AN INTRODUCTION TO MAPLE:
SAMPLE INTERACTIVE SESSION

Bruce W. Char
Keith O. Geddes
Gaston H. Gonnet

University of Waterloo
Research Report CS-83-16
May, 1983
An Introduction to Maple: Sample Interactive Session

Bruce W. Char
Keith O. Geddes
Gaston H. Gonnet

Department of Computer Science
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1

Brief Facts about Maple

What is Maple?

Maple is a system for symbolic mathematical computation which has been under development at the University of Waterloo since December, 1980.

Implementation

The basic system is written in a BCPL-derivative language called Margay, which is then macro-processed into other languages of the BCPL family. Maple has been brought up in versions of C and in B, and there are plans to implement it in other languages such as WSL (Waterloo Systems Language) and Waterloo Port.

Machines

Maple is currently available on VAX computers, Honeywell 6000 series computers, and on MC68000-based microcomputers which support Unix-like operating systems.

Size

The basic Maple system occupies about 100K bytes on a VAX 11/780. Library functions are automatically loaded as required, so the data space grows at a rate that depends on the user's application.

References


Sample Interactive Session

In the interactive Maple session demonstrated on the following pages, lines beginning at the left margin are user input lines and centered lines are system responses.

The Maple system is initiated by a command such as

```
/u/maple/bin/maple
```
on many Unix systems

```
maple/maple
```
on the Honeywell GCOS system.

When Maple is initiated, it displays a maple leaf and the version number of the Maple system that has been loaded.
# One form of arithmetic in Maple is exact rational arithmetic. `**` is the exponentiation operator.

\[ 1 + 1/4 + 1/16 + 1/64 + 1/256; \]

\[ \frac{341}{256} \]

\[ d := (3**50 + 5**20) / 2**80; \]

\[ d := \frac{3589489938993610010205437}{604462909807314587353088} \]

# Constants in Maple may be approximated by floating-point numbers and the user has control
# over the number of digits carried. The `evalr` function causes 'evaluation to a real'.

evalr(d);

.59383129728

Digits := 50;

Digits := 50

evalr(d);

.5938312972841863724294848816626626624277460364851

h := tan(3*Pi/10);

h := tan(3/10 Pi)

evalr(h);

1.3763819204711735382072095819108876795258903360081

# Restore the global variable 'Digits' to its default value.

Digits := 10;

Digits := 10
# Maple supports rational expressions both unexpanded and expanded. The double-quote symbol # refers to the latest expression.

\[(x + 1)^7\]

\(x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1\)

\((y - x)(y^4 + y^3x + y^2x^2 + yx^3 + x^4)\)

\(y^5 - x^5\)

\((xy/2 - (y^2)/3) \ast (x - y)^2;\)

\((1/2 xy - 1/3 y^2)(x - y)^2\)

\(1/2 x^3y - 4/3 x^2y^2 + 7/6 x y^3 - 1/3 y^4\)

# Maple does not force rational expressions into a canonical form.

" * (3*x + y) / (x - y);

\((1/2 x^3y - 4/3 x^2y^2 + 7/6 x y^3 - 1/3 y^4)(3x + y)\)

\(x - y\)

# However, normalizing facilities are available for simplification.

normal(");\]

\(1/6 (3xy - 2y^2)(3x + y)(x - y)\)

" / (x**3 - x**2*xy - x*xy + y**2);

\(1/6 (3xy - 2y^2)(3x + y)(x - y)\)

\(x^3 - x^2y - xy + y^2\)

expr := normal(");

\(expr := (3xy - 2y^2)(3x + y)\)

\(6x^2 - 6y\)
# There are explicit functions for \texttt{gcd} (greatest common divisor) and \texttt{lcm} (least common multiple) computations with polynomials.

\begin{align*}
\text{p} & := 143x^3y - 39x^2y^2 - 11xy + 11x + 3y^2 - 3y; \\
\text{p} & := 143x^3y - 39x^2y^2 - 11xy + 11x + 3y^2 - 3y \\
\text{q} & := 55x^2 + 11x + 11z; \\
\text{q} & := 55xy^3 + 11xy^2 + 11z + 15y^4 - 3y^3 - 3y^2 - 3y \\
\text{gcd}(p, q); & = -11z + 3y \\
\text{lcm}(15*(x-5)y, 9*(x^2-10*x+25)); & = 45x^2y - 450xy + 1125y
\end{align*}

# Maple has facilities for differentiation of expressions.

\begin{align*}
\text{f} & := \sin(x) \ast \cos(x); \\
\text{f} & := \sin(x) \ast \cos(x) \\
\text{fp} & := \text{diff}(f, x); \\
\text{fp} & := \cos(x)^2 - \sin(x)^2 \\
\text{diff}(f, x, x); & = -4 \sin(x) \cos(x) \\
\text{diff}(\sin(x) \ast \cos(x), x) & = \cos(x) \ast \cos(x) \ast \cos(x); \\
\text{sin}(x) & = \left(x^1 \right) \left(x^2 \left(\ln(x) + 1\right) \ln(x) + \frac{x^1}{x}\right)
\end{align*}

# The "subs" command is used below to substitute 1 for x in an expression.

\begin{align*}
\text{subs}(x=1, fp); & = \cos(1)^2 - \sin(1)^2 \\
\text{evalr}(\text{fp}); & = -0.4161468365
\end{align*}
# The "subs" command also can do more general substitutions.

subs(sin(x)**2 = 1 - cos(x)**2, f(p));

\[ 2 \cos(x)^2 - 1 \]

expand( (sin(x) + 1)**2 );

\[ \sin(x)^2 + 2 \sin(x) + 1 \]

subs(sin(x)**2 = 1 - cos(x)**2, x);

\[ 2 - \cos(x)^2 + 2 \sin(x) \]

# In Maple, equations may be manipulated as expressions.

eqn1 := 3*x + 5*y = 13;

eqn1 := 3x + 5y = 13

eqn2 := 4*x - 7*y = 30;

eqn2 := 4x - 7y = 30

3*eqn2 - 4*eqn1;

\[-41y = 38\]

y := 38/(-41);

\[ y := -\frac{38}{41} \]

eqn1;

\[ 3x - \frac{190}{41} = 13 \]

x := (13 + 190/41)/3;

\[ x := \frac{241}{41} \]

eqn1; eqn2;

\[ 13 = 13 \]
\[ 30 = 30 \]

# Unassign the names x and y so that their values will be their own names.

x := 'x'; y := 'y';

\[ z := z \]
\[ y := y \]
# Maple has a solve function which can solve many kinds of equations, including systems of linear
# equations, single equations involving elementary transcendental functions, and polynomial equations.

solve( {eqn1, eqn2}, {x, y} );

\{ y = -\frac{38}{41}, \quad z = \frac{241}{41} \} 

# Note that the use of a colon instead of a semicolon suppresses maple's output in the following.

eqn1 := 3*t + 4*s - 2*t + u = -2;

eqn2 := r - s + 2*t + 2*u = 7;

eqn3 := 4*r - 3*s + 4*t - 3*u = 2;

eqn4 := -r + s + 6*t - u = 1;

solve( {eqn(1..4)}, {r, s, t, u} );

\{ r = 1/2, \quad u = 2, \quad s = -1, \quad t = 3/4 \} 

solve( \cos(x) + y = 9, x );

arccos(9-y)

solve( 2*x*u + G = 0, u );

\frac{\ln(-G)}{\ln(2)}

solve( x**2 - 46*x + 529 = 0, x );

23, 23

solve( 1/2*a*x**2 + b*x + c = 0, x );

\frac{-b + (b^2 - 2ac)^{1/2}}{a}, \quad \frac{-b - (b^2 - 2ac)^{1/2}}{a}

\{ x + y + z = a, \quad x + 2*y - a*z = 0, \quad \sin(a)*z + a*y = 0 \};

solve( " , \{ x, y, z \} );

\{ z = \frac{a \left( a^2 + 2 \sin(a) \right)}{a^2 + \sin(a) + a}, \quad y = \frac{-a \sin(a)}{a^2 + \sin(a) + a}, \quad z = \frac{a^2}{a^2 + \sin(a) + a} \}
# Maple has arrays and also a general "table" facility.

\[ S := \text{array}(1..10); \]
\[ S := \text{array}(1..10, [ ]) \]

\[ S[9] := x^9; \]
\[ S[9] := x^9 \]

\[ T := \text{array}(0..3, 0..3, \text{symmetric}); \]
\[ T := \text{array}(\text{symmetric}, 0..3, 0..3, [ ]) \]

\[ T[1,2] := G12; \]
\[ T[1,2] := G12 \]

\[ T[1,2] + T[2,1]; \]
\[ 2 \, G12 \]

\[ R[\sin(x)] := \cos(x); \quad R[\cos(x)] := - \sin(x); \]

\[ \sin(x) \ast R[\sin(x)] + \cos(x) \ast R[\cos(x)]; \]
\[ 0 \]
# Maple has facilities for computing series expansions of expressions. The series representation
# includes an $O()$ term to indicate the order of truncation of the series. The order of truncation
# can be controlled by the user; the default value is order 6.

expr:

\[
\frac{(3xy-2y^2)(3x+y)}{6x^2-6y}
\]

taylor(expr, x=0);

\[
\frac{1}{3} y^2 + \frac{1}{2} y^2 x + \frac{1}{2} \frac{(9y-2y^2)}{y^2} x^2 + \frac{1}{2} \frac{(9y-2y^2)}{y^2} x^4 + \frac{1}{3} \frac{3x+x^2}{3x^2+1} O(x^4)
\]

\[
r := (x^2+6x-1)/(2x^2+1)^2
\]

\[
s1 := taylor(r, x=0);
\]

\[
s1 := -1+6x+5x^2-24x^2-16x^4+72x^5+O(x^6)
\]

\[
s2 := taylor(r, x=1, 2);
\]

\[
s2 := 2/3 - 8/9 (z-1) + \frac{11}{27} (z-1)^2 + O((z-1)^3)
\]

\[
s3 := taylor(\exp(3x+x^2-x), x=0, 4);
\]

\[
s3 := 1+3/2 x^2 + 19/6 x^4 + \frac{145}{24} x^4 + O(x^5)
\]

taylor(s1*s3, x=0);

\[
-1+5x+15/2 x^2 - 7/6 x^3 - \frac{229}{24} x^4 + O(x^5)
\]

# Maple knows how to compute with asymptotic series as well as Taylor series.

\[
f := n*(n+1)/(2*n-3);
\]

\[
f := \frac{n(n+1)}{2n-3}
\]

asympt(f, n);

\[
1/2 n + 5/4 + 15/8 n^{-1} + \frac{45}{16} n^{-2} + \frac{135}{32} n^{-3} + \frac{405}{64} n^{-4} + \frac{1215}{128} n^{-5} + O(n^{-6})
\]
# There is a limit function to compute the limiting value of an expression as a specified variable approaches a specified value.

\[ \text{limit}(\tan(x)-x)/x^3, x=0); \]

\[ \frac{1}{3} \]

\[ r := (x^2 - 1) / (11*x^2 - 2*x - 9); \]

\[ r := \frac{x^2-1}{11x^2-2x-9} \]

\[ \text{limit}(r, x=0); \]

\[ \frac{1}{9} \]

\[ \text{limit}(r, x=\text{infinity}); \]

\[ \frac{1}{11} \]

\[ \text{limit}(r, x=1); \]

\[ \frac{1}{10} \]

# Maple integers can be arbitrarily long. The following is an example using the factorial operator.

\[ 720!; \]
# Maple can represent and manipulate sets. The set operators are + (union), * (intersection), and - (set difference).

\[
a := \{x, 2y + 1/3\}; \quad a := \{x, 2y + 1/3\}
\]

\[
b := \{z-4, x\}; \quad b := \{z, z-4\}
\]

\[
c := a + b; \quad c := \{z, z-4, 2y + 1/3\}
\]

\[
d := a - b; \quad d := \{2y + 1/3\}
\]

\[
e := a \times b; \quad e := \{x\}
\]

\[
f := b \times d; \quad f := \{\}
\]

# Selection of elements from a set (and more generally, selection of operands from any expression) # is accomplished using the "op" function. op(i,expr) yields the i-th operand. Also, nops(expr) # yields the "number of operands" in expr.

\[
nops(c);
\]

\[
3
\]

\[
g := op(2,c);
\]

\[
g := z-4
\]

\[
h := \{op(3,c),z\};
\]

\[
h := \{z, 2y + 1/3\}
\]
Another data structure in Maple is the list, represented using square brackets. Selection of elements from a list is accomplished using the "op" function. For composition of lists, it is convenient to use the form op(expr) which yields a sequence of all the operands separated by commas. Also, op(i..j, expr) yields the sequence op(i,expr), op(i+1,expr), ..., op(j,expr).

\begin{verbatim}
list1 := [x, 2*y + 1/3];
      list1 := [z, 2*y + 1/3]

list2 := [z-4, x];
          list2 := [z-4, z]

new_list := [op(list1), op(list2)];
      new_list := [z, 2*y + 1/3, z-4, z]

a := [op(1, list1), op(3..4, new_list)];
     a := [z, z-4, z]

F(op(list2));
     F(z-4, z)
\end{verbatim}

Maple can do indefinite summations as well as definite summations.

\begin{verbatim}
sum(i*2, i = 1 .. n-1);
     1/3 n^3 - 1/2 n^2 + 1/6 n

sum((5*i-3)*(2*i+9), i = 1 .. 9876543210);
     3211394431368198288556209751725

sum((5*i-3)*(2*i+9), i = 1 .. n-1);
     10/3 n^3 + 29/2 n^2 - 269/6 n + 27

sum(i*4 * 7**i, i = 1 .. n-1);
     \(-\frac{91}{54} 7^n n + \frac{70}{81} 7^n + \frac{14}{9} 7^n n^2 - \frac{7}{9} 7^n n^3 + \frac{1}{6} n^4 7^n - \frac{70}{81} \)

sum((i-1)/(i+1), i = 1 .. n);
     n - 2 Psi(n+2) + 2 Psi(2)

a := 100: evalr("^n");
     91.60544238
\end{verbatim}
# An important facility in Maple is analytic integration.

\[
f := 1/2 x^2 - 2 + 3/2 x + 7/2 x^2;
\]

\[
f := \frac{1}{2} \frac{1}{z^2} + 3/2 \frac{1}{z} + 2 - 5/2 z + 7/2 z^2
\]

\[
\text{int}(f, x);
\]

\[
-1/2 \frac{1}{z} + 3/2 \ln(z) + 2 z - 5/4 z^2 + 7/6 z^3
\]

\[
\text{int}(f, x = 1..2);
\]

\[
20/3 + 3/2 \ln(2)
\]

\[
\text{evalf(})
\]

\[
7.706387438
\]

\[
\text{int}(x - 7) / (3 x + 11)^2, x);
\]

\[
-1/3 \frac{x - 7}{3 x + 11} + 1/9 \ln(3 x + 11)
\]

\[
g := (x^2 - 3 x + 2) * \sin(x);
\]

\[
g := (x^2 - 3 x + 2) \sin(x)
\]

\[
\text{int}(g, x);
\]

\[
-(x^2 - 3 x + 2) \cos(x) + (2 x - 3) \sin(x) + 2 \cos(x)
\]

\[
\text{expand(})
\]

\[
-x^2 \cos(x) + 3 x \cos(x) + 2 \sin(x) - 3 \sin(x)
\]

\[
h := \sin(t) \times \cos(t);
\]

\[
h := \sin(t) \cos(t)
\]

\[
\text{int}(h, t);
\]

\[
1/2 \sin(t)^2
\]

\[
\text{int}(\exp(x^2), x);
\]

\[
\text{int}(\exp(z^2), z)
\]
An important component of the Maple system is the Maple programming language which may be used to write procedures. Following are some examples of procedures written in Maple.

fibonacci := proc(n)
    option remember;
    if nargs<>1 or not type(n, integer) or n<0 then
        ERROR("wrong number or type of parameters in fibonacci")
    else
        if n<2 then n else fibonacci(n-1) + fibonacci(n-2) fi
    fi
end;

The "option remember" statement at the beginning of the above procedure tells the Maple system to store each computed result in a table. Then the system will retrieve the result rather than re-compute whenever the procedure is called with a previously-computed argument. In this example of a recursive procedure, "option remember" will be very advantageous.

This procedure also includes error-checking on the arguments. The value of the special name "nargs" is the number of arguments with which the procedure was called, <> is the "not equal" operator, and "type" is Maple's type-checking function.

Some examples invoking procedure fibonacci:

fibonacci(101);

573147844013817084101

fibonacci(-1);
ERROR: wrong number or type of parameters in fibonacci
# Tschebysheff(n) : Computes the Tschebysheff polynomials of degrees 0 through n into a table.

Tschebysheff := proc (n)
local p,k;
p[0] := 1; p[1] := x;
for k from 2 to n do
  p[k] := expand( 2*x*p[k-1] - p[k-2] )
od;
RETURN(p)
end;

# An example invoking procedure Tschebysheff:

a := Tschebysheff(5):
a[0], a[1], a[2], a[3], a[4], a[5];

1, x, 2x^2-1, 4x^3-3x, 8x^4-8x^2+1, 16x^5-20x^3+5x

x := 1/4:
a[0], a[1], a[2], a[3], a[4], a[5];

1, 1/4, -7/8, -11/16, 17/32, 61/64

# member: Test for membership in a list.

member := proc (element, List)
local i;
false; for i to nops(List) while not " do
  evalb(element = op(i,List)) od;
end;

# The above is an example of a Boolean procedure — the value returned will be true or false. The
# Maple function "evalb" causes "evaluation as a Boolean", "nops" yields the "number of operands",
# and the double-quote symbol refers to the latest expression.

# Some examples invoking procedure member follow.

member( x*y, [1/2, x*y, x, y] );

true

member( x, [1/2, x*y] );

false

member( x, [ ] );

false
# max: Compute the maximum of a sequence of numbers.

\textbf{max := proc ( )}
\begin{verbatim}
    local i, M, p;
    if nargs = 0 then
        ERROR('function max called with no parameters')
    else
        M := param(1);
        for i to nargs do
            p := param(i);
            if not type(p, rational) and not type(p, real) then FAIL fi;
            if M < p then M := p fi
        od;
        M
    fi
\end{verbatim}
\textbf{end;}

# The above procedure is taken from the Maple system library which defines Maple's "max" 
# procedure to compute the maximum of a sequence of numbers. Unlike the previous procedures, 
# no names are specified for the formal parameters. Rather, the parameters are accessed by the 
# special function param(i). This procedure may be called with any number of parameters.

# Some examples invoking procedure max follow.

\textbf{max(3/2, 1.49)};

\[ \frac{3}{2} \]

\textbf{max(3/5, evalf(ln(2)), 9/13)};

\[ 0.6931471805 \]

\textbf{max(5)};

\[ 5 \]

\textbf{max(-1001, 1/2, -1/2, -9)};

\[ \frac{1}{2} \]

\textbf{max(x, y)};

\[ \text{max}(x, y) \]