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Modelling the Effect of Scattering

A New Lighting Model for Computer Imagery

Research Report

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Modelling the Effect of Scattering

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Abstract

The interaction of light with particles suspended in the air is the cause of some beautiful effects. Among these effects are the colours of the sunset, the blue of the sky, and the appearance of a scene in fog. A lighting model is presented which takes into account the effects of scattering by suspended particles. A method of computing the colours of the sun and the sky, for any sun position above the horizon, is derived from the lighting model. The model is also suitable for rendering fog under general lighting conditions. As an example of the use of the model for rendering fog, the special case of fog lit by the sun, without shadows, is considered.

† This work was originally submitted in fulfillment of the requirements for the degree of Masters of Mathematics to the Faculty of Mathematics of the University of Waterloo.

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List of Illustrations

Introduction

All around us nature beckons. The beauty of creation has inspired many an artist, many a poet, and countless writers through the ages, and now is becoming an inspiration to a number of us in the field of computer graphics. The quest is not necessarily for realism; the illusion of realism is usually sufficient. The simulation of natural appearances is not only an end in itself, but also a means to test our limits in graphical techniques.

The last few years have seen a gradual increase in interest in the simulation of natural phenomena in the computer graphics community. Early work includes that of [Max1981] and [Marshall1980]. With the advent of Fractals and stochastic surfaces, land forms have become much more natural. Texturing has done much for the appearance of water [Norton1982], land [Gardner1984] and even skies [Gardner1984]. Vegetation, in the form of plants [Smith1984], [Reeves1983], trees [Smith1984], [Reeves1985], [Bloomenthal1985], and textured surfaces (for distant viewing) has seen a lot of growth. In the last two years a number of attempts have been made at generating clouds, some with limited success [Voss1983], [Kajiya1984], others with considerable success [Perlin1984], [Gardner1985].

A large number, in fact the majority, of atmospheric effects have not yet been addressed in the computer graphics literature. Many will not be addressed here. Brief mention was made by [Cook1984] of rainbows; Max attempted to portray the change with time of day of the lighting of the sky using simple heuristics [Max1981]. With these exceptions, the only real results to appear have been in the modelling of clouds.

Quite a lot of work has been done in atmospheric physics to explain effects ranging from the colour of the sunset to the green flash, from rainbows to coronae; many of these require taking into account not only refraction by the earth's atmosphere but also dispersion.

The *effect* may sometimes be well modelled without considering the cause: rainbows can be effectively rendered with only the knowledge of the geometry [Cook1984].

Few of the more exotic effects due to refraction are easily modelled. The Chinese lantern effect, caused by layers of air with sharp temperature changes between layers, is just one example of a refraction effect not easily modelled. Another common example is the mirage caused by light being bent as it passes through hot air near the earth, and producing an image of objects above the ground, giving the appearance that they are being reflected from the surface.

Some effects are unfamiliar to most people, the green flash being a good example. For the green flash to occur, a sharply defined (unvegetated) horizon is required, along with the appropriate atmospheric conditions. For this reason the effect only appears at sea and in deserts. Under the right conditions, the top edge of the sun will appear green, and this will be visible when only the top edge is visible (either the start of sunrise or the end of sunset). Since it only lasts a few seconds, it may go unobserved even when it occurs.

A comprehensive model of the atmosphere would produce such effects as the green flash if the right parameters were specified. Such a model, by attempting to model the world as it *is*, would be expensive and more faithful to reality than perception can recognise.

It is not always the case that we perceive reality as it is; hence a model which addresses common effects, presenting the appearance of reality as we believe we know it, is often satisfactory. Tools developed to give the appearance of nature may be later used in systems emphasizing authenticity; however, for the present they will be used to present the illusion of realism. Approximations will always be used: one of the important lessons of physics is learned when the student understands how to decide which approximations are suitable for the problem at hand. If we understand a model which is the best known model for the behaviour of light, then we can make whichever approximations are prudent to produce an image which looks correct, whether or not the techniques used are able to produce all of the more unusual effects.

Rendering packages used in computer graphics have, as time progressed, modelled more and more effectively the interaction of light with surfaces in the environment using the principles of ray optics and the conservation of energy. Light has been assumed to be unaffected by whatever intervening medium may separate the surfaces.

It is not always sufficient to assume that light travels in a vacuum between surfaces. Where there are large amounts of dust or water vapour in the atmosphere, or where the distances are large enough that air molecules themselves have an effect, traditional ray tracing is ineffective. In some cases, it is possible to use simple approximations which apply only to the special case in question. One example of this is fog, which has generally been rendered by varying the contrast as a function of distance [Whitted1981]. While this cannot produce the effect of a searchlight or of shadows in the fog, it has produced quite pleasing results at very little expense. The colour of the sky has been produced using simple heuristics [Max1981], [Duff1985] which are satisfactory in many instances, although unable to predict the colour of the sun for a given sky. Blinn proposed a lighting model for clouds [Blinn1982], which was used by Kajiya and Von Herzen to produce a number of images of clouds [Kajiya1984], but only white ones, and only against a flat blue background.

In order to correctly render scenes in which large numbers of small particles have a noticeable effect on the lighting, it is necessary to know what effect such particles have, and in which regions. We shall begin by describing a general lighting model for use in dispersive media, and then consider a number of cases which, due to the nature of their geometries, do not require ray tracing, but lend considerable justification to the model.

From the physical models used to explain the colour of the sky a lighting model will be developed to give the sky colour as a function of sun position, viewing angle, and haze density.

Since the number of phenomena available is limitless, we will begin with simple problems. From the accepted model for scattering of light by the atmosphere a model of the colour and intensity of the (perceived) sun will be developed.

The lighting model is also useful for rendering scenes with fog. One example of this, fog lit by the sun, will be given to demonstrate that where previous techniques have failed to show the true effect of fog in a scene, this model makes it possible.

It should be noted that the current work is *not* intended as a treatise on physics, but rather a description of one method of improving the realism in computer generated images. While there is a fair amount of physics involved in the motivation for the model, it is intended to give the reader with a background outside of atmospheric optics enough of an understanding of the physics involved to understand why such a model is necessary. The behaviour of light in the presence of scattering media is still a research area in physics and will remain so for some time to come.

The Cook-Torrance model for reflection from metallic surfaces [Cook1981] is a way of using what is known about the reflectance properties of metals while ignoring the physical reasons for different reflectivities among metals. In that model there are approximations made which make the computation easier, while it is hoped that the visible loss of realism is small. Here we present a somewhat simplified model of scattering by small particles to produce some of the effects of the atmosphere on light. Once again there are approximations made which, it is hoped, do not sacrifice very much realism. There are definite limitations to the scattering models used, however the method demonstrates the use of scattering models in portraying the effect of the atmosphere, and it is hoped that more sophisticated models may follow.

A Lighting Model for Scattering Media

Between the parts of opake and colour'd Bodies are many Spaces, either empty, or replenish'd with Mediums of other Densities; as Water between the tinging Corpuscles wherewith any Liquor is impregnated, Air between the aqueous Globules that constitute Clouds or Mists; and for the most part Spaces void of both Air and Water, but yet perhaps not wholly void of all Substance, between the parts of hard Bodies.

– Sir Isaac Newton (1704)

The atmosphere as we know it is made up of molecules, dry and wet haze particles, water vapour and droplets. Visual indications of the presence of these constituents are the colour of the sky, the whitening and loss of contrast of distant objects, loss of visibility in fog and rain, and the appearance of clouds. There are two principal optical effects involved: refraction accounts for the slightly oblate shape of the sun when it's near the horizon [Meinel1983], mirages and other illusions [Fraser1976], and the colours of the rainbow [Khare1974]; scattering is the cause of the remaining effects. Only scattering will be considered here, as the effects of refraction are more commonly known.

Principal Characteristics of Scattering

In all of nature there are three effects which determine the colour of whatever we see. (The term *colour* is normally used to refer to the combined effects of the spectral distribution of the light and of the visual system. We shall use the term to refer only to the spectral distribution of the light.) These are radiation, absorption, and scattering. Radiation is the original production of light by some source, be it a common light bulb, the sun or some distant star, or a firefly in the woods. In some cases light (by which we mean electromagnetic waves in the visible part of the spectrum), is produced by fluorescence, as radiation is absorbed in one part of the spectrum (possibly invisible) and re-radiated as visible light. In any event, light as it leaves the source has a characteristic spectral intensity distribution. This is a function of the temperature and nature of the source, and gives the amount of light emitted as a function of wavelength. The emission spectral intensity distribution of the sun may be found in [ANSI1967].

The colour of light may be altered in two ways after it leaves its source. The light may be differentially absorbed, or it may be scattered. The distinction between scattering and absorption is only in the conservation of electromagnetic energy.

> *Scattering is the process by which a particle — any bit of matter — in the path of an electromagnetic wave continuously (1) abstracts energy from the incident wave, and (2)* [immediately] *re-radiates that energy into the solid angle centred at the particle* [McCartney1976].

Where absorption occurs, not all of the energy is being continuously re-radiated, but rather some is converted to other forms of energy. Absorption and scattering sometimes occur together, with some parts of the spectrum being absorbed and others scattered. Whatever part is absorbed is never seen, and so it is the scattered part that is of interest.

Scattering is responsible, on a microscopic sense, for the reflectivity of most materials [Williamson1983]. It is also responsible for the nearly all of the optical phenomena in the atmosphere. Refraction and reflection are the result of scattering at the atomic level, but are easily described using classical geometric optics. A discussion of the theoretical footings of geometric optics based on scattering is beyond the scope of this work, and may be found in many advanced optics texts, such as [Born1975]. For those phenomena which result from scattering but cannot be described using classical geometric optics, some knowledge of the nature of scattering itself is required. When scattering is sufficiently well understood, its effect on the appearance of a scene may be modelled. The nature of scattering and a method of modelling its effect, are the topics of the remainder of this chapter.

Since the primary medium of interest is the atmosphere, it is helpful to be aware of the sources of scattering there. In Table 1.1 are given the principle particles responsible for atmospheric scattering, along with their radii and concentrations. Note the wide range of sizes and concentrations.

Type	Radius (μm)	Concentration (cm^{-3})
Air molecule	10^{-3}	10^{19}
Haze particle	$10^{-2} - 1$	$10^{3} - 10$
Fog droplet	$1 - 10$	$100 - 10$
Cloud Droplet	$1 - 10$	$300 - 10$
Raindrop	$10^{2} - 10^{4}$	$10^{-2} - 10^{-5}$

Table 1.1 Particles Responsible for Atmospheric Scattering (values taken from [McCartney1976])

Partial Scattering

About any particle the intensity of scattered light forms a 3-dimensional pattern, $\beta_{sc}^{(\lambda)}(\theta, \phi)$, (the angular scattering function) which depends on the size and shape of the particle, and on the wavelength λ of the light involved. Figure 1.1 shows the angles of scattering for a scattering event in which the scattering particle is at the origin. Incident light is travelling in the positive z direction, and light leaves the scattering centre along a line which makes an angle ϕ with the z axis and whose projection onto the x-y plane makes an angle θ with the x axis.

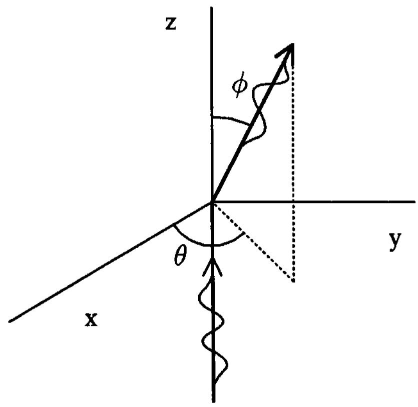

Figure 1.1 Scattering Angles

Individual photons have a probability proportional to $\beta_{sc}^{(\lambda)}(\theta,\phi)$ of being scattered in the direction given by the angles θ and ϕ. Since a large number of photons are influenced by the scattering centre in a very short period of time the total flux in a given direction, and hence the intensity is proportional to $\beta_{sc}^{(\lambda)}(\theta,\phi)$

For spherical particles, and particles small enough with respect to the wavelength to represent point discontinuities in the electric field experienced by the passing light, β_{sc} is symmetric about the line of propagation of the incident light, and so it is sufficient to write $\beta_{sc}^{(\lambda)}(\phi)$. This is the case for air molecules and water vapour (by virtue of their small average size), and wet haze and small raindrops (which are spherical). Non-spherical particles, large with respect to λ, are found in the air in dust storms and heavily polluted environments. Since less is known about the nature of scattering from non-spherical particles, they will not be modelled here. The lighting model that we will give here is sufficient to handle such cases, given an appropriate scattering function, but approximations we will use in some of the treatments of special cases will depend on the nature of the scattering function.

As shown in [Cook1981] the fact that reflectivity is wavelength dependent is very important for those who choose to render metals. Anyone who has seen a prism in action knows of the spectral dependence of refraction. Just as reflection and refraction are wavelength dependent, so is the amount and directionality of scattering (of which both reflection and refraction are macroscopic manifestations). Several examples of angular scattering functions are shown in Figure 1.2.

For small particles, of radius less than about $0.05\,\lambda$, Rayleigh scattering, named after Lord Rayleigh, who was the first to study it [Strutt1871], predominates. Here half of a non-polarized beam becomes polarized parallel to the plane of scattering and the other half perpendicularly. The intensity of the scattered light is the average of the intensities of the two components. The intensity of the parallel portion varies as $\cos^2\phi$, while the remaining portion is independent of angle. This gives a total angular scattering function of $(1 + \cos^2\phi)/2$ [McCartney1976].

Larger particles, of size up to about the wavelength of the light, scatter more light, concentrating it in the forward direction. Still larger particles show extreme concentration of light in the forward lobe, with maxima and minima developing to the sides. The positions and numbers of these maxima and minima vary rapidly with the size of the particle, and since the size of particles in the atmosphere is distributed over a continuum, the effect of the side lobes is negligible in the average [McCartney1976].

For small particles, (Rayleigh scatterers) only the first term in the series from which β_{sc} is derived is needed [Born1975] (pp. 646 ff.). Each term in the series is separable into a product of its azimuthal and polar parts, hence in the limit of small radii, $\beta_{sc}^{(\lambda)}(\phi)$ may be separated into such a product of $\beta_{sc}^{(\lambda)}$ and $\Phi(\phi)$, where $\Phi(\phi)$ has been normalized to have a 0-1 range. In the interest of computational expediency, we will approximate $\beta_{sc}^{(\lambda)}(\phi)$ with product of $\beta_{sc}^{(\lambda)}$ and $\Phi(\phi)$ when computing the amount of light scattered. Since Φ varies with λ, the approximation that is being made here is that, as far as directionality is concerned, all wavelengths of (visible) light are being treated as equal. This is not as bad an approximation as it might sound, since there are many particle sizes in a typical fog or mist, and it is the ratio of particle radius to wavelength which determines Φ.

As indicated above, Φ for large particles has numerous secondary lobes, however, since particles are not of uniform size, these will be averaged together yielding a smooth function. For mono-dispersions (all particles having the same radius) this approximation would neglect the effect of the variation of position and size of side lobes since Φ is, in effect, sampled at one wavelength. Since the effect of these lobes is not important in poly-dispersions, such as are found in natural haze, fogs and clouds, the approximation is somewhat closer to reality than an accurate model of a mono-dispersion.

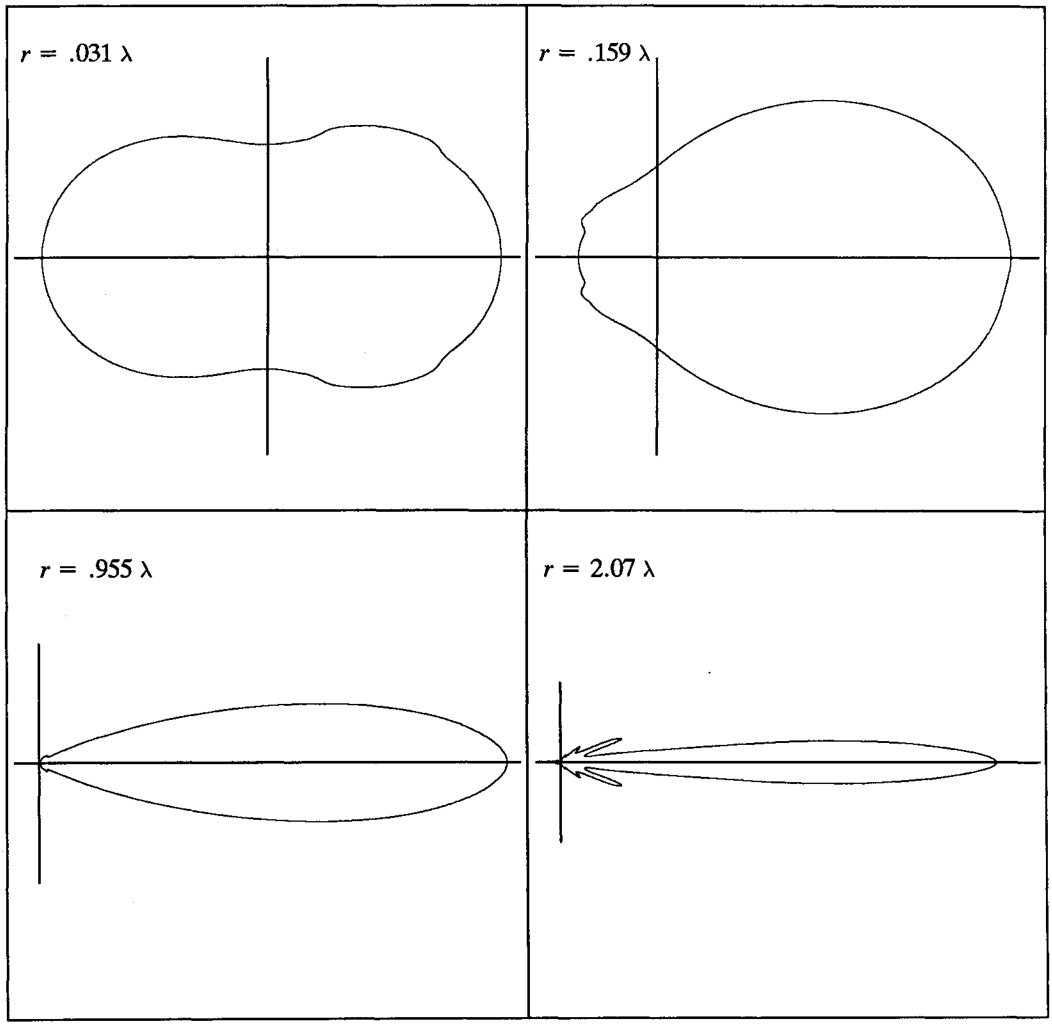

Figure 1.2: Angular scattering functions for spheres of water in which incident light is from the left. The scales differ from the first to the last by a factor of 10^9. Data from [Denman1966].

Complete tabulations of angular scattering functions for spheres may be found in [Denman1966]. These were calculated using a theoretical model developed by Gustav Mie [Mie1908], and discussed in most advanced optics texts, for example [Born1975]. This is the currently accepted model of scattering by small spherical particles, however the method of computation is difficult and time consuming since it requires summing Legendre polynomials of higher order than are generally tabulated. (The series becomes convergent when the order of the polynomial is approximately the same as $2\pi r/\lambda$, which can be greater than 100 in fog) [Denman1966]. Hence the use of tables is by far to be preferred.

Total Scattering and Extinction

The phrase *total scattering* refers to the total amount of light scattered from the beam (in all directions).

The scattering cross section, σ_{sc} , of a particle, which is not, in general the same as the geometric cross section, gives the total amount of light scattered by an individual particle per unit incident irradiance. The scattering efficiency factor, Q_{sc} , is the ratio of the geometric cross section to the scattering cross section. This has considerable variation as shown in Figure 1.3.

Rayleigh particles (particularly air molecules) show a high degree of selectivity, with the amount of scattering being inversely proportional to the fourth power of wavelength, while particles significantly greater in size than the wavelength of the light show no selectivity at all. For large particles, Q_{sc} becomes more nearly proportional to λ^{-2}, until particle size nears $\lambda / 2$, at which it levels off at the value two after some oscillation. That the asymptotic ratio of the scattering cross section to the geometric cross section is two follows from Babinet's principle, which states that the amount of light refracted must be the amount intersected by the geometric cross-section, and the fact that the same amount will be either reflected or refracted [Middleton1952].

While theoretical models do exist for the total scattering efficiency of small spheres, tabulations of experimentally observed values may be found in appendices I and J of [McCartney1976], and they are sufficiently precise for our purposes. It was from there that the data for Figure 1.3 was obtained.

It is useful to define an extinction coefficient, $\beta_{ex}^{(\lambda)}$, which gives the total amount of light lost from the beam per unit distance and depends on both the scattering efficiencies of the particles involved and the density of particles along the path of the beam. As with Q_{sc} , $\beta_{ex}^{(\lambda)}$ will be found through interpolation of tabulated values. Since a natural haze consists of particles of sizes distributed over a continuum, the fine structure on the curve of Figure 1.3 has no effect on the optical properties of real haze, and the effect of the large oscillations is less than it might appear. In fact, fog normally has less than 5% selectivity, with mode droplet radius between 1 and 10μ [McCartney1976].

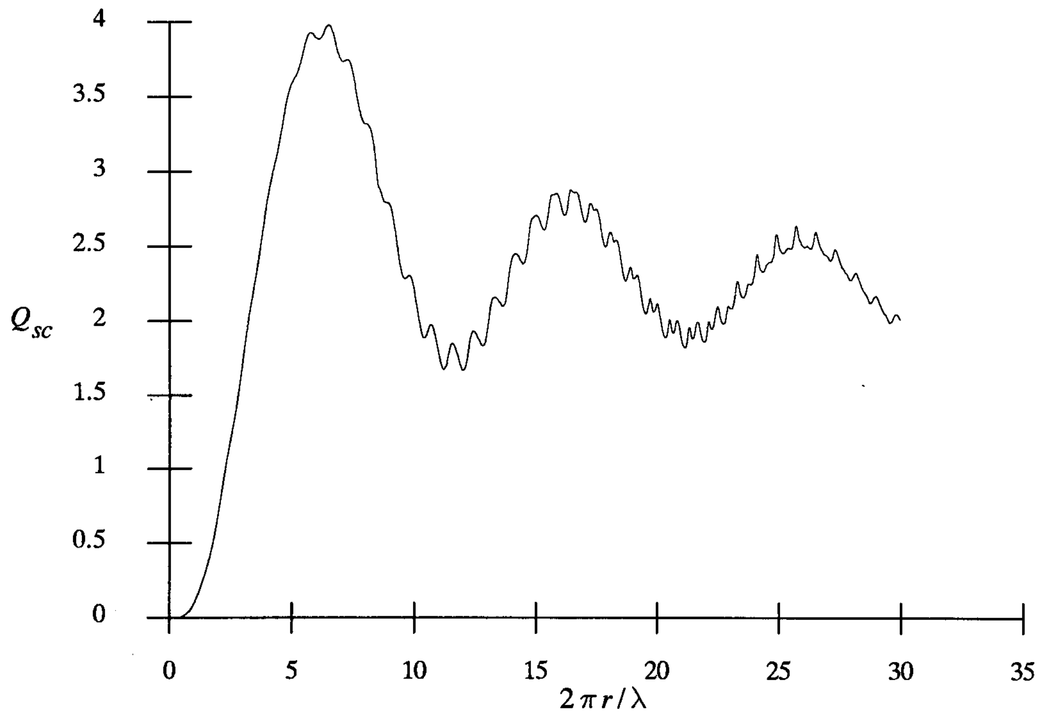

Figure 1.3 Scattering Efficiency of Wet Haze

As light passes through a region containing a scattering medium, some light is scattered out of the beam, reducing the intensity of the direct beam. If particles in the scattering medium have an extinction coefficient $\beta_{ex}^{(\lambda)}$, then in a differential segment of the beam of length dx , where the intensity is $I(\lambda)$, the amount lost from the beam is

$$dI_{\lambda} = \beta_{ex}^{(\lambda)} I_{\lambda}\, dx \tag{1.1}$$

By integrating (1.1) once for each wavelength sampled, the spectral intensity distribution of a beam with original intensity $I_{\lambda 0}$ after travelling a distance r through the medium is found.

$$\int_{I_{\lambda 0}}^{I_{\lambda f}} \frac{d\,I}{I} = \beta_{ex}^{(\lambda)} \int_0^r dx \tag{1.2}$$

$$I_{\lambda f} = I_{\lambda 0} e^{-\beta_{ex}^{(\lambda)} r} \tag{1.3}$$

From (1.3) we may derive the transmittance $T^{(\lambda)}$ and opacity $O^{(\lambda)}$ of a given thickness of a medium of known scattering properties:

$$\mathrm{T}^{(\lambda)} = \frac{I_f}{I_0} = e^{-\beta_{ex}^{(\lambda)} r} \tag{1.4}$$

$$\mathrm{O}^{(\lambda)} = 1 - \mathrm{T}^{(\lambda)} = 1 - e^{-\beta_{ex}^{(\lambda)} r} \tag{1.5}$$

An Approximation Approach

The above discussion has concerned itself with the physical processes that scatter light as it passes through the atmosphere. Of course, light can be scattered many times between entering the atmosphere and being captured by the eye, and each scattering event has the full physical complexity discussed above. Furthermore, the number of scattering events characteristic of different phenomena, such as skylight, clouds, or fog, is bound to be different. How can we capture this physical complexity of these phenomena into a model which can produce the right visual effects?

We would like to make a model that produces good-looking images in a reasonable amount of time, regardless of how well it (physically) models the actual atmosphere. We begin by modelling paths in which light is scattered only once between its entry into the atmosphere and its capture by the eye. This calculation is the simplest and most economical and gives a first order approximation which can be elaborated until visually satisfactory images are produced. Since the scattering geometry is the most difficult part of the calculation we hope to keep single scattering geometry while altering the scattering factors to simulate the effects of multiple scattering.

In choosing scattering functions we begin with measured scattering functions for single scattering, and examine the images they produce. Only if they are unsatisfactory will we elaborate the model.

Computational Methods for Modelling Scattering

We now consider the path of a typical ray of light from a source to the eyepoint. In all that follows the line of sight could just as well be a line from a reflective surface to a source, or the light source could equally well be a reflective surface.

Figure 1.4 shows the path a typical ray might take. It shall be assumed that the majority of the light is scattered once. The transitions from one medium to another are modelled as sharp discontinuities. While the justification for this model is not always satisfactory, it has not been found to produce any artifacts. In some cases, such as at the edge of a fog bank, the density of scattering particles (in this case fog droplets) does change sufficiently rapidly to justify this model.

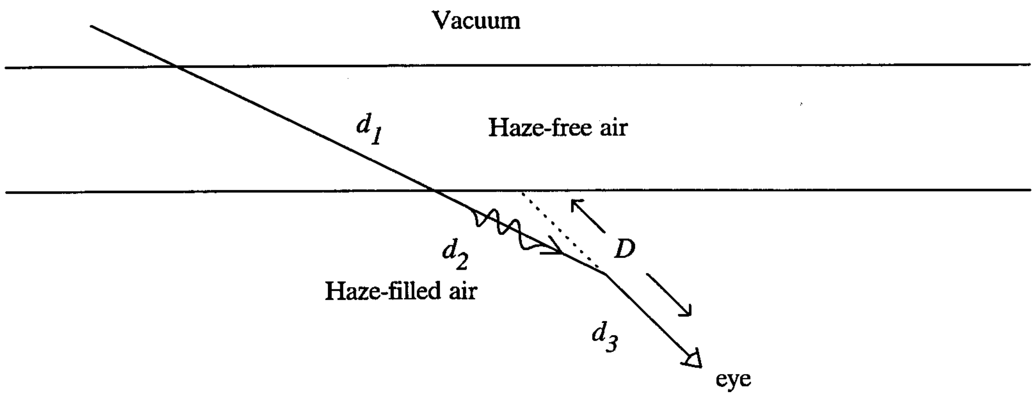

Figure 1.4 The Path of a Typical Ray

Either D or d_1 of Figure 1.4 may be zero: $d_1 = 0$ indicates a source within the haze or fog; $D = 0$ indicates an absence of haze or fog. If both are zero, the problem reduces to ray-tracing in a vacuum. In the most general case, neither is zero.

Consider, then, the most general (single scattering) path. Light enters the atmosphere (either from outside the atmosphere or from a source on the earth) with spectral intensity $I_{\lambda 0}$. Values for $I_{\lambda 0}$ from [Fritz1951] may be used for the sun, values for other sources may be found in handbooks of physical constants such as [AIP1972]. Such references give the intensity at intervals of 5 nm or less, however for our purposes it is sufficient to use 10 nm intervals.

As light travels along its path it is attenuated by the haze it encounters, as well as by the air itself. In the case of sunlight travelling through the atmosphere the effect of haze is negligible at altitudes above 8 - 16 km, while molecular scattering has some effect at altitudes as high as 100 km. After travelling a distance d_1 it enters a region with an increased density of scattering particles. For the sake of discussion let us say it enters the haze layer (in the case of a source on the surface, or at least lower than 16 km, $d_1 = 0$). By this time the intensity is

$$I_1 = I_0 e^{-\beta_{ex_m}^{(\lambda)} d_1} \tag{1.6}$$

The subscript m on β_{ex} is to indicate molecular scattering. From now on, we shall assume that β_{ex} and I are wavelength dependent, even though the argument may be omitted.

After travelling a distance d_2 through the haze some of the light is scattered toward the viewer, scattering at an angle ψ. Before scattering the intensity is

$$I_2 = I_1 e^{-(\beta_{ex_m} + \beta_{ex_p}) d_2} \tag{1.7}$$

Here the subscript p indicates scattering by particles. Of I_2 only a fraction $\beta_{sc}^{(\lambda)}(\psi)$ is scattered toward the eye. The intensity leaving the scattering centre in the direction of the eye is

$$I_3 = I_2 \beta_{sc}^{(\lambda)}(\psi) \tag{1.8}$$

Finally, a portion of the beam is lost on its way to the eye, yielding

$$I_4 = I_3 e^{-(\beta_{ex_m} + \beta_{ex_p}) d_3} \tag{1.9}$$

after travelling a distance d_3 through haze-filled atmosphere. Substituting back and grouping terms yields

$$I_4 = I_0 \beta_{sc}^{(\lambda)}(\psi) \exp(-\beta_{ex_m} d_1 - (d_2 + d_3)(\beta_{ex_m} + \beta_{ex_p})) \tag{1.10}$$

This gives the amount of light arriving at a point from one path. Light reaching a given point from one direction comes from many paths all of whose last segments are collinear. To find the total amount reaching a point from a given direction it is necessary to integrate over all possible values of d_3.

$$I = I_0 \beta_{sc}^{(\lambda)}(\psi) \int_0^D \exp(-\beta_{ex_m} d_1 - (d_2 + d_3)(\beta_{ex_m} + \beta_{ex_p})) d\, d_3 \tag{1.11}$$

where D is the maximum value d_3 can take on. Note that in general the angle ψ depends on d_3. Only for a collimated incoming beam is ψ a constant, and in this case $\beta_{sc}^{(\lambda)}(\psi)$ can be taken out of the integral.

The amount of light striking the final segment, and potentially being scattered, is proportional to the sine of the scattering angle, that is, the angle between the final segment and the previous one. This is because the projection of the final segment onto a plane perpendicular to the direction of propagation varies in this way. For a distributed source such as the sun, the angle will not be defined as a single value, but must be treated as varying from one extreme to the other. The $\sin\psi$ term which is produced from these considerations may be absorbed into $\beta_{sc}^{(\lambda)}(\psi)$, so that the form of equation (1.11) is unchanged.

For some simple geometries this integral may be found symbolically; in general it requires numerical evaluation. Because the integral must be evaluated for each of the wavelengths sampled, symbolic evaluation is by far the preferred approach wherever it can be done.

The integral of (1.11) only gives the contribution of the haze or fog to the light for one line of sight or reflection; this contribution must be combined with any other contributions, such as the light reflected by some object partially obscured by the haze or fog, taking into account the opacity of the haze as given by (1.5).

Summary

In the past, efforts in realistic computer imagery have been concentrated in modelling reflections, while the effect of media between reflective surfaces has received little attention. These effects are very important for some scenes, and cannot be ignored. Recognizing this, some workers have derived the effect of the intervening media for some special geometries, for example clouds [Blinn1982], or made simple approximations which, for some cases yield such pleasing results as the foggy chessmen of [Whitted1981]. In this chapter, a general approach has been given, from which the effects of single scattering can be derived for any geometry. Some scenes, by their nature, will require ray tracing, while a number of significant special cases can be simplified to the point that sophisticated rendering algorithms are not required. In the next chapter the effects of large amounts of air will be considered when finding the colour of the sun and sky, and the colour reflected by clouds at sunset. Following that the simple case of fog lit by the sun will be considered.

The Colour of the Sun and the Sky

It is now, I believe, generally admitted that the light which we receive from the clear sky is due in one way or another to small suspended particles which divert the light from its regular course.
– Lord Rayleigh (1871)

While much work has been done recently on the simulation of natural scenes, including clouds (most notably [Gardner1985], while [Kajiya1984] and [Voss1983] have also made significant contributions) and artificially generated terrain, ([Marshall1980], [Fournier1982], [Mandelbrot1983], [Gardner1984] and [Reeves1985] to name but a few), rather less work has appeared regarding the colour of the sky in which the clouds appear, and the sun, the light source illuminating the clouds and terrain.

Everyday observations tell us that the sky is blue, and the sun is bright white, at least at noon on a clear day. Upon closer inspection the sky appears to vary in colour from a strong blue at the zenith to a very unsaturated shade at the horizon. Evening and morning observations disagree entirely with the conclusions reached at midday. At these two times the sun often appears in shades varying from yellow through orange to deep red, while the sky may be blue, or it may take on pink or purple tints as the sun colours it, particularly near the horizon.

These observations led Nelson Max to vary the colour of the sky with viewing angle from some colour at the horizon to some other colour at the zenith [Max1981]. This led to quite a nice afternoon sky, and a dramatic, although not quite realistic evening sky. This method was not based on a model of the interaction of light with the air, but rather on a simple heuristic.

An even simpler approach was used by Tom Duff in [Duff1985]. His backgrounds consist of two seed colours interpolated from the top scanline to the bottom scanline of an image. For a sky the top colour is some appropriate blue, while the bottom is yellow. The bottom is obscured by the landscape, leaving an unsaturated blue at the horizon.

Both of these approaches are *ad hoc*, in that the colourations of various parts of the scene are uncorrelated. The use of the lighting model presented in Chapter 1 can unite the parts of an image, producing greater realism.

Let us turn our attention, then, to the cause of the colours of the sun and the sky. In the absence of all dust and moisture in the atmosphere, the sky would have a colour approximately a constant blue, that colour being the additive mixture of all the wavelengths represented in the solar spectrum, each weighted by the fraction of that wavelength scattered. On a clear, dry day, just after a summer rain, the sky is as nearly this way as we ever see it when viewed from the ground. From altitudes above the haze the much deeper blue of the unobscured sky may be seen. Naturally as altitude increases, the amount of air present to scatter light towards the eye decreases until the sky becomes totally dark in space.

Both the colour of the sun and that of the sky are at least partially a result of Rayleigh scattering. The effect of haze in the atmosphere is also important in determining the colour of the sky, even at times when the sun is not at all obscured by haze, as it lends the whitish colour to the horizon at noon.

In the remainder of this chapter the details of the effects of scattering on sunlight and sky colour will be considered. The effects of each type of scattering will be considered first with regard to the colour of the sun. Following that the colour of the sky will be explained. As an aid in applying the lighting model the geometry of the sky will then be considered, in particular the distance along an arbitrary line, between two spherical shells (such as the edge of the atmosphere and the start of the haze layer) will be derived. This will lead directly to a description of a method of computing the colours of both the sun and the sky.

The Colour of the Sun

A good approximation for the spectral distribution of sunlight as it enters the atmosphere may be found by using the values in [Fritz1951] as reproduced in Figure 2.1. The distribution peaks in the blue, but since blue is scattered most we see white when the sun is not near the horizon. Even if we saw the sun without the atmosphere, the colour would be so de-saturated that it would be hard to discern it from white.

As light passes through the atmosphere it is scattered both by haze and by molecules. Most dust particles and droplets of water vapour, being much larger than the wavelength of the light involved, essentially diffract the light [Born1975]. There is no significant dependence on wavelength, as long as it is much less than the particle size, so that colours are not differentially scattered.

The colour and intensity of sunlight are determined by the initial colour and intensity, reduced by the effects of scattering along the path. The path of a typical ray is shown in Figure 2.2, in which refraction has been neglected. Refraction only plays a significant rôle when the sun

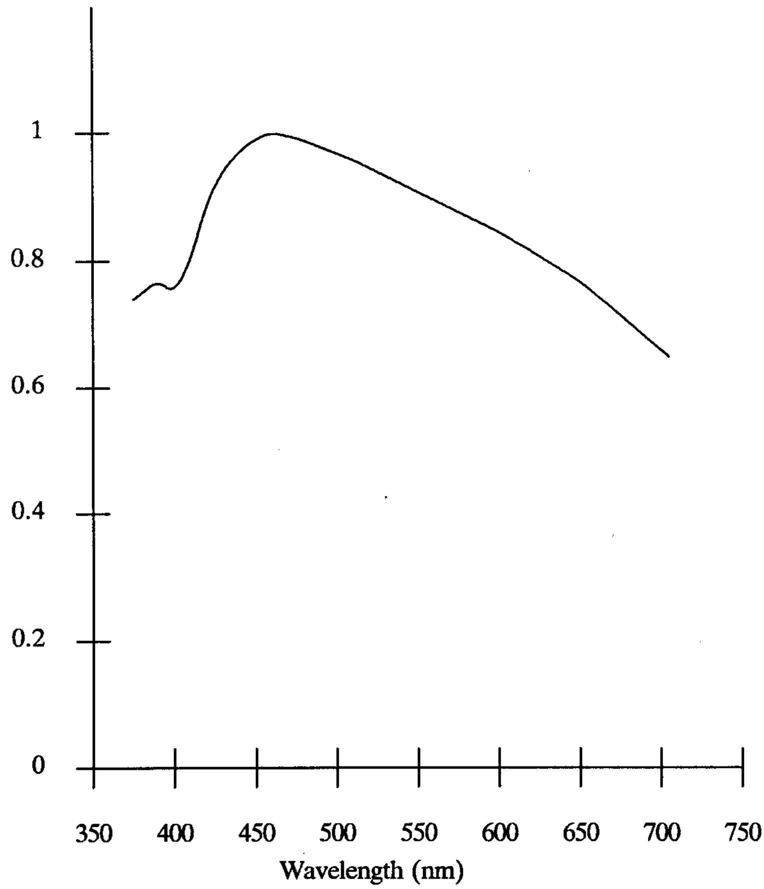

Figure 2.1 The Spectral Distribution of Sunlight

is near the horizon. Even then the bending is not more than a few degrees [McCartney1976].

The remaining effect of the atmosphere on the light from the sun is partial extinction, particularly of the short wavelengths. The effect due to a single type of scattering was given in (1.3) as

$$I_f = I_0 e^{-\beta_{ex}^{(\lambda)} r}$$

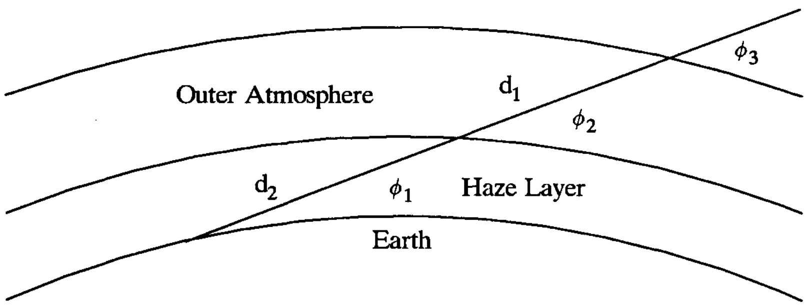

Figure 2.2 The Path of a Ray From the Sun

If the light travels a distance d_1 through the atmosphere before striking an appreciable amount of haze, and then travels a distance d_2 through the haze, the final intensity will be

$$I_\lambda = I_{\lambda 0} e^{\beta_{ex_m}^{(\lambda)} (d_1 + d_2) + \beta_{ex_p}^{(\lambda)} d_2} \tag{2.1}$$

Here $I_{\lambda 0}$ gives the spectral distribution of light entering the atmosphere, and β_{ex_i} are the scattering extinction coefficients (possibly wavelength dependent) of the scatterers. The subscripts, as before, indicate molecular and particulate (haze) scattering.

Note that in the diagram (Figure 2.2), the angles, ϕ_i, at which the ray crosses the arcs differ. These angles would differ most for the case in which one of them is 0, while for the case of $\phi_1 = 90°$ there is no variation. If ϕ_1 is zero (the sun is at the horizon), then we may calculate the maximum error in ϕ_3 as follows: Construct a triangle OPE with O at the centre of the earth, E at the eyepoint, and P at the point where the ray enters the atmosphere, as in Figure 2.3. Because the line of sight is horizontal, angle OEP is a right angle. The angle subtended by OP with the tangent of the outer edge of the atmosphere at P is ϕ_3.
Angle EPO is the complement of ϕ_3, so angle EOP must be ϕ_3. From the law of sines,

$$\cos\phi_3 = \frac{\text{EO}}{\text{OP}}$$

The mean radius of the earth is 6371 km, the atmosphere, beyond 100 km, is too rarefied to have any effect [CRC1981]. With these values, $\phi_3 = 10°$.

Interestingly enough, the angles vary less than this simple geometric argument would suggest: because of the variations in the index of refraction, the light does not follow a straight path, but curves downward, making the angles more nearly equal. In this case, the errors

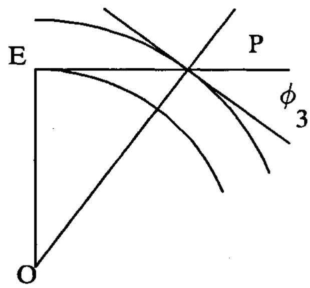

Figure 2.3 The Maximum Error in ϕ_3

introduced by two simplifications partially cancel. Thus far the effect of refraction has been ignored, however it lends some justification to the assumption that for all angles subtended by the line to the sun with the earth's surface, the angles made by the same ray with the outer surfaces of the haze layer and of the atmosphere are the same. Hence ϕ_i shall be assumed to be simply ϕ, the same angle regardless of the spherical shell concerned.

The Colour of the Sky

The colour of the sky found on a given line of sight depends both on the amount of scattering into that line of sight and the colour of light scattered. To simplify matters, we shall assume single scattering. The blue colour of the sky, that due entirely to Rayleigh scattering, will be modelled as a constant colour throughout the sky. This leaves only the colour being scattered by haze particles along the line of sight. The spectral intensity distribution of scattered light depends on the colour of the light incident on the line of sight, and the angle of scattering.

Ignoring refraction we find that for any angle of inclination of the sun, the angle between the line of sight and a line to the sun is effectively the same at all those points along the line of sight that are much closer to the viewer than to the sun. This follows immediately from the approximation of parallel rays from a distant light source. The sun is some 180 million kilometres away. Any scattering which occurs along the line of sight must be somewhere along a line of length much less than 6371 kilometres long (the radius of the earth) [CRC1981]. The difference in scattering angles at two points along such a line is limited to the angle subtended by two rays from the sun which intersect that line. For these distances the maximum difference is .001°.

We shall assume the haze layer to be of constant density, with an abrupt edge at some distance $R_H - R_E$ from the surface of the earth. In reality, the haze layer does end quite suddenly, although not discontinuously. This can be observed from the air, or from high altitudes on the ground. Referring back to Figure 2.1 it becomes clear why the edge of the haze layer is not visible as a sharp discontinuity in the colour of the sky: the distance d_2 of that figure varies smoothly and continuously as the angle of inclination of the sun changes.

Because the haze layer is very thin relative to the radius of the earth, the distance a near-horizontal ray travels before leaving the haze is much greater than the distance a near-vertical one travels. For this reason the effect of the haze on the colour of the sky is much greater for horizontal rays than for vertical ones.

Geometry of the Sky

The basic geometry of the sky is shown in Figure 2.4. The observer is on the surface of the earth, although the surface might equally well be any sphere concentric with the earth. In the diagram the geometry is simplified somewhat: only two concentric spheres are indicated rather than three. The radius of the atmosphere has been artificially increased for visibility. As we shall see, each of the distances d_1 through d_3 required in the lighting model may be obtained through an appropriate substitution of values into the radii and angles shown. The distance $d_i(r)$ is in each case given by y.

Suppose light enters the atmosphere at an angle of ϕ with the horizontal. Before entering the haze layer it travels a distance y through the atmosphere, taking r to be R_H (the radius of the haze layer), R to be R_A (the effective radius of the atmosphere) and η to be ϕ. The atmosphere does not have a uniform density, nor does it simply drop off exponentially as the simplest models assume. Fortunately the distance always appears multiplied by the density in the formulae giving the amount of light scattered or lost from the beam. By reducing the radius of the atmosphere appropriately, the effect of the variance in density may be approximated, without actually modelling it.

Next consider the distance the light travels through the haze before scattering. If R is taken as R_H, and r is allowed to vary from R_E to R_H, while η remains fixed at ϕ, then y gives this distance.

Finally the distance travelled after scattering (through haze and atmosphere) may be found by changing the angle η to that of the angle θ between the line of sight and the horizontal, and letting R vary as r did in the previous instance, while r is fixed at R_E, the radius of the earth.

What remains is to find y in terms of R, r and η. This is found in Appendix 1 to be:

$$y = \sqrt{R^2 - r^2\cos^2\eta} - r\sin\eta \tag{2.2}$$

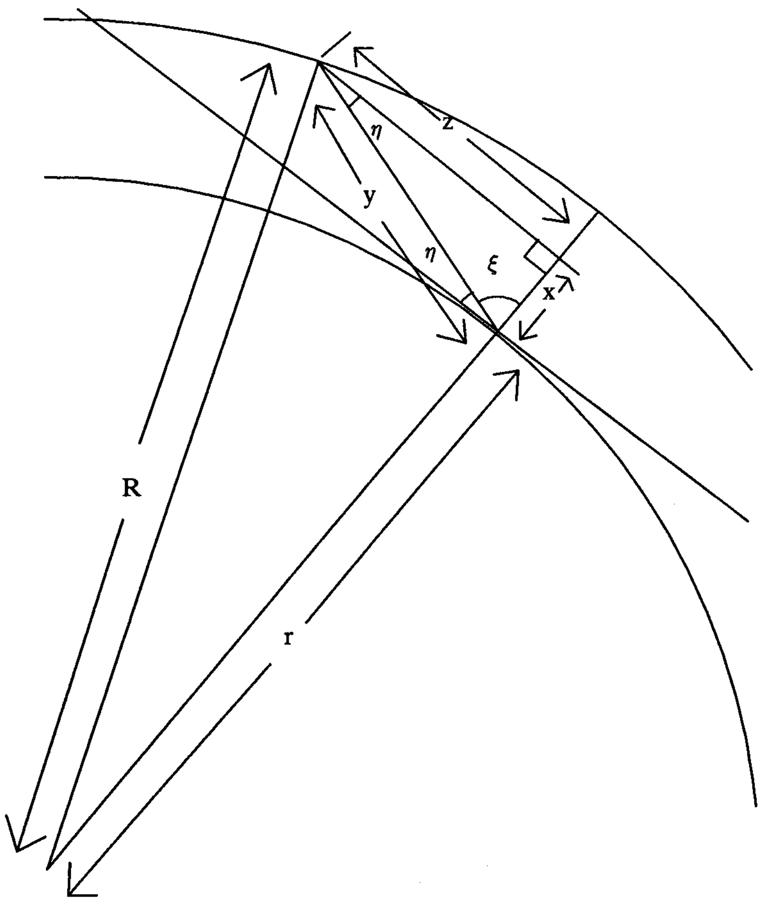

Figure 2.4 The Geometry of the Sky

Computing the Colours of the Sun and Sky

Knowing the distance y of Figure 2.4, the three distances required for computing the colour of the sky and the two needed for the colour of the sun may be found by substituting in values for the angles and radii.

We begin with the colour of the sun. Here we need the distances d_1 and d_2 of (2.1) which may be found from y of (2.2) using the appropriate substitutions. To find d_1 recall the geometry of Figure 2.4 and equation (2.2).

$$y = \sqrt{R^2 - r^2\cos^2\eta} - r\ \sin\phi \tag{2.2}$$

Substitute R_A for R, and the angle of inclination of the sun for η, and

$$d_1 = \sqrt{R_A{}^2 - R_H^2\cos^2\phi} - R_H\ \sin\phi \tag{2.3}$$

Similarly for d_2, use the same equation with R_H replacing R, and R_E in the place of r:

$$d_2 = \sqrt{R_H^2 - R_E{}^2\cos^2\phi} - R_E\ \sin\phi \tag{2.4}$$

This gives a final intensity of

$$I^{(\lambda)} = I_0^{(\lambda)} \exp\Big(\beta_{ex_p}^{(\lambda)} (\sqrt{R_H^2 - R_E{}^2\cos^2\phi} - R_E\ \sin\phi) \tag{2.5}$$
$$+ \beta_{ex_m}^{(\lambda)} (\sqrt{R_A{}^2 - R_H^2\cos^2\phi} + \sqrt{R_H^2 - R_E{}^2\cos^2\phi} - R_H\ \sin\phi - R_E\ \sin\phi) \Big)$$

The colour of the sky is somewhat more difficult to compute. Recalling (1.11) we substitute in the values for the angles and radii and integrate.

$$I = I_0 \beta_{sc}^{(\lambda)}(\psi) \int_0^{l_{MAX}} \exp(-\beta_{ex_m}\, d_1(r) - (d_2(r) + d_3(r))(\beta_{ex_m} + \beta_{ex_p}))\, dl \tag{2.6}$$

$$d_1 = \sqrt{R_A{}^2 - R_H^2\cos^2\phi} - R_H\ \sin\phi \tag{2.7}$$

$$d_2(r) = \sqrt{R_H^2 - r^2\cos^2\phi} - r\ \sin\phi \tag{2.8}$$

$$d_3(r) = \sqrt{r^2 - R_E{}^2\cos^2\theta} - R_E\ \sin\theta \tag{2.9}$$

Here θ is the angle of inclination of the line of sight, while ϕ is that of the line to the sun.

The integral to be performed is a path integral in which the distance along the path may be found from

$$l(r) = \frac{l_{MAX}\, r}{R_H - R_E} - l_{MAX}\, R_E \tag{2.10}$$

$$d\,l = \frac{l_{MAX}}{R_H - R_E}\, d\,r \tag{2.11}$$

The factor $l_{MAX} / (R_H - R_E)$ may be taken out of the integral. Note that this factor is a function of the angle ϕ that the line of sight makes with the horizontal, so it will have to be recalculated for each new line of sight.

With the change in variable, the limits of the integral become R_E and R_H. The final expression for the intensity of the haze is then

$$I^{(\lambda)} = \frac{l_{MAX}}{R_H - R_E} I_0 \beta_{sc}^{(\lambda)}(\psi) e^{-\beta_{ex_m}^{(\lambda)} d_1} \int_{R_E}^{R_H} e^{-(d_2(r) + d_3(r))(\beta_{ex_m}^{(\lambda)} + \beta_{ex_p}^{(\lambda)})}\, dr \tag{2.12}$$

The upper limit of the integral of (2.6) is the distance along the line of sight to the outer edge of the haze layer.

$$l_{MAX} = \sqrt{R_H^2 - R_E^2 \cos^2\theta} - R_E \sin\theta \tag{2.13}$$

From (1.5) and the definition of l_{MAX}, the opacity of the haze layer is simply

$$O = 1 - e^{-\beta_{ex_p} l_{MAX}} \tag{2.14}$$

With the colour and opacity of the haze layer computed, they may be combined with the underlying blue sky yielding the actual colour of the sky. The underlying colour of the blue sky changes little in spectral distribution during the course of a day, however the brightness of a clear sky changes by a factor of 100 from noon to sunset.

Since the underlying colour itself doesn't change, it may be computed once for a typical sky, or estimated by eye. The eye is very forgiving with regard to variations in absolute colour but much more demanding of correct relative colours, hence the base colour of the sky is of little importance [Cowan1985], as long as it is a reasonable shade of blue.

With regard to the change of brightness of the sky as the sun's position changes, the eye also compensates for this, leaving the impression that it is, say, half as bright at sunset as at noon, rather than one percent. Hence, the underlying blue should be reduced to match the reduction in intensity of the haze.

This will give an entire image with intensities in the very bottom of the range of available intensities. Photographers of outdoor scenes have had to deal with this since the invention of photography. If the exposure settings correct at noon were used at dusk, the density of exposed grains of silver in the photograph would be so slight that the image would be unrecognizable. The computation of the colour of the haze layer will give the correct intensity for sunset, and so will produce an image one hundredth as bright as at noon, just as photographic film will produce an image one hundredth as dense. Just as the photographer increases the aperture and exposure time for an evening shot, so must the entire image be brightened by increasing the intensities of all the colours by a constant factor, after the colours of the sky and haze have been combined.

Efficiency Considerations

As presented thus far, computing the colour of a single pixel, that is, the colour along one line of sight, can require over one second of VAX 11/780 processing time. In the interests of efficiency, one might consider converting the scattering coefficient vectors, and the incident intensity vector, to a 3 coordinate colour space, such as CIE XYZ or RGB, before performing such costly operations as the integration. This is *not* recommended. While it does reduce the cost of computing a single image by a factor of eleven, this does not change the inherent complexity of the computation, nor does it yield correct results. The non-linearity of the integrand implies that the operations of evaluating the integral and changing colour spaces do not commute.

A much better method of reducing the cost of the algorithm uses spatial coherency to make the cost of the algorithm less than $O(n \times m)$ for any reasonable values of m and n, the height and width of the image. The cost will vary for different images, but for a particular sky, with all parameters other than resolution held fixed, this reduction in complexity may be obtained.

Consider the fact that the sky is rather much the same all over. In a raster representation with finite-precision representations for the colours there will generally be large spans of constant colour. In order to avoid computing neighbouring pixels of the same colour, the parameters may be checked (particularly the viewing angle) before the integral is performed. If the angle hasn't changed more than some small threshold, the integral need not be performed. Since the predominant cost of the algorithm is floating point function calls, (the cost of writing the output is very small compared to a few integrations), reducing the number of integrations by any factor will reduce the total run time by nearly the same factor.

The threshold may be selected based on the number of bits of precision the final colours will have, or the time permitted to produce an image. The complexity is reduced for such image sizes in which the cost of the function calls continues to dominate the cost of writing the output, since an increase in resolution will increase the separation of pixels with sufficiently different view angles, leaving the number of calls roughly constant. This has reduced run time from an hour for a 128 $\times$ 64 image to a few minutes, with no visible loss of fidelity when the image was displayed on a frame buffer with 8 bits for each of red, green and blue.

The size of an acceptable threshold varies with the complexity of the image. The variance in colour (particularly the amount of red) in a sunset is greater than that in a midday sky, so the colour must be re-evaluated more often to prevent mach bands in the sky.

Because of the nature of the integral, and the variability of the parameters, it was not known at the outset what an appropriate interval for integration would be. An adaptive Simpson's rule evaluator [Burden1978] was implemented with a view toward using it to determine an empirical interval size for use by a non-adaptive Simpson's rule evaluator (with a fixed number of function evaluations). In the adaptive algorithm the integral over an interval is estimated using the function values at the end points and the midpoint. The integral for each of the two halves is then evaluated and the sum is compared with the previous estimate. If the error is less than some threshold, the sum is used as the result. Otherwise the interval is bisected and each half is integrated recursively. In a few trial runs the depth of recursion varied greatly depending on the part of the sky, haze density, and position of the sun. For this reason, the adaptive approach was retained.

In order to avoid re-evaluation of the integral where it is highly unlikely to change, the routine checks the first estimate, compares it with the first estimate from the previous call, and returns the previous result if the first estimates are very nearly the same. This implementation can produce a 256 $\times$ 128 image in less than 400 seconds if the sun is inclined more than 20°, but requires over 4000 seconds for some sunsets.

A cruder method of reducing the time required to produce an image is to render at a low resolution and scale the image up as a post-process. Because sky colours vary particularly smoothly, they lend themselves readily to this technique as well. Even a sky is not smooth enough to be scaled up very far by simple pixel replication, so a more sophisticated technique is needed. Such a technique, using B-splines to represent the image, has been developed, and is presented in [Klassen1986].

No reasonably accurate method will be able to produce a background sky quickly. For this reason it is preferred to pre-compute a sky and store it, either at a low resolution to be later expanded by the technique of [Klassen1986], or at a high resolution, to be used directly. Such an image, computed separately from the remainder of the scene, may be re-used for a large number of scenes, with the cost of the initial computation amortized over time.

Since sky colours vary little horizontally a background image of the sky will compact well under run-length encoding. A single background sky is often sufficient for several minutes of animation, sometimes a large fraction of an hour, as long as the direction of view doesn't change. For an animated sequence in which there is camera motion, one image could be stored per camera direction. The (space) cost of doing so is reasonable if the data is run-length encoded.

Summary

Computer generated imagery has, until now, used *ad hoc* methods of determining the colours of the sun and the sky. With this unified lighting model the sun, sky, clouds and even objects on the ground reflecting light from the sun and the sky can fit together, being computed from the same lighting parameters. In this way images of outdoor scenes need no longer have the appearance of being pasted together, but rather form one harmonious unit.

The cost of computing a scene in this way is not small, however the expensive elements may be precomputed once and used for an entire series of scenes. The major increase in cost is that of computing the sky colour; the colour of the sun may be computed once per scene and used as the illuminant for the entire ground surface, and once per cloud and used for the whole cloud. The parameters controlling the lighting vary little enough over a single cloud or over the visible portion of the earth's surface to make little difference in the colour of their illumination.

Fog

''And do you observe, where those trees slope down the hill,'' (indicating them with a sweep of the hand, and with all the patronising air of the man who has himself arranged the landscape), ''how the mists rising from the river fill up exactly those intervals where we need indistinctness, for artistic effect? Here in the foreground, a few clear touches are not amiss: but a back-ground without mist, you know! It is simply barbarous! Yes, we need indistinctness!''

The orator looked so pointedly at me as he uttered these words, that I felt bound to reply, by murmuring something to the effect that I hardly felt the need myself - and that I enjoyed looking at a thing, better, when I could see it.

– Lewis Carroll, *Sylvie and Bruno*

We now leave the realm of large distances and large amounts of Rayleigh scattering for a look at things closer to the surface of the earth. Since Rayleigh scatterers are so diffuse in the atmosphere, they can only have a significant effect over large distances (50 km or more). The particle density of fog or mist droplets, however, can be sufficient to reduce visibility to anywhere from a metre or so to one or two kilometres.

To those living in relatively dry regions, fog is an interesting and unusual phenomenon, as it was to the orator in *Sylvie and Bruno*. In some places and seasons, fog is a daily occurrence which has a very direct effect on which flora and fauna can survive.

To those in the transportation and communications industries, fog is an obstacle to be overcome, as its ability to reduce visibility can be dangerous for surface transportation and can paralyse air traffic. For this reason, a good deal of research has been done into the optical effects of fog, both in the visible frequencies and in those used for radar and telecommunications. A number of sources are available for those interested in simulating the formation and growth of fog: introductory discussions may be found in [George1951] and [Myers1968]; as usual, the reader is referred to [McCartney1976] for a general discussion and further bibliographic notes.

Whether intended for simulators, in which realism is paramount, or simply for making beautiful pictures, in which the subtle touches fog can impart are unlikely to go unappreciated, any rendering package intended for producing outdoor scenes should be able to include fog in a scene.

In order to model light passing through a dispersive medium, the renderer must have the ability to decide which portions of a ray interact with the medium, and how they are affected. The manner in which they are affected was set forth when the lighting model was introduced. In the discussion of the colours of the sun and the sky several special cases were considered for which the portions of the ray that are affected in particular ways are known from the geometry. In those cases, the effect of Rayleigh scattering was important, although without haze the results would have been quite different. Here we shall consider a simple case as well. The lighting of fog and objects in fog when lit by the sun is simple enough be rendered without the use of ray tracing, but still interesting enough to demonstrate the usefulness of the model, even if shadows are ignored. First we shall consider the general problem of lighting of fog without considering the specifics of rendering with a particular geometry; then we shall consider the specific case of sunlit fog.

Lighting Considerations in the Presence of Fog

Scenes containing fog or mist may be divided into four classes. The scene may be predominantly lit by the sun or there may be artificial lighting. In either case the eyepoint may be within or without the fog. As stated above, we will limit ourselves to one instructive simple case; hence only naturally lit fog will be considered, and even that without shadows. The addition of shadows (specifically shadowed regions of fog) is a more difficult problem which, while it does not require full ray tracing, does require some form of shadow casting.

The prime distinctions between natural and artificial lighting stem from the relative strengths and distances of the sources. Natural light sources consist of the sun, moon and stars. The last of these three may be completely neglected for scenes containing fog, since on a foggy night with no moon, one simply cannot see anything without artificial light. The chief difference between light from the sun and that from the moon is the intensity; the angle subtended by the moon in the sky is, within 3%, the same as that subtended by the sun [CRC1981]. If the sun were a point source (which it isn't), at a mean distance of 1.5×10^8 km (which it is), the angle between rays striking the earth one kilometre apart would be on the order of 10^{-9} radians 10^{-7} degrees. Since the radius of the sun is 6.96×10^5 km, the angle subtended by two rays from opposite edges of the sun striking the same point on the earth is 9.28×10^{-3} radians or .53°, ignoring the magnification caused by refraction. Even this angle is nearly insignificant, except where the size of the sun causes regions to be partially illuminated and partly in shadow, and since the discussion of umbrae and penumbrae belongs elsewhere, the light from all natural light sources shall be assumed to have parallel rays extending throughout the region of interest.

The sun's intensity reaches a maximum of approximately 1.6×10^5 candelas per square centimetre as observed from the earth's surface, and is around $600\,\mathrm{cd\ cm}^{-2}$ at sunset. Contrast this with a 1000 Watt sodium vapour lamp (one of the most efficient types used in street lights) producing $3.5\times10^{-2}\mathrm{cd\ cm}^{-2}$ at a distance of 20 metres. [CRC1981] Clearly the effect of the street lamp is small when the sun is shining.

The light produced by any light source spreads as it propagates. In the absence of scattering influences, the cross sectional area of the beam will increase as the square of distance. This holds both for the point source which directs light uniformly outward in a spherical distribution and for the thin cone of light. Since the total amount of energy passing through any closed surface surrounding the source is constant, the intensity, a measure of the energy passing through a unit area at a given distance, must drop as the square of distance from the source. The distance to the sun is so great that a difference of a few kilometres makes little difference to the total distance to the light source, while artificial lights, if they are near enough to be visible, are near enough that a fraction of a kilometer has a very significant impact on the luminance of the light.

A simple, although uninteresting case is that of artificial light sources within a fog bank, with the observer outside of the fog. If it is not dark, only a very strong source will contrast strongly with its surroundings and have a significant effect on their lighting. Since the intensity of light falls as the square of distance, the observer must be quite close to the source unless there is little ambient lighting. With fog reducing contrast both of the source with its surroundings and of the different parts of the surroundings, the effect of the light source drops still further. For this reason, artificial light sources are generally too weak to be seen from outside a fog bank if the source is within, except when it is quite dark outside the fog and the source is quite near the edge of the fog.

Scattering Due to Fog

Whatever the lighting geometry, the two predominant features of light scattering by fog are wavelength independence and directionality [McCartney1976]. To say that scattering is wavelength independent is generalizing somewhat; the transition from mist to fog has been identified as the point at which wavelength dependence is lost for the visible part of the spectrum [Eldridge1969].

An effect which is quite picturesque and a direct result of the directional nature of scattering is the manner in which fog is lit in a wooded area when the sun is low in the sky. Trees cast shadows in the fog itself leaving the appearance of sharply defined holes in the fog. Such holes are visible only when viewed towards the sun; they disappear when one turns around [Minnaert1954]. A similar effect, showing that the dust particles commonly suspended in the air have a similar scattering behaviour, is the sunbeam which appears when the sun shines through a window into a room. In the case of fog, the beam is only visible if viewed from within an angle of roughly 45° from the direction of the sun, which follows from the angular scattering cross section of water droplets in the size range which make up fog. Sunbeams caused by dust may be seen from other angles as well (try it). To model either of these effect requires either making special assumptions based on the particular geometry [Max1986] or a more complete approach involving either ray tracing or radiosity [Nishita1986]. The unreported work of Eihachiro Nakamae, whose results were shown at the 1986 SIGGRAPH conference and on the back cover of the proceedings [Nakamae1986], has presumably applied one of these approaches.

There are two contributions to the colour of a pixel: the light reflected by the object which is visible in that direction, and the light scattered in the direction of the eye by the fog along the line of sight. The opacity of the fog depends upon the distance to the object being viewed. This is given in (1.5); for fog the extinction coefficient, β_{ex} , is relatively wavelength independent and varies from 15 km^{-1} to 40 km^{-1} , whereas for haze it varies from .02 km^{-1} to .05 km^{-1}. This can be seen from the range of particle sizes for the two classes and Figure 1.3 in Chapter 1.

Assume that the fog is either in a steady state or evolving slowly enough that there is very little change in the internal energy of a unit volume of fog. In that case, all light which enters the unit volume is matched by an equal amount leaving. If light were being absorbed, the internal energy would increase and the temperature rise, leading to a reduction in the number and size of the droplets in the unit volume. Hence, under the steady state assumption, light is only scattered. The amount of absorption of light in the visible range is fairly small even in evolving fog, so this is not a very restrictive assumption.

The angular scattering function, β_{sc} , for fog droplets is much the same as that for wet haze, only more extremely directional. This function depends very strongly on droplet size, but is characterised by strong directionality. Because fogs and mists contain droplets of non-uniform size, the side lobes contribute little to the optical effect of fog.

Knowing something of the optical properties of fog, let us turn our attention to the geometry of naturally lit fog.

The Geometry of Natural Lighting

We assume that the earth is flat. If the sun is not too low in the sky, this approximation gives good results and it does so much more cheaply than a curved-earth model would. A simple case, useful as a test of the model, has fog of uniform density from the surface of the earth to some constant altitude. For simplicity, R_E will be taken as a constant. This corresponds to prairie or ocean in our world, and for the flat earth, a perfectly smooth planar surface. For these assumptions, the illumination of the ground plane and light reaching the eye from it may be easily derived. With this simple case solved, other cases may be solved by substitution of variables. As an example, to find the lighting of an object a height h_O above the ground, the same equations are used as for an object at ground level, with the radius of the earth increased by h_O ; for hills, the radius of the earth is allowed to vary. The assumption that the fog stops at a constant altitude, along with the flat earth approximation implies that the top of the fog has a constant height and that light strikes it at a constant angle. That light from the sun is always from the same direction, regardless of the location of the part of the fog being considered, follows from the fact that rays from the sun are so nearly parallel.

To find the colour perceived along a particular line of sight we must consider both the light reflected by any objects along the line of sight, attenuated by the intervening fog, and the light scattered toward the eye by fog along the line of sight. The former component is coloured by the reflectance of the object while the latter is the colour of the illuminant (here, sunlight). The light scattered toward the eye by fog along the line of sight will be called the *fog* component.

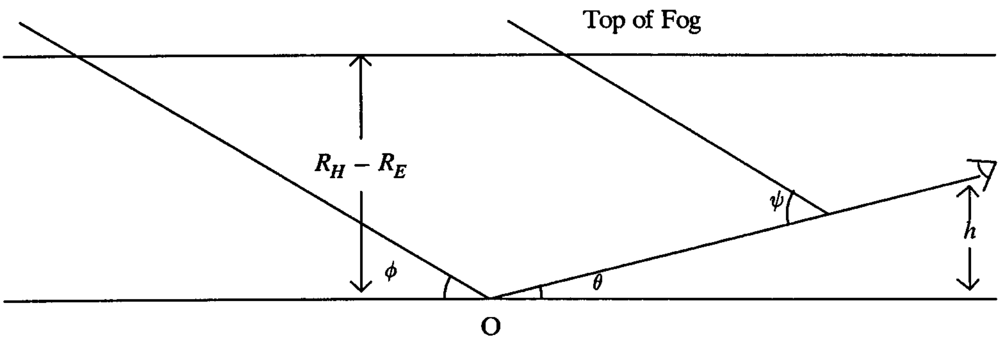

Figure 3.1 Lighting in the Presence of Fog

The Object Component

Consider first the light striking, and hence reflected, from the object (figure 3.1). Suppose the top of the fog to have altitude R_H, and the sun to be at an angle ϕ to the horizontal. An object at O primarily receives light from the sun directly; the intensity of this light is attenuated by a factor depending on the thickness of the fog as measured along a line toward the sun.

$$I = I_0(e^{-\beta_{ex} t}) \tag{3.1}$$

The thickness is easily derived from the geometry:

$$t = (R_H - R_E) \csc \phi \tag{3.2}$$

A secondary source of light incident on the object is that scattered by fog at some small angle χ. Consider Figure 3.2. Long dotted lines show the limits of angles at which light may strike the object. Two rays from the sun are shown: the direct ray, and a typical scattered one. The direct

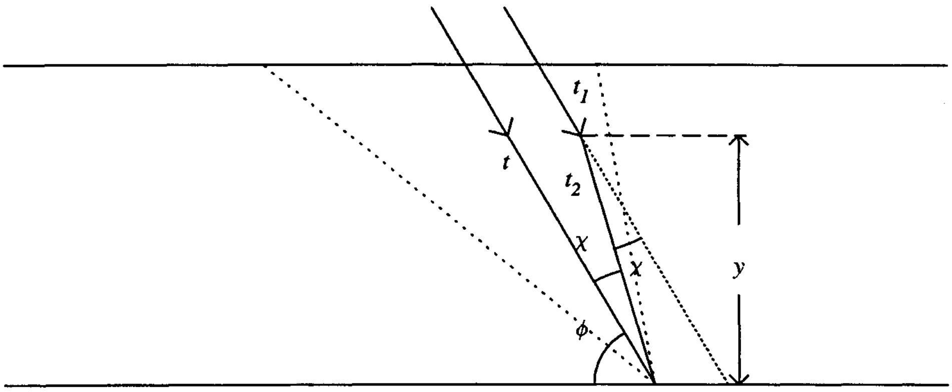

Figure 3.2 Scattered Light Reaching a Reflective Surface

ray travels a distance t through the fog before striking the surface.
The scattered ray, being scattered at a height y above the surface and at an angle χ, travels a distance $t_1(y)$ before scattering and $t_2(y)$ after scattering as given by (3.3) and (3.4):

$$t_1(y) = (R_H - R_E - y)\csc\phi \tag{3.3}$$

$$t_2(y) = y\csc(\phi + \chi) \tag{3.4}$$

So the total distance the scattered ray travels through the fog is

$$t_t = (R_H - R_E)\csc\phi + (\csc(\phi + \chi) - \csc\phi)y \tag{3.5}$$

which is of the form

$$A + By$$

If I_H is the incident intensity reaching the top of the fog then the amount of light scattered toward the object at a scattering angle of χ is

$$I(\chi) = I_H\,\beta_{sc}(\chi)e^{-\beta_{ex}A}\int_0^{R_H} e^{-\beta_{ex}By}\,dy \tag{3.6}$$

$$= I_H\,\beta_{sc}(\chi)e^{-\beta_{ex}A}\,\frac{e^{-\beta_{ex}BR_H} - 1}{-\beta_{ex}B}$$

The effect of this light is particularly important for highly polished surfaces since the specular reflection depends strongly on the angle between the incident ray and the line of sight. In a qualitative sense, the result is a spreading and softening of highlights, and a loss of contrast in reflections. If specular reflections are not important, the net effect of the light scattered toward the object is to produce a slight increase in the illumination of the surface. It remains to be seen just how important the effect is; for the sake of expediency we shall introduce an ambient lighting term to compensate for it.

In (1.10) we gave the amount of light that reaches the eye from a particular scattering centre. Now suppose that a reflecting surface replaces the scattering centre. The path that the light follows has the same parts as the path used in finding (1.10): first through clear atmosphere, then through atmosphere containing scattering particles, then a change in direction accompanied by a loss of intensity, and finally through atmosphere containing scattering particles. Clearly equation (1.10) applies here as well. Rather than having an angular scattering coefficient, we need an angular reflectivity coefficient, which will specify the colour and angular selectivity of the surface. With the addition of the ambient term we see a slight change:

$$I_4 = I_0 R^{(\lambda)}(\psi) \exp(-\beta_{ex_m} d_1 - (d_2 + d_3)(\beta_{ex_m} + \beta_{ex_p}))$$

becomes

$$I_4 = \exp(-d_3(\beta_{ex_m} + \beta_{ex_p})) \left[\text{Ambient} + I_0 R^{(\lambda)}(\psi) \exp(-\beta_{ex_m} d_1 - (\beta_{ex_m} + \beta_{ex_p}) d_2)\right] \tag{3.7}$$

The distances d_i are much more easily found than the distances required for computing the colour of the sky. First, the distance the light travels before reaching the fog depends only on the thicknesses of the fog and the atmosphere and the angle of inclination of the sun.

$$d_1 = (R_A - R_H) \csc \phi \tag{3.8}$$

Here some error is introduced in assuming the earth is flat and the atmosphere a flat-topped volume. In (2.11) the same distance was found to be

$$d_1 = \sqrt{R_A^2 - R_H^2 \cos^2\phi} - R_H \sin\phi$$

In all cases this is less than that given by (3.8). The relative error is given by

$$E(\phi) = \frac{(R_A - R_H) \csc \phi - \sqrt{R_A^2 - R_H^2 \cos^2\phi} + R_H \sin\phi}{-\sqrt{R_A^2 - R_H^2 \cos^2\phi} + R_H \sin\phi} \tag{3.9}$$

Beyond about 100 km the atmosphere is certainly too thin to have much effect. This gives a value of about 6500 km for R_A. A typical value of R_H is 6380 km (29,000 feet thick). For these values

$$E(\phi) = \frac{120\, csc\ \phi - \sqrt{42{,}250{,}000 - 40{,}704{,}400 \cos^2\phi} + 6380 \sin\phi}{-\sqrt{42{,}250{,}000 - 40{,}704{,}400 \cos^2\phi} + 6380 \sin\phi} \tag{3.10}$$

For $\phi = 30°$, the relative error is 2.7%, for smaller angles it grows quickly: for 20° it is 6.5%; for 10°, 24% and for 5°, 71%. Apparently the use of the flat earth approximation is satisfactory for cases in which the sun is inclined from the horizon by at least about 30°. A similar analysis reveals that the flat earth approximation is good to within 1% for fog, as long as ϕ is not less than 15°. In both cases the error is in the *distance*, not the amount of attenuation.

Since the top of the atmosphere, the top of the fog and the surface of the earth are all parallel planes, the second distance has much the same form as the first.

$$d_2 = (R_H - R_E)\, csc\ \phi \tag{3.11}$$

For the distance from the surface to the eye, the angle the line of sight makes with the horizontal is used in the same formula. With the eye at a height h,

$$d_3 = h\ csc\ \theta \tag{3.12}$$

With the values of d_i as found above, the light reaching the eye from a source or a reflective object (the *object* component), is

$$I = \exp(-h\ csc\ \theta\,(\beta_{ex_m} + \beta_{ex_p})) \tag{3.13}$$

$$\left[\text{Ambient} + I_0 R^{(\lambda)}(\psi)\exp(-csc\ \phi(\beta_{ex_m}(R_A - R_E) + \beta_{ex_p}(R_H - R_E)))\right]$$

The Fog Component

To find the fog component we begin with the assumption that the fog is vertically thin, generally a good assumption, which justifies the approximation of single scattering [George1951].

To find the amount of light which is scattered toward the eye by fog between the object and the eye, the integral of (1.11) is used. The distance, d_1, which the light travels before reaching the fog, is the same as that given in (3.8). If the viewer is on the ground and the fog layer is not too thin relative to the height of the eye above the ground, then d_2, the distance the light travels through fog before reaching the line of sight, may be approximated as the constant value given in (3.11). This is equivalent to assuming that no point along the line of sight is far from the ground relative to the height of the fog.

The distance the light travels after striking the line of sight, that is d_3, varies linearly from 0 to $h\ csc\ \theta$.

The actual value of $\beta_{sc}\ (\psi)$ is only needed for one angle.

$$\psi = \theta + \phi \tag{3.14}$$

Substituting the respective distances into (1.11), we find the fog component to be

$$I_f = I_0 \beta_{sc}(\psi) e^{-\beta_{ex_m}^{(\lambda)}(d_1 + d_2) - \beta_{ex_p}^{(\lambda)} d_2} \int_0^{h \csc\theta} e^{-d_3(\beta_{ex_m}^{(\lambda)} + \beta_{ex_p}^{(\lambda)})} d\, d_3 \tag{3.15}$$

This may be evaluated easily enough: The expression may be re-written as

$$I_f = A(\lambda)\ \beta_{sc}(\psi) \int_0^{h \csc\theta} e^{-r B(\lambda)} d\, r \tag{3.16}$$

with

$$B(\lambda) = \beta_{ex_m}^{(\lambda)} + \beta_{ex_p}^{(\lambda)} \tag{3.17}$$

$$A(\lambda) = I_0 e^{-\beta_{ex_m}^{(\lambda)} d_1 - B(\lambda) d_2} \tag{3.18}$$

$$= I_0 e^{-\beta_{ex_m}^{(\lambda)}(R_A - R_H)\csc\phi - B(\lambda)(R_H - R_E)\csc\phi}$$

The integral is easily evaluated:

$$\int_0^{h \csc\theta} e^{-r B(\lambda)} d\, r = \frac{1 - e^{-B(\lambda) h \csc\theta}}{B(\lambda)} \tag{3.19}$$

Combining (3.19) with (3.16) we have

$$I_f = A(\lambda)\ \beta_{sc}(\psi) \frac{1 - e^{-B(\lambda) h \csc\theta}}{B(\lambda)} \tag{3.20}$$

With the evaluation of the integral solved, the entire expression for I_f may be evaluated using two calls to **exp**, and some trig functions. The factors which appear outside of the integral need be evaluated only once for a given scene (assuming a constant height ground surface), so the only trigonometric function called for each line of sight is the **csc** θ which appears in the exponent. Similarly only one of the two calls to **exp** is required for each line of sight.

When the sun is at the horizon (either rising or setting) the problem must be treated differently. In this case the distance the sunlight travels through the fog is less than infinite because of the curvature of the earth.

For small angles of solar inclination (such that the curvature of the earth plays a part) no direct light from the sun reaches visible objects (it is all scattered out of the beam). For this reason the best way to model this situation is to use a constant colour and intensity for the fog component, to represent the light reaching the fog along the line of sight indirectly. No reflections of the light source are possible, but the ambient lighting will be the same colour as the fog component. Both the fog component and the ambient light exist as a result of multiple scattering, predominantly from light striking different parts of the fog directly from the sun. Little absorption takes place as light passes through the fog, but after enough scatterings the direction of propagation is fairly random. Since scattering by fog is nearly wavelength independent, the colour of ambient light is nearly the same as the colour of direct sunlight without the intervening fog, as is the colour of the fog component.

The formulation of the colour (and intensity) of the object component took into account the opacity of the fog between the object and the eye. With both components computed, they may be simply added to each other to yield the total light along a particular line of sight.

If the eye is not actually within the fog, the light may be calculated as if the eye were at the point of intersection of the line of sight with the top surface of the fog. This is the same as taking the above integral in which the integrand was weighted with a step function which drops from 1 to 0 at the boundary, the height of this step function being the density of the fog. This ignores any molecular scattering which might occur between the top of the fog and the eye. If the fog is more than a few kilometres from the viewer, the renderer must take into account Rayleigh scattering between the eye and the fog. This may be done by computing the opacity and light contribution of the air between the fog and the eye, and reducing the contribution of the fog and any objects within the fog by an amount which depends on the opacity of the air, while adding to the total light for that line of sight the light scattered toward the eye by the air. The light scattered by the air, and the opacity of the air may be obtained in exactly the same manner as those for the fog component were, substituting in the appropriate scattering and extinction coefficients.

Efficiency Considerations for Naturally Lit Fog

A number of safe assumptions come to our aid here. To begin with, we are not presuming to deal with cases in which the sun is low in the sky. This means that once the light strikes the top of the fog it travels a sufficiently short distance that significant amounts of Rayleigh scattering cannot occur. The fog makes up only a small part of the atmosphere, so the colour and intensity of the light striking the top of the fog may be computed (once) as if it were striking the earth, without the fog. This problem was solved when the colour of the sun was found.

Since the selectivity of fog is less than 5%, and the extinction coefficient has been measured for red, green and blue lights, there is no need to compute in wavelength space unless Cook-Torrance shading is being used when reflectances are computed. Assuming 33 wavelengths would have been sampled, this is a saving of a factor of 11 in the time that will be needed to compute the image.

The extinction coefficient of fog, β_{ex_p} is much greater than that for air molecules, even though the air molecules are much more densely concentrated [McCartney1976]. This means that $B\ (\lambda)$ is effectively β_{ex_p} .

If the intensity and colour of light reaching the edge of the fog have been computed once and stored in I_t then (3.20) becomes

$$I_f \ = I_t \ e^{-\beta_{ex_p}^{(\lambda)}\ (R_H \ - R_E\ ) csc\ \phi} \beta_{sc}\ (\psi)\ \frac{1 - e^{-\beta_{ex_p}^{(\lambda)} h\ csc\ \theta}}{\beta_{ex_p}^{(\lambda)}} \tag{3.21}$$

In a similar way, (3.13) can be re-arranged as follows:

$$I \ = \text{Ambient} \ \cdot e^{-h\ csc\ \theta\ B\ (\ \lambda)} \tag{3.22}$$

$$+ R^{(\lambda)}(\psi) I_0 e^{-h\ csc\ \theta\ B\ (\ \lambda) - \beta_{ex_m}\ (R_A \ - R_H\ ) csc\ \phi\ +\ B\ (\lambda)\ (R_H \ - R_E\ ) csc\ \phi}$$

$$= \text{Ambient} \ \cdot e^{-h\ csc\ \theta\ \beta_{ex_p}} + I_t\ e^{-h\ csc\ \theta\ \beta_{ex_p}\ -\ \beta_{ex_p}\ (R_H \ - R_E\ ) csc\ \phi} R^{(\lambda)}(\psi) \tag{3.23}$$

Note that some care must be taken when evaluating (3.23) since the object component is zero for distant objects, in which case $h\ csc\ \theta$ is negatively infinite. This term is really intended as the distance from the eye to the object so if h should be zero this distance should be obtained otherwise. If the exponent in the first term is very large, it is safe to assume that the object is sufficiently obscured by the fog as to make no contribution.

In many cases, the fog component is determined solely by the distance from the object to the eye, all other parameters remaining nearly constant throughout an image, and the object component may be computed as if the fog were not there, since its vertical thickness is so small. This is also the case for haze when the sun is well above the horizon. As long as shadows in the fog are not important, there is nothing to prevent a scene from being rendered without fog or haze, with the fog or haze being added later, so long as the distance information is retained. Any rendering algorithm has this information available either implicitly or explicitly and can retain it in a hidden field of the image such as is done in [Duff1985].

Results

"And hast thou slain the Jabberwock?
Come to my arms, my beamish boy!
O frabjous day! Callooh! Callay!"
He chortled in his joy.
— *Jabberwocky,* by Lewis Carroll.

The Sun and the Sky

The angular scattering functions for single scattering seem to be sufficient for modelling the colours of the sun and the sky, as evidenced by the pictures in plates one to six.

Plates one to three show the effect of varying the density and radius of the haze layer, and the mean density of the atmosphere, at sunset. Plate four shows the effect of variations of the haze density when the sun is inclined 40° from the horizon. In these four plates the numbers in the frames are intended as indications of relative values only. Plates five and six show a progression of sky colours in which the solar inclination is varied with all else held fixed. Where the sun is low in the sky the entire image has been brightened, as if the camera aperture had been increased. The numbers in the images indicate the angle of inclination, in degrees, of the sun. The run times vary significantly with the height in the sky of the sun, as is shown in Table 4.1. The times are VAX 11/780 cpu seconds.

Table 4.1
Timings of Various Solar Inclinations

Angle	Time	Angle	Time
0	3875.8	30	655.6
2	975.2	40	653.0
5	599.1	50	787.7
10	670.3	60	634.8
15	650.6	70	651.3
20	648.3	80	635.3

Run times are less than proportional to the number of pixels in the screen as can be seen from Table 4.2 and the graph of Figure 4.1.

Table 4.2
Comparative Running Times for Sky Computations

Pixels	Seconds		Seconds/Pixel	
	Noon	Sunset	Noon	Sunset
128	26.2	159.3	0.204688	1.24453
512	62.4	401.1	0.121875	0.783398
2048	135.2	809.5	0.0660156	0.395264
8192	308.1	1738.9	0.0376099	0.212268
32768	684.8	3756.8	0.0208984	0.114648
131072	1523.7	7892.6	0.0116249	0.0602158

Notice that in Figure 4.1 the slope of the $O(n)$ run time line is quite a bit greater than that of the lines for either sky. Since the plot is a log-log plot, the slopes give the growth factors. From the graph, the empirical run time is approximately $O(n^{.57})$.

Fog

Plates seven and eight show a simple scene lit from the front. In the first case, the scene is shown without fog, and in the second it is with fog. Note the lack of light reflected from the fog toward the eye, especially in the centre, where light would have to be scattered back nearly on its path. Plates nine and ten show the same scene lit from behind. Here the fog scatters considerable light toward the eye, particularly in the region where the light follows a nearly direct path. Because the run times for these scenes are a few minutes at 512x512 resolution, without careful optimization, more exact statistics weren't kept.

Summary

Computer generated imagery has, until now, used *ad hoc* methods of determining the colours of the sun and the sky. With this unified lighting model, the sun, sky, clouds and even objects on the ground reflecting light from the sun and the sky can fit together, being computed from the same lighting parameters. In this way, images of outdoor scenes need no longer have the appearance of being pasted together, but rather form one harmonious unit.

The cost of computing a scene in this way is not small; however some methods of keeping the cost down without sacrificing realism have been discussed. If these methods are used judiciously, the cost is within reason.

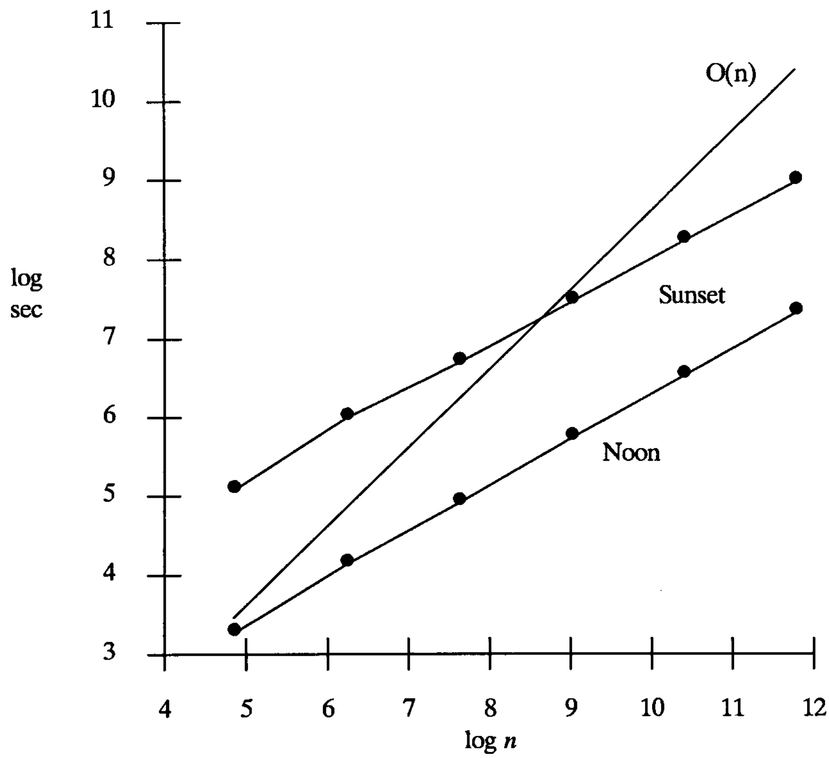

Figure 4.1 Run Times for Computing Skies at Various Resolutions

Many of the problems related to rendering fog realistically remain to be solved. The lighting model presented herein supplies a good foundation on which to build a rendering package which can realistically render fog. While only a simple case has been demonstrated here, rendering techniques can be derived for more general cases. In this way, the general problem of displaying scenes containing fog can be solved.

On the Horizon

This lighting model opens up a variety of possibilities. One application already discussed is that of finding the colour of the sky. From that discussion it should be clear that with care and a good modelling technique, spectacular sunsets could be computed with clouds reflecting the colour of the sun. The sun colour would have to be computed for the altitude of the clouds, taking into account the angle of inclination of the line of sight which strikes them.

A more complete model of the sky could take into account the shadow of the earth as the sun passes beyond the horizon. On a clear (that is cloudless) evening, it is possible to watch the earth's shadow as it is cast on the haze layer for the few minutes it takes to reach the top of the haze layer, after which it ceases to be distinguishable. This is visible longest in the prolonged twilight of winter at high latitudes.

A number of important problems remain to be handled with fog, as well. One application of the lighting model which was not yet addressed was the lighting of fog by artificial sources. Even without shadows, this would be enough to produce some very interesting scenes. The pools of light created by street lights can be quite picturesque. The beams of headlamps provide an excellent proof that fog needs such a sophisticated model as this to be rendered correctly. They are nearly invisible when viewed nearly towards the light and then they become clearest at an angle of about 10-15 degrees, gradually fading out into nothing when viewed in the forward direction from right beside the lights. Max [Max1986], and Nakamae [Nakamae1986] came close, but seem to have neglected the angular dependance of scattering.

Shadows in fog remain as a problem to be solved. In order to compute the light of the fog component, it is necessary to know which part of the fog is lit. This is a problem which could be solved through ray tracing or through shadow casting, generating surfaces at which the fog ceases or begins to be lit by a particular source.

The scattering functions could be modified to reflect the scattering function of quite different materials in order to simulate the sky of Mars or dust underwater. Naturally with that generalization, sunbeams, made from dust in the air, could also be simulated.

One scene that would thoroughly exercise the model would be an evening scene containing distant mountains behind a large body of water which reflects the clouds and sky colours, as well as the setting sun, which, while not directly visible, casts beams through breaks in the clouds. High cirrus clouds would reflect one sun colour, while lower lying cumulus would reflect another colour, except where they are backlit. Another milestone image would contain thick fog, with several street lamps casting pools of light in it, with a spotlight shining through the branches of a tree, which casts shadows in the fog.

Appendix 1

Derivation of Equation 2.2

We construct the right triangle of Figure 2.4, the length of whose hypotenuse is y .
By Pythagoras' theorem

$$(r + x)^2 + z^2 = R^2; \tag{A1.1}$$

From the law of sines

$$z = x \tan\xi; \tag{A1.2}$$

$$x = y \cos\xi; \tag{A1.3}$$

substituting for z from (2.3) into (2.2) gives

$$(r + x)^2 + x^2 \tan^2\xi = R^2; \tag{A1.4}$$

substituting for x from (2.4) into (2.5) and expanding yields

$$r^2 + 2yr \cos\xi + y^2\cos^2\xi + y^2\cos^2\xi\tan^2\xi = R^2; \tag{A1.5}$$

$$y^2 + 2y\, r \cos\xi + r^2 - R^2 = 0; \tag{A1.6}$$

$$y = \frac{-2r \cos\xi \pm \sqrt{4r^2\cos^2\xi + 4(R^2 - r^2)}}{2} \tag{A1.7}$$

$$= -r \cos\xi \pm \sqrt{r^2(\cos^2\xi - 1) + R^2} \tag{A1.8}$$

$$= \sqrt{R^2 - r^2\cos^2\eta} - r \sin\eta \tag{A1.9}$$

in which the positive square root has been taken to yield a positive distance.

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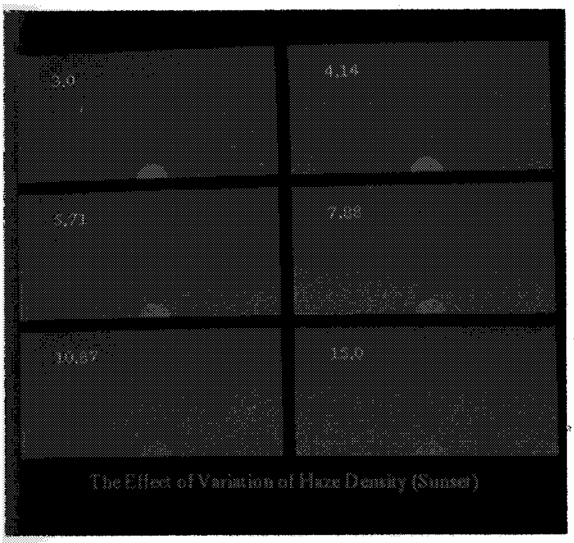
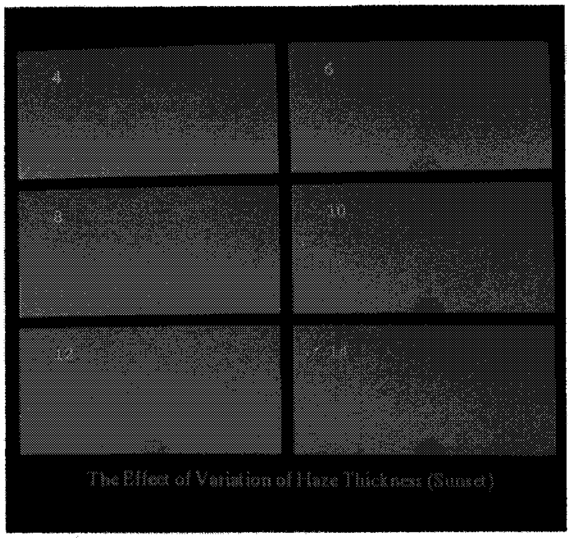


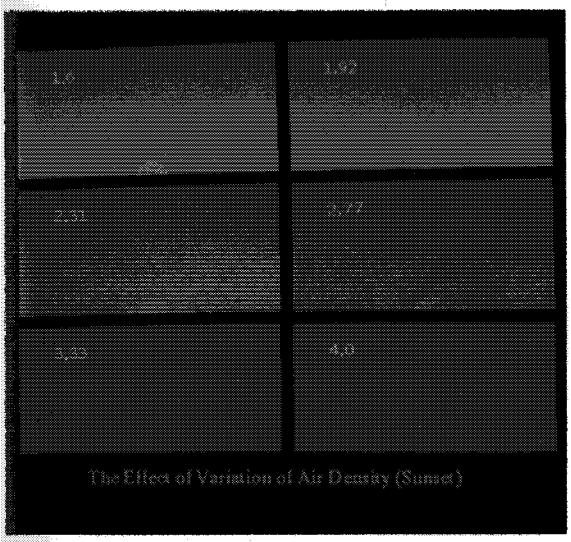
The Effect of Variation of Haze Density (Sunset)

Plate 1



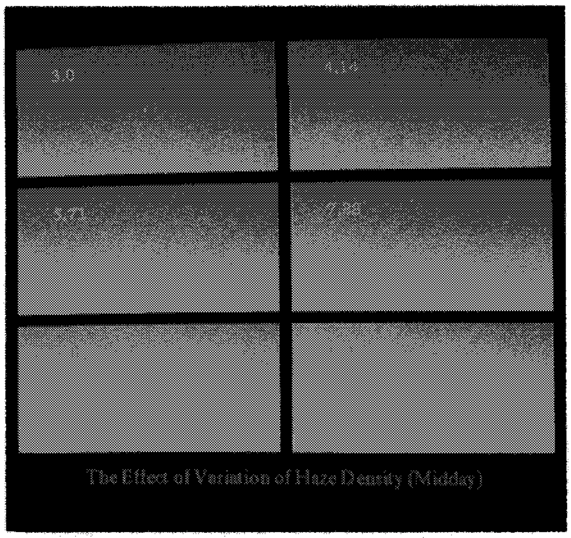
The Effect of Variation of Haze Thickness (Sunset)

Plate 2



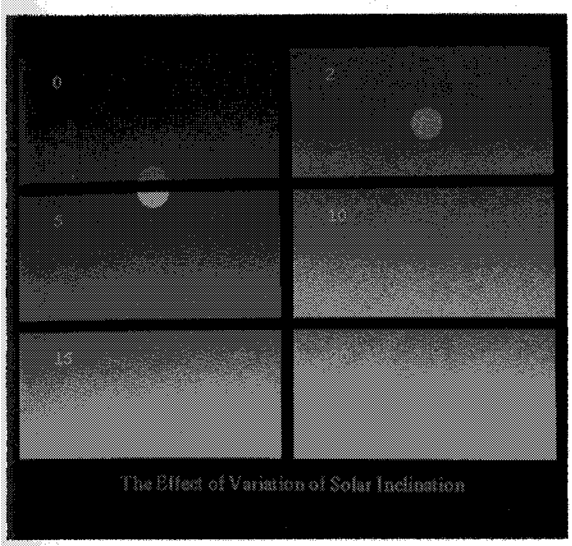
The Effect of Variation of Air Density (Sunset)

Plate 3



The Effect of Variation of Haze Density (Midday)

Plate 4



The Effect of Variation of Solar Inclination

Plate 5

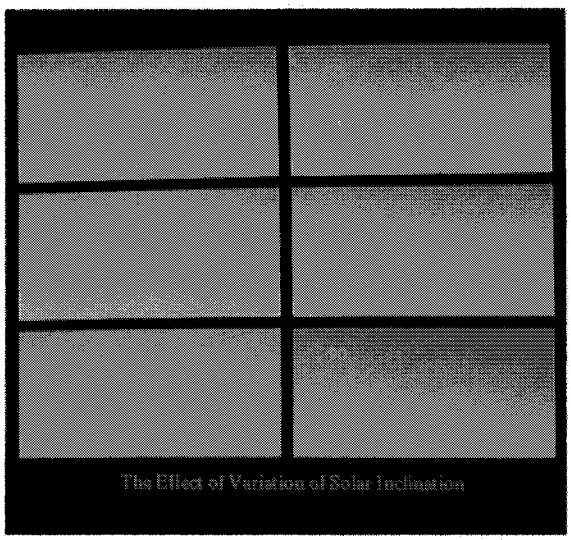
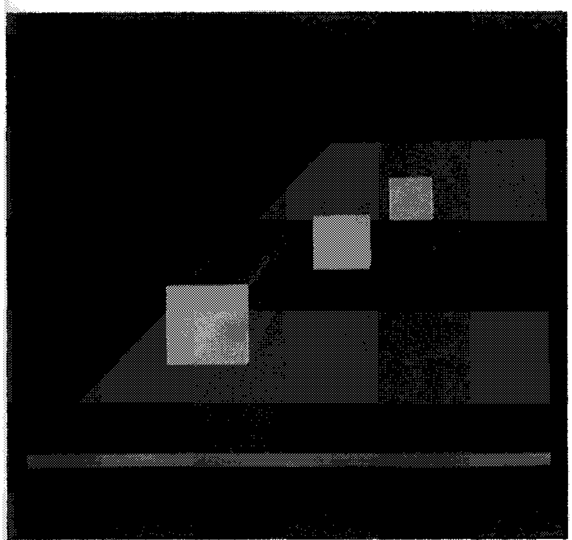

The Effect of Variation of Solar Inclination

Plate 6



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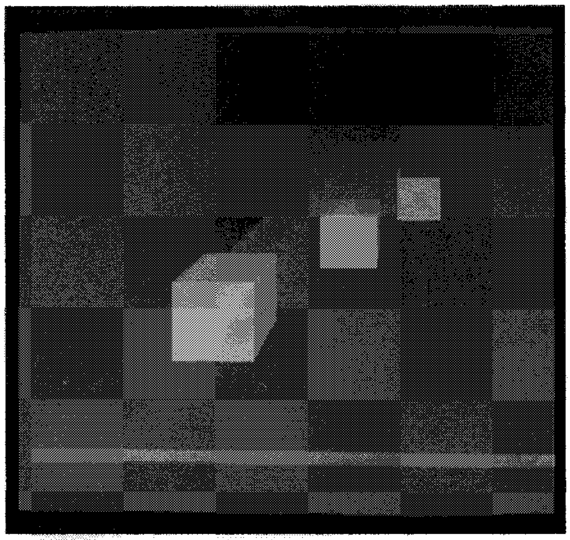

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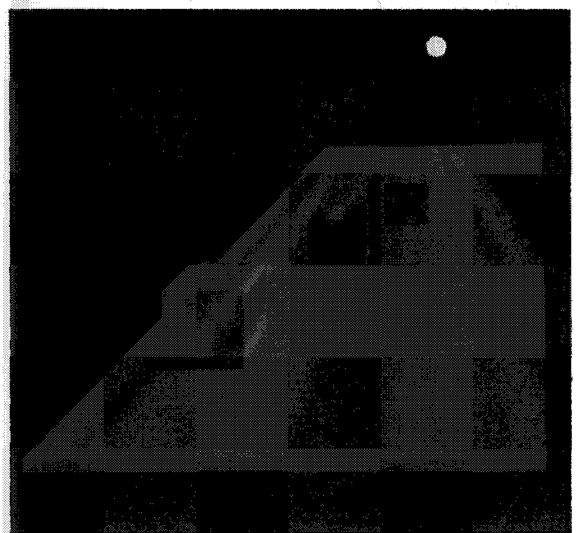

Plate 9

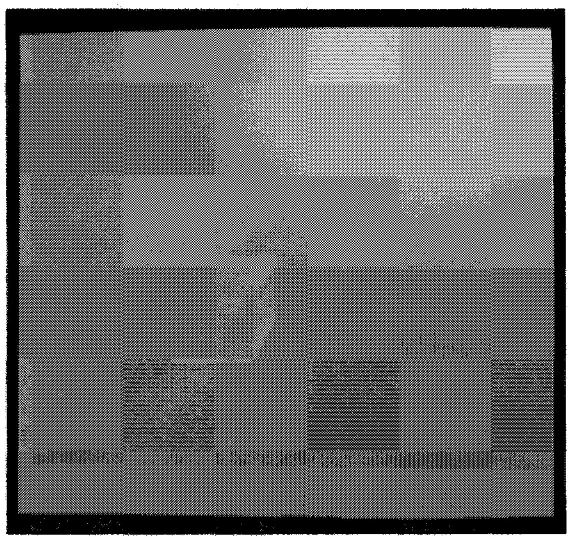

Plate 10