2 \oplus s_{in_1};
(c_{in_2} \rightarrow y_1, m) \oplus y_2 \oplus s_{in_2},
(c_{in_2} \rightarrow y_1, m) \oplus y_2 \oplus s_{in_2}).\]
and reduced to

\[ \text{Dradd} \left( n, m \right) = \lambda \{ \text{in}_1, \text{in}_2 \} \{ c_{\text{in}_1}, c_{\text{in}_2} \} \{ s_{\text{in}_1}, s_{\text{in}_2} \} \cdot \begin{align*}
&\{ m, (c_{\text{in}_2} \to n, m) \oplus ((c_{\text{in}_1} \to \text{in}_1, n) \oplus \text{in}_2 \oplus s_{\text{in}_1}) \oplus s_{\text{in}_2} \} \\
&\{ c_{\text{in}_1}, c_{\text{in}_2} \} \\
&(c_{\text{in}_1} \to \text{in}_1, n) \land (c_{\text{in}_1} \to \text{in}_1, n) + \text{in}_2 \land s_{\text{in}_1}, \\
&((c_{\text{in}_1} \to \text{in}_1, n) \oplus \text{in}_2 \oplus s_{\text{in}_1} ) + s_{\text{in}_2} \land (c_{\text{in}_2} \to n, m) + \\
&(c_{\text{in}_1} \to \text{in}_1, n) \oplus \text{in}_2 \oplus s_{\text{in}_1} ) \land s_{\text{in}_2} \}, \\
\text{Dradd} \left( (c_{\text{in}_1} \to \text{in}_1, n) \oplus \text{in}_2 \oplus s_{\text{in}_1}, \\
(c_{\text{in}_2} \to n, m) \oplus ((c_{\text{in}_1} \to \text{in}_1, n) \\
\oplus \text{in}_2 \oplus s_{\text{in}_1} ) \oplus s_{\text{in}_2} ) \right). \]
8. Multi-Slice Data-Path Definition

Given a data-slice $f$ and a positive integer $n$, an $n$-wide data-path is formed by concatenating $n$ such slices along their control and status ports, as shown in Figure 11.

\[
\begin{align*}
D^n & \xrightarrow{C} \text{1st. Slice} \\
D^n & \xrightarrow{C} \text{nth. Slice} \\
D^{n-1} & \xrightarrow{C} \text{(n-1)th. Slice} \\
D^n & \xrightarrow{C} \text{1st. Slice}
\end{align*}
\]

Figure 11

Therefore, the behavior of an $n$-wide data-path $F$ is defined by:

\[
DP (f, n) = F (S^1, S^2, \ldots, S^n) = \lambda \{ D^1, D^2, \ldots, D^n \}[C].
\]

\[
(n = 1 \rightarrow (\{ f_{emb} (S^1)(D^1)[C](\triangledown) \} \langle f_{emb} (S^1)(D^1)[C](\triangledown) \rangle ),
\]

\[
f_{seq} (S^1)(D^1)[C](\triangledown),
\]

\[
\langle \{ f_{emb} (S^n)(D^n)[C](\langle DP (f, n-1)(D^1, D^2, \ldots, D^{n-1}[C]) \rangle ) \rangle 
\]

\[
\langle f_{emb} (S^n)(D^n)[C](\langle DP (f, n-1)(D^1, D^2, \ldots, D^{n-1}[C]) \rangle ) \rangle,
\]

\[
f_{seq} (S^n)(D^n)[C](\langle DP (f, n-1)(D^1, D^2, \ldots, D^{n-1}[C]) \rangle ) \rangle,
\]

where $S^i = s^i_1, s^i_2, \ldots, s^i_q$, $1 \leq i \leq n$, are the values and $q$ is the length of the state vector of $i$th slice; $D^i = d^i_1, d^i_2, \ldots, d^i_m$, $1 \leq i \leq n$, are the $m$ data input values; $C$ is the control input vector of each slice; and $\triangledown$ is the first slice's status initialization vector.

A few observations are in order here.

- Since the slices are identical\(^\dagger\), concatenations can be realized through the abutment\(^\dagger\) of the corresponding layouts.

\[^\dagger\] In practice $f$ can be parametrized and the $i$th slice can receive $i$ as its parameter.
• We have assumed that the status information is passed from the lower indexed slices to the higher indexed ones. Assuming that the smallest indexed slice is also the least significant slice (under some number representation scheme), this formulation satisfies the requirements of certain functionals, such as the carry propagations in a sliced adder.

• A similar formulation exists for the case where the status signal has to propagate in the opposite direction, for example a sliced comparator. Since control signals are simply passed through the slices without any modification, simultaneous flows of both formulations will not lead to infinite recursion, as might be feared.

• Definition (24) can be used to extend certain slice properties to that of the data-path itself, through structural induction proof techniques. In the past, designers have assumed this in an implied way and have used the properties of the slice and the corresponding $n$-wide data-paths in an interchangeable form. We also do this in the next example by defining a single slice, and applying the control part to the slice, assuming that the multi-bit version of the data-path leads to the identical behavior.
9. A Complete Example

The SDC-based graphical representation of a circuit designed to calculate the greatest common divisor (GCD) of two values at its data-input ports ‘in₁’ and ‘in₂’, and producing the result at its data-output port ‘out’ is shown in Figure 12.
The input values are sampled at the last assertion of the ‘r’ (reset) control input and the availability of the result is signaled by the first assertion of the ‘f’ (finish) status output. The hardware follows the usual GCD algorithm of repeatedly subtracting the smaller value from the larger value until the two values match. It is the purpose of this section to develop the functional models of the data-path and the control parts independently, and to combine them to form the total module’s behavioral model.

We start by applying the composition rule (11) to the data-path. Given functional primitives:

\[ \text{eql} = \lambda\{a, b\}. \langle a \equiv b \rangle \]

\[ \text{gt} = \lambda\{b, a\}. \langle a > b \rangle \]

\[ \text{sub} = \lambda\{a, b\}. \{a - b\} \]

and the composite register module

\[ \text{reg} (a) = \lambda\{\text{in}\}[ld]. (\{a\}, \text{reg} (ld \rightarrow \text{in}, a)) \]

the gcd_path is defined by:

\[ \text{gcd} \_\text{path} (a, b) = \lambda\{\text{in}_1, \text{in}_2\}[j, k, la, lb]. (\text{rec} (\]

\[ \{y_1\} = \text{sel} \_\text{cmb} \{y_7, \text{in}_1\}[j]; \]

\[ \{y_2\} = \text{sel} \_\text{cmb} \{y_7, \text{in}_2\}[j]; \]

\[ \{y_3\} = \text{reg} \_\text{cmb} (a) \{y_1\}[la]; \]

\[ \{y_4\} = \text{reg} \_\text{cmb} (b) \{y_2\}[lb]; \]

\[ \langle s_1 \rangle = \text{eql} \_\text{cmb} \{y_3, y_4\}; \]

\[ \langle s_2 \rangle = \text{gt} \_\text{cmb} \{y_3, y_4\}; \]

\[ \{y_5\} = \text{sel} \_\text{cmb} \{y_4, y_3\}[k]; \]

\[ \{y_6\} = \text{sel} \_\text{cmb} \{y_3, y_4\}[k]; \]

\[ \{y_7\} = \text{sub} \_\text{cmb} \{y_5, y_6\} \text{in} (\]

\[ \{y_3\}(s_1, s_2), \text{gcd} \_\text{path} (\text{reg} \_\text{seq} (a)\{y_1\}[la], \]

\[ (\text{reg} \_\text{seq} (b)\{y_2\}[lb])). \]

Please note that in this example we have used a single slice and a data-path of those slices in an interchangeable form. As the result, we have assumed that the status inputs to the data-path receive proper initialization without explicit reference to them.
After expansion and simplifications, \texttt{gcd\_path} behavior reduces to:

\[
\texttt{gcd\_path} (a,b) = \lambda \{ \texttt{in}_1, \texttt{in}_2 \} [ j, k, \texttt{la}, \texttt{lb} ] . \left( \{ a \} \left( a \equiv b, a > b \right), \right.
\]
\[
\texttt{gcd\_path} \left( ( \texttt{la} \rightarrow ( j \rightarrow \texttt{in}_1, ( k \rightarrow (a-b),(b-a))), a \right),
\]
\[
( \texttt{lb} \rightarrow ( j \rightarrow \texttt{in}_2, ( k \rightarrow (a-b),(b-a)), b )))
\]

This completes the definition of the data-path part.

The control part is made of two sub-modules: the PLA and the unit-delay parts. The PLA realizes the microprogram to be executed by the module. The unit-delay holds the state of the control-part. We start by first defining the PLA part, called \texttt{pla}, and combine it with a unit-delay element to form the complete control-part, called \texttt{contunit}. Following are these two steps.

\[
\texttt{pla} = \lambda [ r ] \langle s_1, s_2, c \rangle . ( [ c', j, k, \texttt{la}, \texttt{lb} ] \langle f \rangle )
\]

which is expanded to:

\[
\texttt{pla} = \lambda [ r ] \langle s_1, s_2, c \rangle . ( [ ( \bar{r} \land \bar{c} \land s_1 ) \lor (\bar{r} \land c) ,
\]
\[
r , ( \bar{r} \land s_1 \land s_2 \land \bar{c} ) , ( r \lor ( \bar{r} \land \bar{s}_1 \land s_2 \land \bar{c} ) ,
\]
\[
( r \lor ( \bar{r} \land \bar{s}_1 \land \bar{s}_2 \land \bar{c} ) ) ] \langle \bar{r} \land c \rangle )
\]

and

\[
\texttt{contunit} ( p ) = \lambda [ r ] \langle s_1, s_2 \rangle . ( \text{rec} ( \left[ y_1, j, k, \texttt{la}, \texttt{lb} \right] \langle f \rangle = \texttt{pla\_cmb} [ r ] \langle s_1, s_2, y_2 \rangle ;
\]
\[
\left. \langle y_2 \rangle = \texttt{del\_cmb} ( p ) [ y_1 ] \right] \text{in} ( \left[ j, k, \texttt{la}, \texttt{lb} \right] \langle f \rangle , \texttt{contunit} ( \texttt{del\_seq} ( p ) [ y_1 ] ) .
\]

\texttt{contunit} can be reduced to

\[
\texttt{contunit} ( p ) = \lambda [ r ] \langle s_1, s_2 \rangle . ( [ r , \bar{r} \land \bar{s}_1 \land s_2 \land \bar{p} ,
\]
\[
r \lor ( \bar{r} \land \bar{s}_1 \land s_2 \land \bar{p} ) , r \lor ( \bar{r} \land \bar{s}_1 \land \bar{s}_2 \land \bar{p} ) ]
\]
\[
\langle \bar{r} \land p \rangle , \texttt{contunit} ( ( \bar{r} \land \bar{p} \land s_1 ) \lor (\bar{r} \land p ) )
\]
Combining \texttt{gcd-path} and \texttt{contunit}, as shown in Figure 13, to form the complete module, called \texttt{gcd}, leads initially to:

\[
gcd(a, b, p) = \lambda_{\{in_1, in_2\}}[r].(\text{rec (}
\{out\}(s_1, s_2) = \texttt{gcd-path}_{\text{cmb}}(a, b)\{in_1, in_2\}[j, k, la, lb];
\[j, k, la, lb\}(f) = \texttt{contunit}_{\text{cmb}}(p)[r](s_1, s_2))
\text{in (}\{out\}(f),
\text{gcd (gcd-path}_{\text{seq}}(a, b)\{in_1, in_2\}[j, k, la, lb],
\texttt{contunit}_{\text{seq}}(p)[r](s_1, s_2))));
\]

this expands to:

\[
gcd(a, b, p) = \lambda_{\{in_1, in_2\}}[r].(\text{rec (}
\text{out} = a;
\text{s}_1 = a \equiv b;
\text{s}_2 = a > b;
\text{j} = r;
\text{k} = \bar{r} \land \text{s}_1 \land \text{s}_2 \land \bar{p};
\text{la} = r \lor (\bar{r} \land \text{s}_1 \land \text{s}_2 \land \bar{p});
\]
\]
\[ lb = r \lor ( \bar{r} \land \bar{s}_1 \land \bar{s}_2 \land \bar{p} ); \]
\[ f = \bar{r} \land p ) \text{ in } ( \{ \text{out} \} ( f ), \]
\[ \text{gcd} (( la \rightarrow ( j \rightarrow \text{in}_1, ( k \rightarrow ( a - b ), ( b - a ))) , a ), \]
\[ (( lb \rightarrow ( j \rightarrow \text{in}_2, ( k \rightarrow ( a - b ), ( b - a ))) , b ), \]
\[ ( \bar{r} \land \bar{p} \land s_1 ) \lor ( \bar{r} \land p ) ))), \]

and can eventually be reduced to:

\[ \text{gcd}( a, b, p ) = \lambda\{\text{in}_1, \text{in}_2\}[ r ]. ( \{ a \} ( \bar{r} \land p ), \text{gcd}( \]
\[ (( r \lor q ) \rightarrow ( r \rightarrow \text{in}_1, ( q \rightarrow ( a - b ), ( b - a ))) , a ), \]
\[ (( r \lor q ') \rightarrow ( r \rightarrow \text{in}_2, ( q \rightarrow ( a - b ), ( b - a ))) , b ), \]
\[ ( \bar{r} \land \bar{p} \land s_1 \lor ( \bar{r} \land p ) )) \]

where
\[ q = \bar{r} \land ( a \equiv b ) \land ( a > b ) \land \bar{p} \]
\[ q' = \bar{r} \land ( a \equiv b ) \land ( a > b ) \land \bar{p} \]

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11. References

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