THE DESIGN OF MAPLE: A COMPACT, PORTABLE, AND POWERFUL COMPUTER ALGEBRA SYSTEM

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ABSTRACT

The Maple system has been under development at the University of Waterloo since December 1980. The kernel of the system is written in a BCPL-like language. A macro-processor is used to generate code for several implementation languages in the BCPL family (in particular, C). Maple provides interactive usage through an interpreter for the user-oriented, higher-level, Maple programming language.

This paper discusses Maple's current solution to several design issues. Maple attempts to provide a natural syntax and semantics for symbolic mathematical computation in a calculator mode. The syntax of the Maple programming language borrows heavily from the Algol family. Full "recursive evaluation" is uniformly applied to all expressions and to all parameters in function calls (with exceptions for only four basic system functions).

Internally, Maple supports many types of objects: integers, lists, sets, procedures, equations, and power series, among others. Each internal type has its own tagged data structure. "Dynamic vectors" are used as the fundamental memory allocation scheme. Maple maintains a unique copy of every expression and subexpression computed, employing hashing for efficient access. Another feature relying upon hashing is the "remembering" facility, which allows system and user-defined functions to store results in internal tables to be quickly accessed in later retrieval, thus avoiding expensive re-computation of functions.

The compiled kernel of the Maple system is relatively compact (about 100K bytes on a VAX under Berkeley Unix). This kernel includes the interpreter for the Maple language, basic arithmetic (including polynomial arithmetic), facilities for tables and arrays, print routines (including two-dimensional display), basic simplification, and basic functions (such as *coeff, degree, map,* and *divide*). Some functions (such as *expand, diff* (differentiation), and *taylor*) have a "core" in the kernel, and automatically load external user-language library routines for

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extensions. The higher-level mathematical operations (such as *gcd, int* (integrate), and *solve*, are entirely in the user-language library and are loaded only when called.

The approach to portability of the Maple system is also discussed. Maple currently runs in C under Berkeley Vax/Unix, and B under a Honeywell GCOS operating system. Maple is currently being ported to Motorola 68000 microprocessor systems on "Unix-like" operating systems.

1. Motivation for Designing a New System

Maple is a language and system for symbolic mathematical computation, under development at the University of Waterloo since December, 1980. (The name "Maple" is not an acronym but rather it is simply a name with a Canadian identity.) The type of computation provided by Maple is known by various other names such as "algebraic manipulation" or "computer algebra". The Maple system can be used interactively as a mathematical calculator, and computational procedures can be written using the high-level Maple programming language.

With so many languages and systems already developed and being developed, the question arises: "Why develop yet another system?". We will explain our motivation for developing the Maple system and the goals we are trying to achieve with Maple.

The primary motivation can be described as *user accessibility.* This concept has several aspects. The state of the art in 1980 was such that in order to have access to a powerful system such as MACSYMA (or Vaxima)[Mos74a, Fod81a] it was necessary to have a large, relatively costly mainframe computer and then to dedicate it to a small number of simultaneous users. In the university setting, this meant it was not feasible to offer symbolic computation to large classes for student computing. In a broader context, this meant that a large community of potential users of symbolic mathematical computation remained non-users. The development of the MUMATH[Ric79a] and PICOMATH[Sto80a] systems showed that a significant symbolic computation capability could be provided on low-cost, small-address-space microcomputers. It seemed clear that it should be possible to design a symbolic system with a full range of capabilities for symbolic mathematical computation which was neither restricted by the small address space of the early microcomputers nor "inaccessible to the masses" because of unreasonable demands on computing resources. In particular, it seemed possible to design a modular system whose demands on memory would grow gracefully with the needs of the application program.

Portability was another of our earliest concerns, partly because we found ourselves users of a computing environment in transition, and partly because it was clear that a wide variety of computer systems would be coming onto the market in the decade of the 1980's. It was also recognized that "user accessibility" is greatly affected by the quality of user interface which a system provides.

Thus the primary design goals of the Maple system are: *compactness,* a *powerful set of facilities* for symbolic mathematical computation, *portability,* and

a *good user interface.* These issues are discussed in more detail in the following sections.

2. Syntax and Semantics

Part of our attempt to provide a good user interface has been to try to design a syntax which is mathematically natural. This goal is conditioned by our current assumption that most users will be accessing Maple from "ordinary" terminals using one-dimensional ASCII input. (An interesting direction for the future would be to address the design of a good user interface based upon more sophisticated peripherals, building upon previous work such as [Hof79a].) Under the current assumption, many mathematical operations are specified by the traditional function-call syntax common to many programming languages. However, Maple's syntax is enriched with mathematical constructs such as *equations,* and *ranges* (e.g. 1..3).

2.1. Sample Maple Statements

The following sample statements serve to illustrate some of Maple's syntax. (Note that the double-quote operator " is used as a "ditto" symbol to specify the latest expression.)

```
taylor( exp(3*x**2 + x), x=0, 4 );
```

$$1 + x + 7/2\,x^2 + 19/6\,x^3 + \frac{145}{24}x^4 + O(x^5)$$

```
sum( (5*i-3)*(2*i+ 9), i = 1..n );
```

$$10/3\,(n+1)^3 + 29/2\,(n+1)^2 - 269/6\,n - 107/6$$

```
expand(");
```

$$10/3\,n^3 + 49/2\,n^2 - 35/6\,n$$

```
eqn1 := 3*x + 5*y = 13;   eqn2 := 4*x - 7*y = 30;   solve( {eqn.(1..2)}, {x, y} );
```

$$eqn1 := 3\,x + 5\,y = 13$$

$$eqn2 := 4\,x - 7\,y = 30$$

$$\{y = -\frac{38}{41},\ x = \frac{241}{41}\}$$

```
limit( (tan(x)-x)/x**3, x=0 );
```

$$1/3$$

```
fibonacci := proc (n)
        option remember;
        if not type(n,integer) or n<0 then
                ERROR(`invalid argument to procedure fibonacci`)
        else
                if n<2 then n else fibonacci(n-1) +  fibonacci(n-2) fi
        fi
end;

fibonacci(101);
```

573147844013817084101

2.2. Control Structures

Many of the control structures in the Maple language have been borrowed from other languages. Specifically, from Algol 68 we borrowed the repetition statement:

```
for <name> from <expr> by <expr> to <expr> while <expr>
    do <statement sequence> od
```

and the selection statement:

```
if <expr> then <statement sequence>
elif <expr> then <statement sequence>
...
else <statement sequence>
fi
```

From C we borrowed the break statement for breaking out of a loop, and RETURN(expr) for returning from a procedure. The ERROR(string) construct, similar to a feature in MACSYMA, is a special function which causes an immediate return to the top level of Maple with "ERROR: string" printed out as a message. However, a procedure may be given the "errortrap" option to allow it to "catch" an ERROR condition in it or in one of the subprocedures it calls -- this is useful for error-checking in library functions, for example.

2.3. Some Semantic Features

An important semantic feature is that Maple applies *full, recursive* evaluation of expressions as the standard evaluation rule. For example, the sequence of statements

```
a := x;
x := 3;
a;
```

yields the value 3, not x. The quoting facility for preventing the evaluation of an expression is to surround the expression with single-quotes, as in 'a+ b'.

Another semantic feature in Maple is the general rule that all parameters to all functions (system-supplied or user-defined) are fully evaluated from left to right before being passed. (Again, the quoting facility can be used to explicitly prevent evaluation). We have allowed precisely four exceptions to this general rule, for four specific system functions: *assigned* (which returns true or false depending on whether the name passed as its argument is assigned or not), *evaln* (which evaluates its argument to a name), *evalb* (which evaluates its argument as a Boolean expression), and *remember* (which is a function used to place the result of a computation in an internal table for later retrieval). Another important feature of Maple is the set of powerful primitive functions that are available when writing procedures in the user-level Maple language. Some examples of such primitive functions are *degree, coeff, lcoeff* (to extract the leading coefficient), *op* (to pick operands from an expression), and *map* (to apply a procedure onto each of the operands of an expression, separately).

2.4. Types in Maple

Maple provides a *type* function for run-time type-checking. For example, if a procedure f has a parameter x then a common construct in the procedure body is a statement such as:

```
if not type(x, algebraic) then
        ERROR(`invalid argument to procedure f`) fi
```

The Maple language has been designed to avoid obligatory type declarations, a principle that we think is important if we are to have a convenient interactive system. Furthermore, we think that the syntax and semantics which applies when writing Maple procedures should be identical with the syntax and semantics of Maple's interactive mode. Consequently, no type declarations are required in Maple and writing type-independent Maple code comes naturally.

On the other hand the Maple language is not type-less. Every object has a precise type and the type information is coded in the data structure. Our concept of "objects" and "types" applies not only to the conventional objects such as integers and lists, but also to mathematical objects such as sums and products, and to objects such as procedures and tables (arrays). As an illustration of the concept of a procedure as an object, a definition of the function *abs* in Maple could take the form:

```
abs := proc (x)
        if not type(x,rational) and not type(x,real) then
                ERROR(`invalid argument to procedure abs`)
        else
                if x<0 then -x else x fi
        fi
       end;
```

This is an ordinary assignment statement, where the procedure definition (the proc...end construct) on the right-hand-side is a valid Maple expression (i.e., an *object* with its own data structure of type *procedure*). The name *abs* could later

be re-assigned any other value (of any type). It is also possible to have a procedure definition which has not been assigned to any name, as in the following expression to reverse the left and right hand sides of a list of equations:

```
map( proc (x) op(2,x)=op(1,x) end,    [ a=b, c=d, e=f ] ) .
```

3. Data Structures

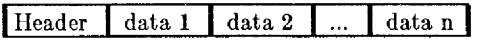
Maple has a rich set of data structures designed into it, currently about 36 different structures. Approximately one-quarter of these data structures correspond to programming language statements: assignment, if, read, etc. The remaining data structures correspond to the various types of expressions, including expressions formed using standard arithmetic and logical operators, and structures for numbers, lists, sets, tables, (unevaluated) functions, procedure definitions, equations, ranges, and series. All of these structures are represented internally as dynamic arrays (vectors), similar to the approach taken by[Nor82a].

3.1. Advantages of Dynamic Vectors

This approach using dynamic vectors at the machine level and a rich set of data structures at the abstract level has significant advantages in improved compactness and efficiency of the resulting system code. Firstly, in Maple there is only one level of abstraction above the system-level objects. It is clear that in symbolic mathematics there are many data types. The fewer and more direct the mappings between the abstract objects and the system-level objects, the simpler and more efficient will be the code that manipulates these objects. Secondly, we believe that the design of data structures should be related, if possible, to the language that describes the data objects. In our case we have a simple BNF language with the LALR(1) property, and it is natural to relate the data structures to the productions in the language. This immediately suggests the need for many data structures since there are many productions in the language. Thirdly, dynamic vectors allow us, in many cases, to have direct access to each of the components of the structure at about the same cost. This is highly desirable in some circumstances over the sequential access required when all objects are represented as lists. Fourthly, dynamic vectors are more compact than structures linked by pointers. In summary, an important part of the compactness and efficiency of Maple is due to the use of proper data structures.

3.2. Examples of Maple's Data Representation

All of the internal data structures in Maple have the same general format:

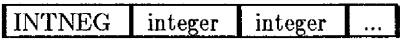


The *header* field encodes the length $(n+1)$ of the structure, the type, one bit to indicate simplification status, and two bits to indicate garbage collection status. Every data structure is created with its own length and this length will not change during its entire existence. Data structures are typically not changed after creation since it is not predictable how many other data structures are

pointing to a given structure. The normal procedure to modify structures is to create a copy and modify the copy, hence returning a new data structure.

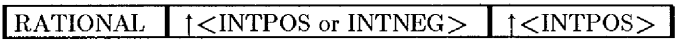
The following are some specific examples of data structures in Maple. The notation ↑<xxx> will be used to indicate a pointer to a structure of type xxx.

Negative integer



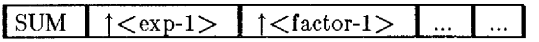
Here the INTNEG header includes both the tag for INTNEG and the length of the data structure, which depends upon the size of the negative number being represented. Each integer field of an INTNEG contains one base BASE digit. BASE=10000 for 32-bit machines and BASE=100000 for 36-bit machines; that is, BASE is the largest power of 10 that will fit into a half word on the host machine.

Rational number



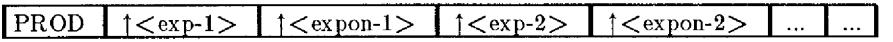
The second integer is always positive and different from 0 or 1. The two integers are relatively prime.

Sum of several terms



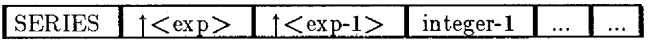
This structure should be interpreted as pairs of expressions and their constant factors. The simplifier lifts all explicit constant factors from each expression and places them in the <factor> entries. A term consisting only of a rational constant is represented with factor 1.

Product/quotient/power



This structure should be interpreted as a product of $<\text{exp-i}>^{<\text{expon-i}>}$. Rational number or integer expressions to an integer power are expanded. If there is a rational constant in the product, this constant will be moved to the first entry by the simplifier.

Series



The first expression is the "taylor" variable of the series, the variable used to do the series expansion. The remaining entries have to be interpreted as pairs of coefficient and exponent. The exponents are integers (not pointers to integers) and appear in increasing order. A coefficient O(1) (function call to the function "O" with parameter 1) is interpreted specially by Maple as an "order" term.

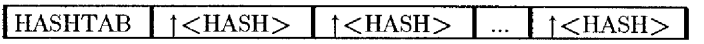
4. The Use of Hashing in Maple

Maple handles all table searching in a uniform way. All of the searching is done by an algorithm which is a slight modification of direct-chaining hashing. Although it is not obvious, the internal tables play a crucial role; they are used for: locating variable names, keeping track of simplified expressions, keeping track of partial computations, mapping expression trees into sequential files for internal input/output, and for storing arrays and tables. It is immediately obvious that the searching in these tables has to be fast enough to guarantee overall efficiency.

The algorithm used for these tables can be understood as an implementation of direct-chaining where instead of storing a linked list for each table entry, we store a variable-length array. This requires a versatile and efficient storage manager, but without one, symbolic computation would not be feasible regardless.

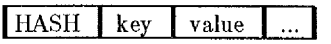
The two data structures used to implement tables are:

Table entry



Each entry points to a HASH entry or it is 0 if no entry was created. The size of HASHTAB is constant for the implementation. For best efficiency, the number of entries should be prime.

Hash-chain entry



Each entry in the table consists of a consecutive pair, the first one being the hashing key and the second the stored value. A key cannot have the value 0 as this is the indicator for the end of a chain. For efficiency reasons, the HASH entries are incremented by 5 entries at a time and consequently some entries may not be filled. Keys may be any integer or pointer which is representable in one word. In many cases the key is itself a hashing value (two step hashing).

4.1. The Simplification Table

All simplified expressions and subexpressions are stored in the simplification table. The main purpose of this table is to ensure that expressions appear internally only once. Every expression which is entered into Maple or which is internally generated is checked against this table, and if found, the new expression is discarded and the old one is used. This task is done by the simplifier which recursively simplifies (applies all the basic simplification rules) and checks against the table.

The task of checking for equivalent expressions within thousands of subexpressions would not be possible if it was not done with the aid of a "hashing" concept. Every expression is entered in the simplification table using its *signature* as a key. The signature of an expression is a hashing function itself, with

one very important attribute: it is order independent. For example, the signatures of the expressions a+ b+ c and c+ a+ b are identical; the signatures of a**b and b**a are also identical. Searching for an expression in the simplification table is done by:

- Simplifying recursively all of its components;
- Applying the basic simplification rules.
- Computing its signature and searching this signature in the table. If the signature is found then we perform a full comparison (taking into account that additions and products are commutative, etc.) to verify that it is the same expression. If the expression is found, the one in the table is used and the searched one is discarded.

The number of times that we have to do a full comparison on expressions is minimal; it is only when we have a "collision" of signatures. Some experiments have indicated that signatures coincide once every 50000 comparisons for 32-bit signatures. (Notice that the signatures are still far from uniform random numbers). The resulting expected time spent doing full comparisons is negligible. Of course, if the signatures disagree then the expressions cannot be equal at the basic level of simplification.

4.2. The Partial Computation Table

The partial computation table is responsible for handling the option remember in function definitions in its explicit and implicit forms. Basically, the table stores function calls as keys and their results as values. Since both these objects are data structures already created, the only cost (in terms of storage) to place them in the table is a pair of entries (pointers). Searching these hashing tables is extremely efficient and even for simple functions it is orders of magnitude faster than the actual computation of the function.

The change in efficiency due to the use of the remembering facility may be dramatic. For example, the Fibonacci numbers computed with

```
f := proc(n)
        if n<2 then n else f(n-1)+ f(n-2) fi end;
```

take exponential time to compute, while

```
f := proc(n) option remember;
        if n<2 then n else f(n-1)+ f(n-2) fi end;
```

requires linear time.

Besides the facility provided to users, the internal system uses the partial computation table for diff, taylor, expand, and evalr. The internal handling of expand is straightforward. There are some exceptions with the others, namely:

- diff will store not only its result but also its inverse; in other words, if you integrate the result of a differentiation the result will be "table-looked up" rather than computed. In this sense, integration "learns" from differentiation.
- taylor and evalr need to store some additional, environment, information (Degree for taylor and Digits for evalr). Consequently the entries in these cases are extended with the precision information. If a result is requested with less

precision than what is stored in the table, it is retrieved anyway and "rounded". If a result is produced with more precision than what is stored, the table entry is replaced by the new result.

- evalr only remembers function calls; it does not remember the results of arithmetic operations.

Arrays are implemented using internal tables, with the address of the (simplified) expression sequence of indices used as the hashing key. (Note that since simplified expressions appear only once, we can use their addresses as keys.) Since arrays are treated just like tables at the internal level, dense and sparse arrays are handled equally efficiently.

5. Compact Size as a Design Goal

The kernel of the Maple system (i.e., the part of the system which is written in the systems implementation language) is kept intentionally small -- for example, it occupies about 100K bytes on a VAX. The kernel system includes only the most basic facilities: the user programming language interpreter, numerical, polynomial and series arithmetic, basic simplification, facilities for handling tables and arrays, print routines, and some fundamental functions such as *coeff, degree, subs* (substitute), *map, igcd* (integer gcd computation), *lcoeff* (leading coefficient of an expression), *op, divide, imodp/imods* (integer modular operations using positive/symmetric representation), and a few others. Some of the fundamental functions have a small "core" coded in the kernel and an interface to the Maple library for extensions. The interface is general enough so that additional power, such as the ability to deal with new mathematical functions of interest to a particular user, can be obtained by user-defined Maple code. Some examples of functions which have such a "core" and a user interface are *diff, expand, taylor, type,* and *evalr* (for evaluation to a real number). Other functions supplied with the system are entirely in the Maple library, including *gcd, factor, normal* (for normalization of rational expressions), *int,* and *solve.*

The compactness of a system is affected by many different design decisions. The following points outline some of the design decisions which have contributed to the compactness of the Maple system.

1. *The use of appropriate data structures.* As we have pointed out in section 3, an important factor in compactness is the design of a rich set of data structures appropriate to the mathematical objects being manipulated, with a direct mapping between these abstract structures and the machine-level "dynamic arrays". This data structure design avoids the introduction of an intermediate "artificial" level of structure such as lists. One level of compactness is thus achieved because the number of pointers is reduced compared with a linked-list representation. Significantly, another level of compactness is achieved because the code required to manipulate these data structures is generally shorter than the code which must deal with a list representation.

2. *The use of a viable file system.* By having an efficient interpreter and by placing much of the code for system functions into the user-level library, Maple has the property that "you only pay for what you use". Writing functions in the user-level Maple language has the additional advantages of readability, maintainability, and portability. This necessarily depends upon having a file system that (at least through efficient simulation) has some desirable properties such as a tree-structured directory system and variable-length records. It may have been unreasonable a decade ago to make such assumptions about the file system, but these assumptions are (or will be) satisfied by many current and future mainframe and micro computer systems.

3. *Avoiding a large run-time support system.* Providing an "integrated programming environment" or a large run-time support system can lead to non-trivial memory requirements. For example, Franz Lisp on Berkeley Unix starts off at almost 500K bytes. We view Maple as just one of many software tools that a user may employ to solve problems, regardless of which system it may be used on. We see no need to provide all of these tools within Maple itself, not only because they greatly increase the problems of porting without providing any greater algebraic computation power, but also because many computing environments will allow their native software tools to be easily connected to Maple (say, as communicating processes) once Maple has been ported to that environment. For example, Unix EMACS[Gos81a] can invoke Maple as a subprocess on Berkeley Unix, providing some screen managing and editing facilities for Maple. Thus we do not view the basic Maple system, which provides minimal programming support (e.g., only a simple trace package and no editor), as lacking a programming environment. Rather, we see Maple as being easy to integrate into an environment chosen by the user. We certainly think that having a good user/programming interface to Maple is important. Indeed, we look forward toward developing a "personal algebra machine" in the near future. However, we envision this kind of work as building upon the basic Maple system rather than building more into it.

4. *A policy of treating main memory as a scarce resource.* We believe that this point of view is important if we are to achieve the goal of providing a symbolic computation system to "the masses". Because we have adopted such a point of view, we are constantly concerned about which functions belong in the Maple kernel and which functions can be supplied as user-level code in the Maple library. Since we have an efficient mechanism to retrieve Maple functions from the library, and an efficient interpreter, we are not forced to abandon computational power for the sake of compactness.

5. *The choice of the BCPL family of systems implementation languages.* Implementing Maple in systems languages from the BCPL family has helped us to achieve the compactness goals outlined in the above points. These languages typically produce relatively compact and efficient object code, thus contributing directly to the goal of treating main memory as a scarce resource. The support of "dynamic arrays" in the implementation language allows the

creation of compact data structures for the higher-level objects. Furthermore, an implementation language in the BCPL family typically has a run-time library that is small, selectively included, and yet provides the desired functionality.

Although the availability of inexpensive memory and hardware support for large address spaces makes it possible to design a programming system which has all of its routines contained within a large (virtual) main memory, we consider such a design to be inefficient both on mainframe timesharing systems and on the arriving generation of inexpensive but powerful microprocessor systems. It will continue to be true, in our view, that a more efficient design can be achieved by treating main memory as a scarce resource. Maple's design with a relatively small kernel interfacing to an external library takes the latter point of view.

6. Computational Power through Libraries of Functions

Another goal of the Maple system is to provide a powerful set of facilities for symbolic mathematical computation. In other words, we are not willing to achieve compactness by sacrificing the computational power of the system. Thus while the number of functions provided in the kernel system is kept to a minimum, many more functions for symbolic mathematics are provided in the system library, to be loaded as required. The functions in the system library are written in the high-level Maple programming language and are therefore readily accessible to all users of the Maple system. A load module for each library procedure is stored in "Maple internal format" which is a quick-loading expression-tree representation of the procedure definition. When a library function is invoked, its load module is read into the Maple environment (if not already loaded) and the expression tree is interpreted by the Maple interpreter.

Since run-time loading of compiled code is not (yet) a portable feature for BCPL-family languages on most systems, the execution speed of the system is seen to depend on the interpreter for the Maple language. Maple's interpreter is relatively efficient; for example, an experiment performed by running the tak function[Gri82a] shows Maple's interpreter to be about four times faster than Vaxima's interpreter on that particular benchmark. Consequently, the tradeoff between "user-level" and "system-level" code is not as great in Maple as in other systems. When a critical function has been identified as causing a serious degradation in execution time, it has been moved into the compiled kernel system*. Undoubtedly, there would be some gain in execution speed if all of the Maple functions were coded entirely in the compiled kernel but the resulting loss of compactness, and hence of user accessibility, outweighs such gains in execution speed.

* This was done, for example, with the function for polynomial division which was first placed in the system library and then later moved into the kernel. On the other hand, some functions such as solve and int have been moved from the kernel out to the system library without causing a significant degradation in performance.

7. Portability

As part of the general goal of "user accessibility", the Maple system is not tied to one operating system, nor to one programming language. Maple is intended to be portable across several languages, descendants of BCPL. To achieve this level of portability and to have a *single* source code (multiple copies are viewed as a disastrous scenario) we use a general purpose macro-processor called Margay. Our current Margay macros define a language very similar to B or C except for the places where the languages differ, where we do one of the following:

(i) Write a new macro which can be easily mapped onto every language. (Most of the time the macro will have some additional information which may be redundant for some languages but used by others). This is possible since the whole internal maple is relatively small (5500 lines) and we are willing to modify the code to improve portability.

(ii) Avoid using a particular feature if it is too peculiar to a single language.

(iii) Avoid, whenever possible, constructs that may be ambiguous across different languages.

The macro-processor is used not only as a way of providing a higher level of readability of the source code, as M6 was used with Altran [Hal71a], but also as a way to make Maple portable across several languages.

Maple is currently running under the GCOS operating system on a Honeywell 66/80 (110K words maximum address space) and under Berkeley Unix on VAX 11/780's. We have begun experiments porting to C on various operating systems on MC68000-based microcomputers, such as Xenix, Unisoft Unix, and the WICAT operating system. We have plans to port Maple into other BCPL-derivative languages in the near future, such as the locally-developed languages WSL[Bos80a] and PORT[Mal82a].

8. Notes on Software Development

Maple development started on a Honeywell system in B when the project began in 1980. When Waterloo acquired a VAX in 1981, we ported Maple to C. At that time, we were forced to demonstrate portability between languages and operating systems out of necessity, since Maple had to continue to work on the Honeywell for student use.

8.1. Choice of BCPL-derivatives as implementation language

While Maple's behaviour is based as much on our coding of algorithms and data structures as on our choice of implementation language, it seems clear to us that a general-purpose system based on a BCPL-family language can be compact, yet have reasonable performance on interesting problems. The software tools available (parser-generators, execution profilers, etc.) have made the implementation process proceed in a timely fashion with a small staff. While we don't think any final conclusions should or can yet be drawn about the relative merits of Lisp or BCPL-family languages as vehicles for symbolic systems, we do suggest that the choice of system implementation languages now seems less limited than in the

early '70s when the last generation of algebraic systems were being designed. Our approach towards portability is of course tied to the health and propagation of BCPL-family languages, but we feel this is assured, at least for the next few years, given the interest of the larger computer science community in such languages. We feel that our approach frees us to concentrate on providing algebraic computation power, as opposed to worrying about machine code generators, portable subsets, or porting programming environments.

8.2. Breadboarding

Our mode of operation up to this point is akin to "breadboarding" an electrical design, in that we can observe real, not merely theoretical, performance over a long period of time, and yet be in a position to make possibly incompatible changes in a timely fashion. The compactness of the Maple kernel is what makes breadboarding feasible for us, in that someone modifying the kernel must deal with only 5500 lines of code. As a consequence of this approach, version 1 of the Maple system is almost unrecognizable as a predecessor of version 2, although version 3 (currently under development) is characterized mainly by added facilities rather than by fundamental design changes compared with version 2.

We do not claim to have worked out all of the design and implementation issues facing us. Maple has changed since the inception of the project, not only through the introduction of additional features, but also through incompatible changes made because we changed our minds. Nevertheless, we have willingly subjected the system to significant usage at every stage of its development. The first version of the Maple system was running within a week of the first discussions on its design, with significant "real-world" problems solved using it within a month. Hundreds of students at Waterloo have already used Maple in undergraduate and graduate classes *. We are continuing to operate in a mode where there is a short time period between ideas and their implementation, with the result that the practical, real, applications of "great ideas" are soon found, and the "great ideas that are not-so-great" are modified or discarded. In this, we are grateful for the flexibility of our academic environment (and students!), and for the vigour of workers in algebraic manipulation of the past decade who have provided us with a wealth of implemented algorithms and applications problems that are obvious tests for Maple.

To some extent, the breadboarding approach means that we have had to proceed slowly on the design of "large features" such as user-directed simplification, but we think that by tying the design of Maple closely to its implementation and usage we have gained invaluable experience and feedback. Furthermore, we think that doing so has kept us from designing beyond our immediate capacity to remain faithful to maintaining efficiency and portability.

*Maple is used in an undergraduate data base class (its support of sets and tuples was used for a relational data base package), as well as courses in algebraic manipulation. It has also been used for "real" formula manipulation by some of our departmental colleagues, and as an algebraic calculator by students on a casual basis.

9. Conclusions

We expect several more cycles of building, using, and learning for Maple. Nevertheless, we believe that our accomplishments so far affirm the validity of our approach towards data representation and manipulation, towards portability, and towards making algebraic manipulation generally available. David Stoutemyer once said that one way to make computer symbolic math economically feasible for the masses would be to encourage the University of Waterloo to develop a compact "WATALG" system [Sto79a]. With the Maple system, we have taken up the spirit of that challenge.

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MAPLE

User's Manual

Second Edition

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PREFACE

The design and implementation of the Maple system are currently in progress. This manual is a working document for the project. The version of the Maple system which has been completed at the time of publication of this second edition of the manual (December 1982) is version 2.2.

The Maple project was first conceived in the fall of 1980 as the logical outcome of discussions on the state of symbolic computation at the University of Waterloo. The authors wish to acknowledge many fruitful discussions with colleagues at the University of Waterloo, particularly Morven Gentleman, Michael Malcolm and Frank Tompa. It was recognized in these discussions that none of the locally-available systems for symbolic computation provided the environment nor the facilities that should be expected for symbolic computation in the 1980's. We concluded that since the basic design decisions for current symbolic systems such as ALTRAN, CAMAL, Reduce, and MACSYMA[Hal71a, Bou71a, Hea71a, Mar71a] were made more than ten years ago, it would make sense to design a new system from scratch taking advantage of the software engineering technology that has become available since then, as well as drawing from the lessons of experience.

Like other algebraic manipulation systems, Maple's basic features (e.g. elementary data structures, input/output, rational number arithmetic, and elementary simplification) are coded in a systems programming language for efficiency. For users, there is a high-level language with an Algol68-like syntax more suitable for describing algebraic algorithms. An important property of Maple is that most of the algebraic facilities are defined using the high-level user language. The basic system is sufficiently compact and efficient to be practical to use in a present-day time-sharing environment while providing a useful array of facilities. To this basic system can be added successive levels of 'function packages', each of which adds more facilities to the system as may be required, such as polynomial factorization or the Risch integration algorithm. The modularity of this design should allow users latitude in selecting which algebraic facilities they wish to have.

The basic system is written in a language belonging to the BCPL/B/C family. The Margay macro processor[Joh83a] is used to generate versions of the source code in B (for Honeywell TSS) and C (for the Vax UNIX† system). It is anticipated that very high level use of Maple (e.g., the Risch integration algorithm) will be impractical in a heavily-used time-sharing environment. Such use will be more practical on a dedicated microprocessor with one or more megabytes of main memory. Current plans are to use a Motorola 68000-based microsystem for the first implementation on dedicated hardware.

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†UNIX is a Trademark of Bell Laboratories.

CONTENTS

1. INTRODUCTION

Maple is a mathematical manipulation language. (The name can be said to be derived from some combination of the letters in the preceding phrase, but in fact it was simply chosen as a name with a Canadian identity). The type of computation provided by Maple is known by various other names such as 'algebraic manipulation' or 'symbolic computation'. A basic feature of such a language is the ability to, explicitly or implicitly, leave the elements of a computation unevaluated. A corresponding feature is the ability to perform 'simplification' of expressions involving unevaluated elements.

In Maple, statements are normally evaluated as far as possible in the current 'environment'. For example the statement

```
a := 1;
```

assigns the value 1 to the name a. If this statement is later followed by the statement

```
x := a + b;
```

then the value 1+ b is assigned to the name x. Next if the assignments

```
b := -1;  f := sin(x);
```

are performed then x evaluates to 0 and the value 0 is assigned to the name f. (Note that sin(0) is automatically 'simplified' to 0). Finally if we now perform the assignments

```
b := 0;  g := sin(x);
```

then x evaluates to 1 and the value sin(1) is assigned to the name g. (Note that sin(1) cannot be further evaluated or simplified in a symbolic context, but there is a facility to 'evaluate to real' which will yield the decimal expansion of sin(1) to a number of digits controlled by the user).

2. LANGUAGE ELEMENTS

2.1. Character Set

The Maple character set consists of letters, digits, and special characters. The letters are the 26 *lower case letters*

a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

and the 26 *upper case letters*

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z.

The 10 *digits* are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

and the 25 *special characters* are

	blank	[	left bracket
;	semicolon	]	right bracket
:	colon	{	left brace
=	equal	}	right brace
+	plus	`	grave accent
-	minus	'	single quote
*	asterisk	"	double quote
/	slash	<	less-than
!	exclamation	>	greater-than
.	period	_	underscore
,	comma	@	at-sign
(	left parenthesis	#	sharp
)	right parenthesis		

leaving the following ASCII characters as yet unused:

$	dollar	|	vertical bar
&	ampersand	%	percent
~	tilde	?	question mark
\	back slash	^	circumflex

2.2. Tokens

The tokens consist of keywords, operators, strings, natural integers, and punctuation marks.

The *keywords* are the following reserved words:

break	by	do	done
elif	else	end	fi
for	from	if	local
od	option	options	proc
quit	read	save	stop
then	to	while	

The *operators* consist of the *binary* operators

+	addition; set union
-	subtraction; set difference
*	multiplication; set intersection
/	division
**	exponentiation
<	less than
<=	less than or equal
>	greater than
>=	greater than or equal
=	equal
<>	not equal
and	logical and
or	logical or
:=	assignment
.	concatenation; decimal point
..	ellipsis (more generally, ...*)

the *unary* operators

+	unary plus (prefix)
-	unary minus (prefix)
!	factorial (postfix)
not	logical not (prefix)
.	decimal point (prefix or postfix)

* two *or more* consecutive periods are parsed as an ellipsis.

and the *nullary* operators

%	last expression
%%	penultimate expression
%%%	before penultimate expression

(Note that three of the operators are reserved words: and, or, not).

The simplest instance of a *string* is a letter followed by zero or more letters, digits, and underscores. Another instance of a string is the at-sign (@) followed by zero or more letters, digits, and underscores. More generally, a string can be formed by enclosing in grave accents any sequence of characters (except a grave accent). In all cases, the maximum length of a string in Maple is 499 characters. A string is a valid name (e.g. a variable name or a function name) but the user should not indiscriminately use names involving the at-sign (@) since such names are used globally by the Maple system. A Maple string is also used in the sense of a 'character string', usually by enclosing a sequence of characters in grave accents.

A *natural integer* is any sequence of one or more digits. The basic constants in Maple (integers, rational numbers, and reals) are formed from the natural integers using operators. The length of a natural integer (and hence the length of integers, rational numbers, and reals) is arbitrary (i.e. the length limit is system dependent but generally much larger than users will encounter).

The *punctuation marks* are

;	semicolon	,	comma
'	single quote	[	left bracket
`	grave accent	]	right bracket
{	left brace	(	left parenthesis
}	right brace	)	right parenthesis

The semicolon is used to separate statements and the comma is used to separate expressions in an expression sequence (as in a function call or in specifying a list or a set). Enclosing an expression in a pair of single quotes specifies that the expression is to be unevaluated. The grave accent is used in forming strings. The left and right parentheses have their familiar uses in grouping terms in an expression and in grouping parameters in a function call. The left and right brackets are used to form subscripted names. The left and right brackets are also used to form lists and the left and right braces are used to form sets.

2.3. Blanks, Lines, and Comments

The *blank* is a character which separates tokens, but is not itself a token. Blanks cannot occur within a token but otherwise blanks may be used freely. The one exception to the free use of blanks is when forming a string by enclosing a sequence of characters within grave accents, in which case the blank is significant like any other character.

Input to the Maple system consists of a sequence of statements separated by semicolons. The system operates in an interactive mode, executing statements as they are entered. A *line* consists of a sequence of characters followed by <return>. A single line may contain several statements or it may contain an incomplete statement (i.e. a statement to be completed on succeeding lines), or it may contain several statements followed by an incomplete statement. A statement will normally be recognized as complete only when a semicolon is encountered, except in the cases of the quit statement, the break statement, the if ... fi construct and the do ... od construct. When a line is entered, the system evaluates (executes) the statements (if any) which have been completed on that line.

When a sharp (#) is encountered, it and all subsequent characters on the line are considered to be a *comment*. The comment is echoed by the system.

2.4. Files

The file system is an important part of the Maple system. The user interacts with the file system either explicitly by way of the read and save statements, or implicitly by specifying a function name corresponding to a file which the system will read in automatically.

A *file* consists of a sequence of statements either in 'Maple internal format' or in 'user format'. If the file is in user format then the effect of reading the file is identical to the effect of the user entering the same sequence of statements. The system will display the result of executing each statement which is read in from the file. On the other hand, if the file is in Maple internal format then reading the file causes no information to be displayed to the user but updates the current Maple environment with the contents of the file. Maple assumes that a file will be in Maple internal format when its file name ends with the characters '.m' . For example, some typical names for files in user format are:

```
temp
`/lib/src/gcd`
```

while some typical names for files in internal format are:

```
`temp.m`
`/lib/gcd.m`
```

(Note that file names involving characters such as '/' or '.' must be enclosed in grave accents in order to be interpreted properly as <name>s).

The contents of a file in user format are written into the file either from a text editor external to Maple or else from Maple by using the *save* statement (no '.m' suffix in the file name). The contents of a file in Maple internal format are written into the file from Maple by using the *save* statement ('.m' suffix in the file name). Either type of file may be read into a Maple session by using the Maple *read* statement. Some Maple functions are not part of the basic Maple system which is loaded in initially, but rather reside in files in the Maple library. When one of these functions is invoked in Maple, the corresponding file is automatically read into the Maple session in Maple internal format. (See section 7 – Library Functions).

3. STATEMENTS AND EXPRESSIONS

3.1. Types of Statements

There are nine types of statements in Maple. They will be described informally here. The formal syntax is given in section 4.2.

3.1.1. Assignment Statement

The form of this statement is

<name> := <expression>

and it associates a name with the value of an expression.

3.1.2. Expression

An <expression> is itself a valid statement. The effect of this statement is that the expression is evaluated.

3.1.3. Read Statement

The statement

read <expression>

causes a file to be read into the Maple session. The <expression> must evaluate to a <name> which is a valid file name in the host system. The file name may be one of two types as discussed in section 2.4. A typical example of a read statement is

read `/u/dmackenz/lib/f.m`

where the grave accents are necessary so that the <expression> evaluates to a <name>.

3.1.4. Save Statement

The statement

save <expression>

causes the current Maple environment to be written into a file. The <expression> must evaluate to a <name> which is a valid file name in the host system. If the file name ends with the characters '.m' then the environment is saved in Maple internal format, otherwise the environment is saved in user format.

3.1.5. Selection Statement

The selection statement takes one of the following general forms. Here <expr> is an abbreviation for <expression> and <statseq> stands for a sequence of statements.

if <expr> then <statseq> fi
if <expr> then <statseq> else <statseq> fi
if <expr> then <statseq> elif <expr> then <statseq> fi
if <expr> then <statseq> elif <expr> then <statseq> else <statseq> fi

Wherever the construct 'elif <expr> then <statseq>' appears in the above forms, this construct may be repeated any number of times to yield a valid selection statement. The sequence of statements in the branch selected (if any) is executed.

3.1.6. Repetition Statement

The syntax of the repetition statement is as follows, where <expr> and <statseq> are as above.

for <name> from <expr> by <expr> to <expr> while <expr>
do <statseq> od

where any of 'for <name>', 'from <expr>', 'by <expr>', 'to <expr>', or 'while <expr>' may be omitted. The sequence of statements in <statseq> is executed zero or more times. The 'for <name>' part may be omitted if the index of iteration is not required in the loop, in which case a 'dummy index' is used by the system. If the 'from <expr>' part and/or the 'by <expr>' part are omitted then the default values 'from 1' and/or 'by 1' are used. If the 'to <expr>' part and/or the 'while <expr>' part are present then the corresponding tests for termination are checked at the beginning of each iteration, and if neither is present then the loop will be an infinite loop unless it is terminated by the execution of the **break** statement (see section 3.1.7), or the **quit** statement (see section 3.1.8), or by the execution of a return from a procedure (see section 5.5).

3.1.7. Break Statement

The syntax of the break statement is

break

and it causes an immediate exit from the innermost repetition statement within which it occurs. It is an error if the break statement occurs at a place which is not within a repetition statement.

3.1.8. Quit Statement

The syntax of the quit statement is any one of the following three forms:

quit
done
stop

The result of this statement is to terminate the Maple session and return the user to the system level from which Maple was entered. (In the Vax UNIX and Honeywell TSS versions of Maple, hitting the break/interrupt key twice in rapid succession will also exit

from Maple).

3.1.9. Empty Statement

The empty statement is syntactically valid in Maple. For example

```
a := 1; ; quit
```

is a valid statement sequence in Maple consisting of an assignment statement, the empty statement, and the quit statement. Of course since blanks may be freely used, any number (including zero) of blanks could appear between the semicolons here yielding a syntactically identical statement sequence.

3.2. Expressions

Expressions are the fundamental entities in the Maple language. The various types of expressions are described informally here. The formal syntax is given in section 4.2.

3.2.1. Basic Constants

The basic constants in Maple are integers, rational numbers, and reals. A <natural integer> is any sequence of one or more digits of arbitrary length (i.e. the length limit is system dependent but generally much larger than users will encounter). An *integer* is a <natural integer> or a signed integer (i.e. + <natural integer> or - <natural integer>). A *rational number* is of the form

<integer> / <natural integer>

(Note that a rational number is always simplified so that the denominator is unsigned, and it will also be reduced to lowest terms).

An <unsigned real> is one of the following three forms:

<natural integer> . <natural integer>
<natural integer> .
. <natural integer>

A *real number* is an <unsigned real> or a signed real (i.e. + <unsigned real> or - <unsigned real>). The **evalr** function is used to force an expression to be evaluated to a real number (if possible). The number of digits carried in the 'mantissa' when evaluating reals is determined by the value of the global name 'Digits' which has 10 as its initial value. Note that the current version of Maple displays real numbers with very small or very large magnitudes using the notation Real(mantissa,characteristic) which corresponds to the internal data structure. For example, evalr(exp(-10)) yields Real(4539992971,-14) which represents the number

```
4539992971 * 10**(-14)
```

while evalr(exp(-2)) yields .1353352832 .

3.2.2. Names

A <name> in Maple has a value which may be any expression or, if no value has been assigned to it, then it stands for itself. A <name> is usually a <string>, which in its simplest form is a letter followed by zero or more letters, digits, and underscores (with a maximum length of 499 characters). Note that lower case letters and upper case letters are distinct, so that the names

g G new_term New_Term x13a x13A

are all distinct. Another type of <string> is formed by the at-sign (@) followed by zero or more letters, digits, and underscores. Names beginning with the at-sign are used as global variable names by the Maple system and therefore should not be used indiscriminately by users.

A <string> can also be formed by enclosing in grave accents any sequence of characters (except the grave accent). The following are valid strings (and hence names) in Maple:

`This is a strange name:` `2D` `n-1`

The grave accents do not themselves form part of the string so they disappear when the string has been input to Maple. For example, if n has the value 5 then the statement

```
`n-1` := n-1;
```

will yield the following response from Maple:

```
n-1 := 4
```

The user should beware of misusing this facility for string (name) formation to the point of writing unreadable programs!

More generally, a <name> may be formed using the *concatenation operator* in one of the following three forms:

<name> . <natural integer>
<name> . <string>
<name> . (<expression>)

Some examples of the use of the concatenation operator for <name> formation are:

v.5 p.n a.(2*i) V.(N.(i-1)) r.i.j

The concatenation operator is a binary operator which requires a <name> as its left operand. Its right operand is evaluated and then concatenated to the left operand. For example if n has the value 4 then p.n evaluates to the name p4, while if n has no value then p.n evaluates to the name pn. Similarly if i has the value 5 then a.(2*i) evaluates to the name a10. As a final example if N4 has the value 17 and i has the value 5 then V.(N.(i-1)) evaluates to the name V17, while V.N.(i-1) evaluates to the name VN4 (assuming that N has no value).

Another type of <name> in Maple is a *subscripted name* which takes the form

<name> [<expression sequence>] .

For example, the following are valid <name>s in Maple:

```
a[3,1,5]; a[-4]; a[1];
i := 2; j := 1; a[i,j];
```

and all of the names listed here may be used in the same Maple session. The subscripts appearing in a subscripted name must evaluate to integers in order to form a valid <name>. This construct is a special case of the concatenation construct. For example, if i and j evaluate to integers then the name a[i,j] is precisely equivalent to the following name formed using Maple's concatenation operator:

```
a . `[` . i . `,` . j . `]`
```

Therefore for the case of single subscripts, the constructs

```
a.i , for i = 1, 2, . . . , n
a[i] , for i = 1, 2, . . . , n
```

are equally general. However when using two or more subscripts the subscripted names have advantages. For example if i = 1, j = 27, m = 12, and n = 7 then

```
a[i,j]; a[m,n];
```

evaluate to the distinct names a[1,27] and a[12,7] while

```
a.i.j; a.m.n;
```

both evaluate to the single name a127. Finally note that since a[1,27], for example, is a valid name it follows that the construct

```
a[1,27][5]
```

is also valid. In general, subscripted names may be freely combined with additional subscripts, and with the concatenation construct, as desired.

(Note that arrays are not defined in the present version of Maple. A definition of an array data structure is currently under discussion and is expected to be introduced in version 3.0 of Maple.)

3.2.3. Sets and Lists

A *set* is an expression of the form

```
{ <expression sequence> }
```

and a *list* is an expression of the form

```
[ <expression sequence> ] .
```

Note that an <expression sequence> may be empty so that the empty set is represented by { } and the empty list is represented by [ ]. A set is an *unordered* sequence of expressions and the user should not assume that the expressions will be maintained in any particular order. (The Maple system will use a particular ordering that is convenient from the implementation point of view). A list is an *ordered* sequence of expressions so the order of the expressions will be the order specified by the user. For example, if the user inputs the set {x,y,y} the system might respond with the representation {y,x} while

if the user inputs the list [x,y,y] then the representation used by the system will be precisely this list.

3.2.4. Algebraic Operators

There are nine *algebraic operators:*

", "", """, !, +, -, *, /, **

The nullary operator " has as its value the latest expression, the nullary operator "" has as its value the penultimate expression, and the nullary operator """ has as its value the expression preceding the penultimate expression. The unary operator ! is used as a postfix operator and it denotes the factorial function of its operand. + and - may be used as prefix operators representing unary plus and unary minus. The latter five operators all may be used as binary operators, representing addition, subtraction, multiplication, division, and exponentiation, respectively.

The operators +, -, and * have a different semantics when their operands are sets, in which case they denote set union, set difference, and set intersection, respectively. For example, if the following statements are executed:

```
set1 := {x+y, x, y}; set2 := {y, y-x};
a := set1 + set2; b := set1 - set2; c := set1 * set2;
```

then the value of a is {y, x, y-x, x+y}, the value of b is {x, x+y}, and the value of c is {y}.

The order of precedence of all operators is described in section 3.2.9 below. However, any expression may be enclosed in parentheses yielding a new valid expression and this mechanism can be used to force a particular order of evaluation.

3.2.5. Relations and Logical Operators

A new type of expression can be formed from ordinary algebraic expressions by using the *relational operators* <, <=, >, >=, =, <>. The semantics of these operators is dependent on whether they occur in an *algebraic* context or in a *boolean* context.

In an algebraic context, the relational operators are simply 'place holders' for forming equations or inequalities. Addition of equations and multiplication of an equation by a constant are fully supported in Maple. In the case of adding or subtracting two equations, the addition or subtraction is applied to each side of the equations yielding a new equation. In the case of multiplying an equation by a constant, the multiplication is distributed to each side of the equation. Other operations on equations can be performed, using the 'expand' function as required. No operations on inequalities are currently supported in Maple.

In a boolean context a relation is evaluated to the value 'true' or the value 'false'. In the case of the operators <, <=, >, >= the difference of the operands must evaluate to a constant and this constant is compared with zero. In the case of either of the relations

op1 = op2
op1 <> op2

the operands can be arbitrary algebraic expressions.

More generally, an expression can be formed using the *logical operators*

and
or
not

where the first two are binary operators and the third is a unary (prefix) operator.

In Maple, the names 'true' and 'false' have special meanings when they occur in boolean contexts but they are ordinary <name>s which may be freely manipulated. Any arbitrary expression may be used in a boolean context and if the expression does not evaluate to either the value of 'true' or the value of 'false' then a semantic error will be reported. Note that since 'true' and 'false' are ordinary names, it is possible to assign values to them. For example, a user could assign

true := 1; false := 0;

and thereafter expressions which previously evaluated to 'true' or 'false' will evaluate to '1' or '0'. Normally users will leave the names 'true' and 'false' unassigned so that their values are their own names.

3.2.6. Ranges

Yet another type of expression is a *range* which is formed using the ellipsis operator:

<expression> .. <expression>

(the operator here can be specified as two *or more* consecutive periods). The ellipsis operator simply acts as a 'place holder' in the same manner as when the relational operators are used in an algebraic context.

Two common uses of ranges are in Maple's built-in functions sum and int. For example, in the function call

sum(i**2, i = 1..n)

the sum function interprets this to mean that the lower and upper limits of summation are 1 and n, respectively. Similarly, in the function call

int(exp(2*x), x = 0..1)

the integration function interprets this as a definite integration with lower and upper limits of integration 0 and 1, respectively. The range construct is also used by Maple's built-in function op, which extracts operands from an expression. For example, if

a := [x,y,z,w];

then op(2,a) yields y, op(3,a) yields z, and op(2..4,a) yields the expression sequence y,z,w (which might be formed into a new list as [op(2..4,a)] since an <expression sequence> is

not itself a valid <expression> in Maple). A final example of the use of a <range> is the construct

<name> . (<range>)

which is a generalization of the name-formation construct using the concatenation operator. This construct produces an <expression sequence> which, as we have noted, is not itself a valid <expression> but it can be used wherever an <expression sequence> is valid. For example,

```
print(p.(1..5))
```

is exactly equivalent to

```
print(p1,p2,p3,p4,p5) .
```

3.2.7. Unevaluated Expressions

An expression enclosed in a pair of single quotes is called an *unevaluated expression.* For example, the statements

```
a := 1;  x := a + b;
```

cause the value 1+ b to be assigned to the name x while the statements

```
a := 1;  x := 'a + b';
```

cause the value a+ b to be assigned to the name x. The latter effect can also be achieved (if b has no value) by the statements

```
a := 1;  x := 'a' + b;
```

The effect of evaluating a quoted expression is to strip off (one level of) quotes, so in some cases it is useful to use nested levels of quotes. Note that there is a distinction between 'evaluation' and 'simplification' (see section 6) so that the statement

```
x := '2 + 3';
```

will cause the value 5 to be assigned to the name x even though the expression appearing here is quoted. This is because the 'evaluator' simply strips off the quotes but it is the 'simplifier' which transforms the expression 2 + 3 into the constant 5. Simplification can be avoided in a case like this by using two levels of single quotes:

```
x := ''2 + 3'';
```

in which case the result of evaluating the right hand side will be the unevaluated expression '2 + 3' which will be left unchanged by the simplifier.

A special case of 'unevaluation' arises when a name which may have been assigned a value needs to be unassigned, so that in the future the name simply stands for itself. This is accomplished by assigning the quoted name to itself. For example, if the statement

```
x := 'x';
```

is executed, then even if x had previously been assigned a value it will now stand for itself in the same manner as if it had never been assigned a value.

3.2.8. Procedures

Another valid expression in Maple is a *procedure definition* which takes the form

proc (<nameseq>) local <nameseq>; options <nameseq>; <statseq> end

where the 'local <nameseq>;' part and/or the 'options <nameseq>;' part may be omitted, and where <nameseq> stands for a (possibly empty) sequence of <name>s. This construct has some similarities with the concept of unevaluated expressions, but in this case it is more generally a <statseq> (i.e. a sequence of statements) which is unevaluated. Note that the keywords 'proc' and 'end' serve a purpose similar to the single quotes in unevaluated expressions (except that evaluation of this expression does not cause these keywords to be stripped off). An example of a procedure definition is

```
max := proc (a,b) if a>b then a else b fi end
```

which is syntactically an assignment statement where the <expression> on the right hand side is a procedure definition.

A procedure is invoked by using the syntax

<name> (<expression sequence>)

which is another instance of an expression. For example if max is defined as above then the expression max(1,2) causes a procedure invocation in which the 'actual parameters' 1 and 2 are substituted for the 'formal parameters' a and b, respectively, and then the 'procedure body' is executed yielding the value 2 in this case. The syntax of a procedure invocation may also be used in cases where the <name> has not been assigned, in which case the result is an *unevaluated function*, such as sin(x) or exp(x**2). (A more general discussion of procedures will be postponed until section 5).

3.2.9. Precedence of Operators

The order of precedence of all unary and binary operators is listed in the following table, from highest to lowest binding strengths. In parentheses it is stated whether the operators are left associative, right associative, or non-associative.

.	(left associative)
!	(left associative)
**	(non-associative)
*, /	(left associative)
+, -	(left associative)
..	(non-associative)
<, <=, >, >=, =, <>	(non-associative)
not	(right associative)
and	(left associative)
or	(left associative)
:=	(non-associative)

Thus the concatenation or decimal point operator '.' has the highest binding strength and the assignment operator ':=' has the lowest binding strength. Note that the exponentiation operator '**' is defined to be non-associative and therefore a**b**c is syntactically invalid in Maple. (The user must use parentheses to state his intentions).

The evaluation of expressions involving the logical operators proceeds in an intelligent manner which exploits more than the simple associativity and precedence of these operators. Namely, the left operand of the operators 'and' and 'or' is always evaluated first and the evaluation of the right operand is avoided if the truth value of the expression can be deduced from the value of the left operand alone. For example, the construct

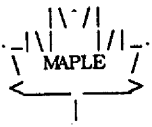
```
if d <> 0 and f(d)/d > 1 then . . . fi
```

will not cause a division by zero because if d=0 then the left operand of 'and' becomes false and the right operand of 'and' will not be evaluated.

3.3. Sample Maple Session

This section presents a sample interactive session using the Maple system. Maple is initiated on the Vax UNIX system by issuing the command '/u/maple/bin/maple' and is initiated on Honeywell TSS by issuing the command 'maple/maple'. In the following presentation of the Maple session, all lines containing italic characters are user input lines and all other lines are system responses. Each user input line must be terminated by <return>.

```
# Initiate the Maple system using the command
#      /u/maple/bin/maple    if on Vax UNIX,
#      maple/maple    if on Honeywell TSS.
```



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```
# Integers, rational numbers, and reals.

254 + 5280*99999;
527994974
9!;
6
9!!;
720
1 + 1/4 + 1/16 + 1/64 + 1/256;
341/256
evalr(");
1.33203125000
a := (5**40 + 3**50) / 2**90;
a := 45478324578584871158695804 37/6189700196426901374495621 12
evalr(a);
7.3474196060
Digits := 40;
Digits := 40
evalr(a);
7.3474196060154781089322604350878881161323
evalr(a, 60);
7.347419606015478108932260435087888116132378478690193035482991
b := 2**90;
b := 1237940039285380274899124224
a*b;
90956649157169742317391608 74
```

```
# Names, including the concatenation operator.

g := 52;  G := 4;
g := 52
G := 4
g*G;
208
```

```
for i to 5 do p.i := i**2 od;
p1 := 1
p2 := 4
p3 := 9
p4 := 16
p5 := 25
print(p.(1..5));
1  4  9  16  25
x := 999;  x := 'x';
x := 333
x := x

# Sets and lists.

set1 := {z, 2*y+1/3};  set2 := {z-4, z};
set1 := {x,2*y+1/3}
set2 := {x,z-4}
set1 + set2;
{x,z-4,2*y+1/3}
set1 * set2;
{x}
set3 := set1 - set2;
set3 := {2*y+1/3}
set2 * set3;
{}
list1 := [z, 2*y+1/3];  list2 := [z-4, z];
list1 := [x,2*y+1/3]
list2 := [z-4,x]
new_list := [op(list1), op(list2)];
new_list := [x,2*y+1/3,z-4,x]

# Polynomials and rational functions.

p := x**2 - x - 2;
p := x**2-x+(-2)
q := (x+1)**2;
q := (x+1)**2
r := "" * "";
r := (x**2-x+(-2))*(x+1)**2
expand(r);
x**4+x**3-3*x**2-5*x+(-2)
s := p/q;
s := (x**2-x+(-2))*(x+1)**(-2)
normal(s);
(x+(-2))/(x+1)
```

```
r*s;
(x**2-x+(-2))**2

# Equations.

eqn1 := 3*p + 5*q = 13;
eqn1 := 3*p+5*q=13
eqn2 := 4*p - 7*q = 80;
eqn2 := 4*p-7*q=30
3*eqn2 - 4*eqn1;
-41*q=38
q := 38/(-41);
q := -38/41
eqn1;
3*p+(-190/41)=13
p := (13 + 190/41)/3;
p := 241/41
eqn1; eqn2;
13=13
30=30
solve( {5*x + 10*y = 97, x - y = 12}, {x,y} );
{y=37/15,x=217/15}

# Unevaluated expressions and procedures.

a; b;
45478324578584871158695804376189700196426901374495621112
12379400392853802748991242224
f := 'b * (a+ 5)';
f := b*(a+ 5)
f;
152853651121438756062347819 94
max := proc (a,b) if a>b then a else b fi end;
max := proc (a,b)if param(2)<param(1) then param(1) else param(2) fi end
max(a,b);
12379400392853802748991242224
max(25/7, 525/149);
25/7
```

```
#Integers can be arbitrarily long.  Here is one that almost fills one screen
#  on a typical video terminal.  Recall from above that 9!! (i.e. 6!) is 720
#  so the following statement yields the same result as 720! .

9!!!;

26012189435657951002049032270810436111915218750169457857275418378508356311569473
82240678577958130457082619920575892247259536641565162052015873791984587740832529
10524469038881188412376434119195104550534665861624327194019711390984553672727853
70993456298555867193697740700037004307837589974206767840169672078462806292290321
07161669867260548938445514257193985499448939594496064045132362140265986193073249
36977047760606768067017649166940303481996188145562519559256691883082551494294759
65372748456246288242345265977897377408964665539924359287862125159674832209760295
05696699927284670563747137533019248313587076125412683415860129447566011455420749
58995256354306828863463108496565068277155299625679084523570255218622235813001670
08345234432368219357931847019565107297818043541738905607274280485839959197290217
26612291298420516067579036232337699453964191475175567557695392233803056825308599
97744167578435281591346134039460490126954202883834710136373382448450666009334848
44407119312925376946573543373757247722301815340326471775319845373414786743270484
57983786618703257405938924215709695994630557521063203263493209220738320923356309
92326750440170176057202601082928804233560664308988871029738079757801305604957634
28388630571906622052911748225105366977566030295740433879834715185526028053338663
57139101046336419769097397432285994219837046979109956303389604675889865795711176
56667003915674815311594398004362539939973120306649060132531130471902889849185620
37666691644687911252491937544258458950003115616829743046411425380748972817233759
55380661719801404677935614793635266265683339509760000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000
00000000000000000000000000000000000000000000000000000000000000000000000000000000

quit
```

4. DATA TYPES AND FORMAL SYNTAX

4.1. Data Types

Every expression in Maple is represented internally by an expression tree where each node is a particular data type. While some data types are strictly for internal use, most of the data types corresponding to expressions are accessible to the user and can be tested for via the *type* function. The user can examine the components of such a data type by using the *op* function. In this section we discuss the data types that are accessible to the user. For a more detailed description of the internal data types, see section 6.

4.1.1. Integer

An expression is of type 'integer' if it is an (optionally signed) sequence of one or more digits of arbitrary length (i.e. the length limit is system dependent but generally much larger than users will encounter). The 'op' function considers this data type to have only one operand, so if n is an integer then the value of op(n), and also the value of op(1, n), is the integer n.

4.1.2. Rational Number

A rational number (called type 'rational') is represented by a pair of integers (numerator and denominator) with all common factors removed and with a positive denominator. Like integers, rational numbers are of arbitrary length. The 'op' function considers this data type to have two operands, where the first operand is the numerator and the second operand is the denominator.

4.1.3. Real Number

An expression of type 'real' is a number represented externally as a sequence of digits with a decimal point (e.g. 1.5, 15000., .15). Real numbers are represented internally by a pair of integers (the mantissa and the characteristic), which represent the number $mantissa \times 10^{characteristic}$. Thus op(150.1) yields the expression sequence 1501, –1. Arithmetic with real numbers is performed via the *evalr* function. The number of digits carried in the mantissa when evaluating reals is determined by the value of the global name *Digits* which has 10 as its initial value. Note that the current version of Maple displays real numbers with very small or very large magnitudes using the notation Real(mantissa,characteristic) which corresponds to the internal data structure. For example, evalr(exp(–10)) yields Real(4539992971,–14) while evalr(exp(–2)) yields .1353352832 .

4.1.4. Name

An expression is of type 'name' if it is a <string> as defined by the Maple grammar. For example, if x has not been assigned a value then x is of type 'name' and op(x) has the value x.

4.1.5. Expression Sequence

There is an internal data type in Maple for an <expression sequence>, which is a sequence of <expression>s separated by commas. An <expression sequence> is not, by itself, a valid <expression> but it occurs in many places as a component of an <expression>. There is no type name known to the type function for this data type.

When the op function is used to extract parts of an expression, the result is often an expression sequence. For example,

```
a := [x,y,z,w]; op(a);
```

yields the expression sequence x,y,z,w. An important special case of an <expression sequence> is the null expression sequence and there is a global name in Maple, NULL, whose value is the null expression sequence. The value of the global name NULL is equivalent to the value of the operation op([ ]).

4.1.6. Set and List

Two more data types are the 'set' and the 'list'. Each of these types consists of a sequence of expressions and if expr is an object of either of these two types then op(expr) yields the expression sequence. The external representation of a set uses braces '{', '}' to surround the expression sequence and the external representation of a list uses brackets '[', ']' to surround the expression sequence.

4.1.7. Addition, Multiplication, and Power

An expression can be composed using the algebraic operators +, –, *, /, **. Such an expression is of type `+`, type `*`, or type `**`. Thus the expression a – b is of type `+` and op(a – b) yields the expression sequence a, –b. Similarly the expression a/b is of type `*` and op(a/b) yields the expression sequence a, b**(–1). Of course, b**(–1) is an example of an expression of type `**`. The representation used for these algebraic expressions is often referred to as sum-of-products form.

4.1.8. Series

The 'series' data type in Maple is a special data type which represents an expression as a (truncated) power series in one specified indeterminate. This data type is created by a call to the *taylor* function. For this data type, the 0th operand is defined to be the name of the indeterminate, the 1st, 3rd, . . . operands are the coefficients (generally expressions), and the 2nd, 4th, . . . operands are the corresponding exponents. The exponents are ordered from least to greatest. Usually, the final pair of operands in this data type are the special 'order' symbol O(1) and the integer n which indicates the order of truncation. (Note: The print routine displays the final pair of operands using the notation O(x**n) rather than more directly as O(1)*x**n, where x represents the 0th operand). However, if the series is known to be exact then there will be no 'order' term in the series. An example of this occurs when the 'taylor' function is applied to a polynomial whose degree is less than the truncation degree for the series.

4.1.9. Equation and Inequality

An expression of type 'equation' (also called type `=`) has two operands, the left-hand-side expression and the right-hand-side expression. An equation is represented externally using the binary operator '='.

There are three internal data types for inequalities, corresponding to the operators '<>', '<', and '<='. Inequalities involving the operators '>' and '>=' are converted to the latter two cases for purposes of representation. Correspondingly, only three names are known to the type function for inequalities: `<>`, `<`, `<=` . Like an equation an inequality has two operands, the left-hand-side expression and the right-hand-side expression.

4.1.10. Boolean Expression

The simplest cases of Boolean expressions are the <name>s *true* and *false*[*]. Equations and inequalities (formed using the relational operators =, <>, <, <=, >, >=) are also treated as Boolean expressions if they appear in a 'Boolean context'. More complicated Boolean expressions can be built out of these simple expressions with the logical operators **and**, **or**, and **not**. The built-in function *evalb* can be called with a Boolean expression as argument in order to cause the expression to be evaluated as a Boolean. For example, the equation a = b is an algebraic equation if it appears alone but evalb(a = b) will evaluate this equation as a Boolean. However, an equation or inequality will be recognized as being in a Boolean context if it appears in the 'while part' of a repetition statement or in the 'if part' of a selection statement. In addition to the type names for equations and inequalities, the following type names are also known to the type function: `and`, `or`, `not` .

4.1.11. Range

An expression of type 'range' (also called type `..`) has two operands, the left-hand-side expression and the right-hand-side expression. A range is represented externally using the binary operator '..' which simply acts as a place-holder.

4.1.12. Procedure Definition

A procedure definition in Maple is a valid expression and its type is called 'procedure'. The external representation of a procedure definition is

proc (<nameseq>) local <nameseq>; options <nameseq>; <statseq> end

The internal data structure represents each <nameseq> in the order shown above followed by the statement sequence <statseq>. Since <statseq> is not a valid expression in Maple, this part of the data structure is not retrievable by the *op* function. There are three operands defined for the *op* function applied to this data structure: the first operand is the <nameseq> of formal parameters, the second operand is the

[*] *true* and *false* are just Maple names that the system returns as the result of Boolean evaluation. Users can use *true* and *false* just like any other name, but to be safe it is best to avoid assigning values to these names.

<nameseq> of local variables, and the third operand is the <nameseq> of option names. Therefore, if

```
f := proc (a,b)
  local c;
  options remember;
  c := a/b;
  if type(c, integer) then c else FAIL fi
end;
```

then

```
op(1,f);        yields    a,b
op(2,f);        yields    c
op(3,f);        yields    remember
op(f);          yields    a,b,c,remember
```

4.1.13. Unevaluated Function Invocation

A function invocation takes the form

<name> (<expression sequence>)

and if <name> is undefined then the result is an unevaluated function invocation, called type 'function'. Typical examples of the type 'function' are sin(x), exp(x**2), g(a,b) where none of sin, exp, and g has been defined. For the *op* function applied to this data type, operand 0 is defined to be the <name> of the function and the remaining operands are the elements of the <expression sequence>. For example,

```
op(0,g(a,b));     yields    g
op(1,g(a,b));     yields    a
op(2,g(a,b));     yields    b
op(g(a,b));       yields    a,b
```

4.1.14. Unevaluated Factorial

The factorial function is invoked in the form <expression>! and if <expression> does not evaluate to an integer then the result is an unevaluated factorial, called type '!'. For the *op* function applied to this data type there is only one operand defined and its value is the <expression>.

4.1.15. Unevaluated Concatenation

An expression which consists of an unevaluated concatenation is said to be of type '.' . Normally, the concatenation operator is evaluated to form a name but an example of an expression of type '.' would be the unevaluated expression 'a.i' . In the current version of Maple, if the name 'i' does not evaluate to an integer then the expression a[i] is another example of type '.' (i.e. an unevaluated concatenation).

4.2. Formal Syntax

This section presents the BNF grammar which describes the syntax accepted by Maple. In the following grammar, where a sequence of symbols is enclosed in a pair of "§" symbols it indicates that this portion of the statement is optional. Where *empty* occurs in the grammar, no symbol is required. A Maple *session* consists of a <statseq>, which is a sequence of statements separated by semicolons.

```
<statseq> ::=     <statseq> ; <stat> |      <stat>

<stat> ::=        <exp>
                | <name> := <exp>
                | read <exp>
                | save <exp>
                | § for <name> §  § from <exp> §  § by <exp>§  § to <exp> §
                        § while <exp> §
                                do
                                <statseq>
                                od
                | break
                | if <exp> then <statseq> <elsepart>
                | quit †
                | empty

<exp> ::=         <exp> or <exp>          /*Boolean expressions*/
                | <exp> and <exp> |       not <exp>

                | <exp> < <exp>   |       <exp> <= <exp>
                | <exp> > <exp>   |       <exp> >= <exp>
                | <exp> <> <exp>  |       <exp> = <exp>

                | <exp> ..‡ <exp>         /*range sequence*/

                | <exp> + <exp>           /*algebraic expressions*/
                | <exp> - <exp>
                | + <exp>         |       - <exp>
                | <exp> * <exp>   |       <exp> / <exp>
                | <exp> ** <exp>
```

†done or stop can be used as synonyms for quit . In the VAX UNIX and Honeywell TSS versions of Maple, hitting the break/interrupt key twice in rapid succession will also exit from Maple.

‡Actually, two *or more* consecutive periods are permitted.

```
|   proc ( § <nameseq> § )                          /*procedure definition*/
          § local <nameseq> ; §
          § options <nameseq> ; §
          <statseq>
          end

|   <natural> . <natural>                           /* real numbers*/
|   <natural> .        | . <natural>

|   <natural>                                       /*unsigned integer*/

|   { <expseq> }                                    /*set*/

|   [ <expseq> ]                                    /*list*/

|   <name>                                          /*variable name*/

|   <exp> !                                         /*factorial*/

|   ( <exp> )                                       /*parenthesized expression*/

|   ' <exp> '                                       /*unevaluated expression */

|   <name> ( <expseq> )                             /*function call*/

|   "                                               /*previously computed expressions*/
|   ""        |        """

<expseq> ::=    <explist> |      empty

<explist> ::=   <explist> , <exp> |      <exp>

<name>  ::=     <string> |       <name> . ( <exp> )
                |               <name> . <string>  |     <name> . <natural>
                |               <name> [ <expseq> ]

<nameseq>       ::=     <nameseq> , <string>       |      <string>

<elsepart>      ::=     fi        |       else <statseq> fi
                        |       elif <exp> then <statseq> <elsepart>

<natural>       ::=      § <natural> §   <digit>

<digit>  ::=    0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

```
<string> ::=      <letter>   § <alphanumeric> §
                  |          @  § <alphanumeric> §
                  |          ` <charstring> `

<alphanumeric>    ::=      § <alphanumeric> §  <letter>
                  |          § <alphanumeric> §  <digit>
                  |          § <alphanumeric> §  _

<letter> ::=      /*Any lower-case or upper-case letter a-z or A-Z.*/

<charstring>      ::=      <anychar>  § <charstring> §

<anychar>         ::=      /*Any character in the supported character set.*/
```

5. PROCEDURES

5.1. Procedure Definitions

One instance of an expression in Maple is a procedure definition, which has the general form

proc (<nameseq>) local <nameseq>; options <nameseq>; <statseq> end

Such a procedure definition may be assigned to a <name> and it may then be invoked using the syntax

<name> (<expseq>) .

When a procedure is invoked the statements in <statseq> are executed sequentially (and some of the names have special semantics as described below). The *value* of a procedure invocation is the value of the last statement in <statseq> that is executed.

It is possible in Maple to define and invoke a procedure without ever assigning the procedure definition to an explicit <name>, as in the following example:

```
proc (x) x**2 end;
"(2);
```

where the value of the procedure invocation "(2) is 4. Another example of using a procedure definition without a name is when a simple function is to be *mapped* onto an expression, as in:

```
a := [1,2,3,4,5];
map( proc (x) x**2 end, a );
```

which causes each element of the list 'a' to be squared, yielding the new list [1,4,9,16,25].

The keywords 'proc' and 'end' may be viewed as brackets which specify that the <statseq> is to remain unevaluated when the procedure definition is evaluated as an expression. The simplest instance of a procedure definition involving no formal parameters, no local variables, and no options can be seen in the following definition of a procedure called max:

```
max := proc () if a>b then a else b fi end
```

Executing the statements

```
a := 25/7;  b := 525/149;  max();
```

yields 25/7 as the value of the procedure invocation max(). This procedure is making use of the names 'a' and 'b' as *global names*. In Maple, all names are global names unless otherwise specified. One instance of non-global names is the case of *formal parameters* which are specified within the parentheses immediately following the keyword 'proc'. A more useful definition of the above procedure max can be obtained by making the names 'a' and 'b' formal parameters:

```
max := proc (a,b) if a>b then a else b fi end
```

This procedure may now be invoked in the form max(expr1, expr2) where expr1 and expr2 are expressions. For example, max(25/7, 525/149) evaluates to 25/7. The names 'a' and 'b' are now local to the procedure, so that if these names have values external to the procedure the external values neither effect, nor are affected by, the invocation of the procedure.

5.2. Parameter Passing

The semantics of parameter passing are as follows. Suppose the procedure invocation is of the form

<name> (<expr1>, <expr2> ,..., <exprn>) .

Firstly, <name> is evaluated and let us suppose for now that it evaluates to a procedure definition with formal parameters

<par1>, <par2> ,..., <parn> .

Next, the *actual parameters* <expr1> ,..., <exprn> are evaluated in order from left to right. Then every occurrence of <pari> in the <statseq> which makes up the body of the procedure is substituted by the value of the corresponding actual parameter <expri>. It is important to note that these parameters will not be evaluated again during execution of the procedure body. (The consequences of this fact are explained in section 5.4 below). In terms of traditional parameter passing mechanisms used by various computer languages, Maple's parameter passing could be termed 'call by evaluated name'. In other words, all actual parameters are first evaluated (as in 'call by value') but then a strict application of the substitution rule is applied to replace each formal parameter by its corresponding actual parameter (as in 'call by name').

It is possible for the number of actual parameters to be either greater than, or less than, the number of formal parameters specified. If there are too few actual parameters then a semantic error will occur if (and only if) the corresponding formal parameter is referenced during execution of the procedure body. The case where the number of actual parameters is greater than the number of specified formal parameters is, on the other hand, fully legitimate. Maple allows an alternate mechanism for referencing parameters within a procedure body; namely, the special name 'param' when used as a function call such as

```
param(i)
```

references the i-th actual parameter. For example, the above procedure max could be defined without any specified formal parameters as follows:

```
max := proc () if param(1) > param(2) then param(1) else param(2) fi end
```

This procedure may now be invoked exactly as before with two actual parameters and the semantics are identical to the previous definition. The user will notice that, when displaying procedure definitions, the current version of Maple uses the 'param' syntax for specifying formal parameters even if the user specified formal parameter names. For

example, if the input to Maple is

```
max := proc (a,b) if a>b then a else b fi end;
```

then the response from Maple is

```
max := proc (a,b)if param(2)<param(1) then param(1) else param(2) fi end
```

(where, as a minor point, note also that Maple chooses to represent inequalities using the '<' relation rather than the '>' relation). There is no restriction against having extra actual parameters appear in a procedure invocation; if they are never referenced they are simply ignored (but they will be evaluated).

In addition to the special name 'param' there are two other special names that Maple understands within a procedure body: 'nargs' and 'paramseq'. The value of the name 'nargs' is the number of actual parameters (i.e. the number of arguments) with which the procedure was invoked. The value of the name 'paramseq' is the expression sequence

```
param(1), param(2) ,...., param(nargs)
```

of the actual parameters with which the procedure was invoked.

As an example of the use of the name 'nargs', let us generalize our procedure max so that it will be defined to calculate the maximum of an arbitrary number of actual parameters. Consider the following procedure definition:

```
max := proc ()
  result := param(1);
  for i from 2 to nargs do
    if param(i) > result then result := param(i) fi
  od;
  result
end;
```

With this definition of max we can find the maximum of any number of arguments. Some examples are:

```
max(25/7, 525/149);          yields        25/7
max(25/7, 525/149, 9/2);     yields        9/2
max(25/7);                   yields        25/7
max();                       causes an error
```

where the latter case is an example of a procedure being called with too few actual parameters. If we wish to change our definition of max so that the procedure invocation max() with an empty parameter list will return the null value then we may check for a positive value of nargs in a selection statement as in the following definition of max.

```
max := proc ()
  if nargs > 0 then
    result := param(1);
    for i from 2 to nargs do
      if param(i) > result then result := param(i) fi
    od;
    result
  fi
end;
```

The user will notice that, when displaying procedure definitions, the current version of Maple prints the function call param(0) in place of the special name 'nargs' and it prints the function call param(-1) in place of the special name 'paramseq'. This is a reflection of the internal implementation, but if the user creates the function call param(0) or param(-1) a semantic error will occur.

5.3. Local Variables and Options

The mechanism for introducing *local variables* into a Maple procedure is to use the 'local part' of a procedure definition. The 'local part' must appear immediately following the parentheses enclosing the formal parameters, and its syntax is

```
local <nameseq>;
```

The semantics are that the names appearing in <nameseq> are to be local to the procedure. In other words, this can be viewed as causing a syntactic renaming of every occurrence of the specified names within the procedure body. As an example, let us reconsider the latest definition of max appearing above. There are two global variables appearing in the procedure definition which we would almost certainly want to make local: result and i. This is effected by the following version of the procedure definition.

```
max := proc ()
  local result, i;
  if nargs > 0 then
    result := param(1);
    for i from 2 to nargs do
      if param(i) > result then result := param(i) fi
    od;
    result
  fi
end;
```

The user will notice that, when displaying procedure definitions, the current version of Maple uses a function syntax of the form loc(i) for the various local variables that have been specified just as it uses the param(i) syntax for all formal parameters. The use of the loc(i) function calls is a reflection of the internal implementation, but the user is not able to refer to local variables in this way.

There is a facility to specify *options* for a procedure by using the 'options part' of a procedure definition. The 'options part' must appear immediately after the 'local part' and its syntax is either of the following two forms:

```
option <nameseq>;
options <nameseq>;
```

The only <name> that is currently recognized as an option is the name 'remember'. The semantics of specifying 'option remember' or 'options remember' as the options part of a procedure definition are as follows. After executing the procedure and obtaining the value of a particular procedure invocation, the Maple system makes an entry in a table called the *partial computation table* which associates the result with that particular procedure invocation. If there is ever another invocation of this procedure with actual parameters that have the same values then the Maple system will immediately retrieve the result from the partial computation table. In this way, it is possible to avoid redundant executions of procedures that may be very costly. (See also the remember function in section 7).

5.4. Assigning Values to Parameters

Let us now consider an example of a procedure where we may wish to return a value into one of the actual parameters. Recall that the integer quotient q and the integer remainder r of two integers a and b must satisfy the 'Euclidean division property'

$$a = bq + r$$

where either $r = 0$ or $abs(r) < abs(b)$. This property does not uniquely define the integers q and r, but let us impose uniqueness by choosing

$$q = trunc(a/b)$$

using the built-in Maple function trunc. The remainder r is then uniquely specified by the above Euclidean division property. (Note: This choice of q and r can be characterized by the condition that r will always have the same sign as a). The following definition of the procedure 'rem' returns as its value the remainder after division of the first parameter by the second parameter, and it also returns the quotient as the value of the third parameter (if present).

```
rem := proc (a,b,q)
  local quot;
  quot := trunc(a/b);
  if nargs > 2 then q := quot fi;
  a - quot * b
end;
```

The procedure rem as defined here may be invoked with either two or three parameters. In either case the value of the procedure invocation will be the remainder of the first two parameters. The quotient will be returned as the value of the third parameter if it appears. At this point recall that the semantics of parameter passing specify that the actual parameters are evaluated and then substituted for the formal parameters.

Therefore, an error will result if an actual parameter which is to receive a value does not evaluate to a valid name. It follows that when a name is being passed into a procedure for such a purpose it should usually be explicitly quoted (to avoid having it evaluated to some value that it may have had previously). The following statements will serve to illustrate.

```
rem(5, 2);                yields    1
rem(5, 2, 'q');           yields    1
q;                        yields    2

rem(-8, 3, 'q');          yields    -2
q;                        yields    -2
rem(8, -3);               yields    2
rem(8, 3, q);             yields    System error (in evalname)
```

The latter error message arises because the actual parameter q has the value -2 from a previous statement, and therefore the value -2 is substituted for the formal parameter q in the procedure definition yielding an invalid assignment statement. The solution to this problem is to change the actual parameter from q to 'q'.

When values are assigned to parameters within a procedure, a restriction which must be understood is that parameters are evaluated only once. Basically this means that formal parameter names cannot be freely used like local variables within a procedure body, in the sense that once an assignment to a parameter has been made that parameter should not be referred to again. The only legitimate purpose for assigning to a parameter is so that on return from the procedure the corresponding actual parameter has been assigned a value. As an illustration of this restriction, consider a procedure get_factors which takes an expression expr and, viewing it as a product of factors, determines the number n of factors and assigns the various factors to the names f.i for i = 1 ,..., n. Here is one attempt at writing a procedure for this purpose.

```
get_factors := proc (expr,f,n)
  local i;
  if type(expr, `*`) then
    n := nops(expr);
    for i to n do
      f.i := op(i,expr)
    od
  else
    n := 1;
    f.1 := expr
  fi
end;
```

If this procedure is invoked in the form

```
get_factors(x*y, 'f', 'number');
```

the result is 'ERROR: unable to execute for statement'. What has happened is that the

third actual parameter is a name (as it must be because it is to be assigned a value within the procedure) and when execution reaches the point of executing the for-statement, the limit n in the for-statement is the name 'number' that was passed in. The point is that the formal parameter n is evaluated only once upon invocation of the procedure and it will not be re-evaluated. A general solution to this type of problem is to use local variables where necessary, and to view the assignment to a parameter as an operation that takes place just before returning from the procedure. For our example, the following procedure definition follows this point of view.

```
get_factors := proc (expr,f,n)
  local i, nfactors;
  if type(expr, `*`) then
    nfactors := nops(expr);
    for i to nfactors do
      f.i := op(i,expr)
    od
  else
    nfactors := 1;
    f.1 := expr
  fi;
  n := nfactors
end;
```

Another solution to the problem in this example is to change the limit in the for-statement to the operator ", which will yield the desired value. This leads to the following procedure definition.

```
get_factors := proc (expr,f,n)
  local i;
  if type(expr, `*`) then
    n := nops(expr);
    for i to " do
      f.i := op(i,expr)
    od
  else
    n := 1;
    f.1 := expr
  fi
end;
```

5.5. Error Returns and Special Returns

The most common return from a procedure invocation occurs when execution 'falls through' the end of the <statseq> which makes up the procedure body, and the value of the procedure invocation is the value of the last statement executed. There are three other types of returns from procedures.

An *error return* occurs when the special function call

ERROR(<string>)

is evaluated. This function call causes an immediate exit to the top level of the Maple system and the following error message is printed out:

ERROR: <string> .

An *explicit return* occurs when the special function call

RETURN(<expseq>)

is evaluated. This function call causes an immediate return from the procedure and the value of the procedure invocation is the value of the <expseq> given as the argument in the call to RETURN. In the most common usage <expseq> will be a single <expression> but a more general <expseq> (including the null expression sequence) is valid. It is an error if a call to the function RETURN occurs at a point which is not within a procedure definition.

A *fail return* occurs when the special name

FAIL

is evaluated. The effect of evaluating this name is to cause an immediate return from the procedure. The value of the procedure invocation is the procedure invocation itself, as an unevaluated expression. It is an error if the name FAIL occurs at a point which is not within a procedure definition. The effect of FAIL can also be achieved by the construct

RETURN('<name>(paramseq)')

where <name> is the name by which the procedure was invoked.

As an example of a procedure which includes an error return and a fail return, consider the function 'max' which is supplied in the Maple library. The latest definition that we developed in section 5.3 for the function 'max' has a property which makes it unacceptable as a library function. Namely, if a user calls this function with an argument that does not evaluate to a constant, such as in max(x, y) where x and y have not been assigned any values, then the result is an error message from the Maple system: 'ERROR: cannot evaluate boolean'. This error results from attempting to execute an if statement of the form

if y>x then . . .

where x and y are indeterminates. Since this call to the function 'max' may have occurred from a procedure nested several levels, the resulting error message will not be very informative to the user. In order to improve this situation, type-checking should be done within the 'max' function and appropriate action should be taken if an invalid argument is encountered. The following code from the Maple library shows how this can be done using the FAIL return facility, so that the result of the function call max(x, y) will be the unevaluated function max(x, y). This code also shows the use of the ERROR return facility to return an error message if the function is called with no arguments.

```
max := proc ()
  local i, M, p;
  if nargs = 0 then
    ERROR(`function max called with no parameters`)
  else
    M := param(1);
    for i to nargs do
      p := param(i);
      if not type(p, rational) and not type(p, real) then FAIL fi;
      if M < p then M := p fi
    od;
    M
  fi
end;
```

5.6. Boolean Procedures

It was noted in section 3 that the names 'true' and 'false' may be freely manipulated as names even though these names have a special significance when they arise in a Boolean context. It follows that Boolean procedures may be written like any other procedures. As an example of a Boolean procedure, consider the following definition of a function called 'member' which tests for list membership.

```
member := proc (element, l)
  local i;
  false; for i to nops(l) while not " do
                    evalb( element = op(i, l) ) od;
  "
end;
```

Some examples invoking this procedure follow.

member(x*y, [1/2, x*y, x, y]);	yields	true
member(x, [1/2, x*y]);	yields	false
member(x, [ ]);	yields	false

A few points about this procedure should be noted. Firstly, note that the equation

```
element = op(i, l)
```

is to be evaluated as a Boolean expression and therefore it is necessary to apply the function 'evalb' to it. Otherwise, this expression would be treated as an algebraic equation. Secondly, note the use of the nullary operator " in the while-part of the loop to refer to the 'latest expression'. Alternatively, this could be coded with the use of another local variable but in this case it seems preferable to use the " operator. Finally, it should be noted that the final " appearing in this procedure would be redundant in some contexts but is necessary here. If it were left out then the value of the procedure invocation would be the value of the last statement executed, which would be the value

of the for-loop. This value will be null in the case where 'l' is the empty list, but the correct value to return in such a case is 'false' rather than the null value. However, the value of " is never updated by a null value and this fact is exploited in the above procedure definition.

5.7. Reading and Saving Procedures

It is usually convenient to use a text editor to develop a procedure definition and to write it into a file. The file can then be read into a Maple session. For example, the max procedure might be written into a file named /u/gahill/max . In a Maple session the statement

```
read `/u/gahill/max`;
```

will read in the procedure definition. Since this procedure is in 'user format' Maple will echo the statements as they are read in. Once the procedure is debugged it is desirable to save it in 'Maple internal format' so that whenever it is read into a Maple session the reading is very fast (and no time is spent displaying the statements to the user). To accomplish this one must use a file with a name ending in the characters '.m' . Within Maple the 'user format' file is read in and then Maple's save statement is used to save the file in 'Maple internal format'. For example, suppose that we have saved our procedure definition in a file named /u/gahill/max. If we then enter the Maple system and execute the statements

```
read `/u/gahill/max`;
save `/u/gahill/max.m`;
```

we will have saved the internal representation of the procedure in the second file. This file may be read into a Maple session at any time in the future by executing the statement

```
read `/u/gahill/max.m`;
```

which will update the current Maple environment with the contents of the specified file. (It is often convenient to place the 'save' statement at the end of the 'user format' file so that simply reading in the file will cause it to be saved in 'Maple internal format'). The user will quickly discover the time-saving advantages of saving procedure definitions in 'Maple internal format'.

A special case of reading procedure definitions in 'Maple internal format' can be accomplished using the built-in function *readlib*. Specifically, the function invocation readlib(pname) will cause the the following **read** statement to be executed:

```
read `` . libname . pname . `.m`
```

where 'libname' is a global name in Maple which is initialized to the pathname of the standard Maple system library on the host system. For example, on the UNIX system the value of 'libname' is `/u/maple/lib/`. (The value of 'libname' on any host system can be determined by entering Maple and simply displaying its value). The complete pathname being specified in the above **read** statement is a concatenation of the values of 'libname',

'pname', and the suffix `.m`, which could alternatively be specified by

```
cat(libname, pname, `.m`) .
```

In order to specify this concatenation using only Maple's concatenation operator '.' it is necessary to concatenate these values to the null string ``, because the left operand of Maple's concatenation operator is not fully evaluated but is simply evaluated as a <name>. (See section 3.2.2.)

The *readlib* function is more general than this. If it is called with two arguments then the second argument is the complete pathname of the file to be read, and the first argument 'pname' is the procedure name which is to be defined by this action. Thus the following two function calls are equivalent:

```
readlib('f')
readlib('f', `` . libname . `f.m`)
```

but if the procedure definition for 'f' is not in the standard Maple system library then the second argument is required to specify the correct file. (Even more generally, the *readlib* function can be called with several arguments in which case all arguments after the first are taken to be complete pathnames of files to be read, and the first argument is a procedure name which is to be defined by this action). The definition of the *readlib* function involves more than just the execution of one or more **read** statements. This function will also check to ensure that after the files have been read, 'pname' has been assigned a value and this value is returned as the value of the *readlib* function. In other words, the *readlib* function is to be used when the purpose of the **read** is to define a procedure named 'pname' (and some other names may or may not be defined at the same time).

The most common application of the *readlib* function is to cause automatic loading of files. For this purpose, the value of 'pname' is initially defined to be an unevaluated *readlib* function, as in one of the following assignments:

```
pname := 'readlib('pname')';
pname := 'readlib('pname', filename)';
```

where the single quotes around the argument 'pname' are required to avoid a recursive evaluation. Then if there is subsequently a procedure invocation pname(...), the evaluation of 'pname' will cause the *readlib* function to be executed, thus reading in the file which defines 'pname' as a procedure, and the procedure invocation will then proceed just as if 'pname' had been a built-in function in Maple. Indeed, this method is precisely how the names of Maple's system-defined library functions are initially defined so that the appropriate files will be automatically loaded when needed. (For example, enter Maple and display op('gcd') to see what the name 'gcd' is defined to be and readlib('gcd') will be the response).

For completeness, the following is a definition in Maple code of Maple's built-in function *readlib*.

```
readlib := proc (pname)
  local i, errmsg;
  errmsg := `wrong number (or type) of parameters`;
  if nargs=0 or not type(pname, name) then
    print(`In function readlib;`); ERROR(errmsg)
  fi;
  pname := 0;
  if nargs=1 then
    read `` . libname . pname . `.m`
  else
    for i from 2 to nargs do
      if not type(param(i), name) then
        print(`In function readlib;`); ERROR(errmsg)
      else
        read param(i)
      fi
    od
  fi;
  if op(pname) = 0 then  print(`` . pname . `:`);
                                        ERROR(`ineffective readlib`)  fi;
  pname;
  "
end;
```

6. INTERNAL REPRESENTATION AND MANIPULATION

6.1. Internal Organization

Maple appears to the user as an interactive "calculator". This mode is achieved by immediately executing any statement which is typed at the user level. It is in this context where we can define Maple as a *parser-driven* program. The parser is effectively the main program; its task is to read input, parse statements and call the statement evaluator each time a statement is input.

The parser accepts the Maple language which has been kept simple enough to have the LALR(1) property. The parser, being the main program, retains control throughout the session. For each production which is successfully reduced, it creates the appropriate data structure. Additionally the reduction of the nonterminal <stat> produces a call to the statement evaluator, the main Maple evaluator. Maple will read an infinite number of statements; its normal conclusion is achieved by the evaluation (not the parsing) of the <quit> statement. Thus it is possible to write a statement like:

if <condition> then quit fi;

which will terminate execution conditionally.

The initialization phase is normally called before the parser. In some sense we may say that both initialization and parser are at the topmost level of control. This is particularly true for some parsers, like yacc, which provide a "canned" main program whose only task is to call sequentially the initialization, parser, and possibly a finalization routine.

The internal functions in Maple can be divided into four distinct groups.

(1) Evaluators. The evaluators are the main functions responsible for evaluation. There are five types of evaluations: statements (done by evalstat); algebraic expressions (eval); boolean expressions (evalbool); name forming (evalname), and real arithmetic (evalr). Although the parser calls only evalstat, thereafter there are many interactions between the evaluators. For example, the statement

if a>0 then b.i := 3.14/a fi;

is first analyzed by evalstat which calls evalbool to resolve the if-condition. Once this is done, say with a true result, evalstat is invoked again to do the assignment, for which evalname has to be invoked with the left-hand-side and eval with the right-hand expression. Finally evalr will be called to evaluate the result. Most of the time the user will not directly invoke any of the evaluators; these are invoked automatically as needed. In some circumstances, when a different type of evaluation is needed, the user can directly call evalr, evalbool (evalb for the user), and evalname (evaln).

(2) Algebraic functions. These are functions which are directly identified with a function available at the user level, and are commonly called "basic". Some examples clarify this immediately: taking derivatives (diff), picking parts of an expression (op), dividing polynomials (divide), finding coefficients of polynomials (coeff), series

computation (taylor), mapping a function (map), substitution of expressions (subs, subsop), expansion of expressions (expand), finding indeterminates (indets), etc. Some functions in this group may migrate to the Maple level (Maple library) and vice versa due to tradeoffs between size and efficiency.

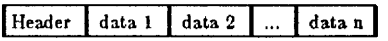
(3) Algebraic service functions. These functions are algebraic in nature, but serve as subordinates of the functions in the above group. Most frequently these functions cannot be explicitly called by the user. Examples of functions in this group are: the arithmetic (integer, rational, and real) packages (const, consti) the basic simplifier (simpl), printing (print), the series package (polyn), the set-operations package (sets), retrieval of library functions (retrieve), etc.

(4) General service functions. Functions in this group are at the lowest hierarchical level; i.e., they may be called by any other function in the system. Their purpose is general, and not necessarily tied to symbolic computation. Some examples are: storage allocation and garbage collection (storman), table manipulation (hash,pc), internal input/output (put), non-local returns, and various error handlers.

The flow of control within the basic system is not bound to remain at this level. In many cases, where appropriate, a decision is made to call functions written in Maple and residing in the library. For example, most uses of the function expand(...) will be handled by the basic system; however, if an expansion of a sum to a power greater than 4 is required, the internal expand will call the external (Maple library) function `expa/large` to resolve it. Functions such as diff, evalr, taylor, and type make extensive use of this feature. (For example, the basic function diff does not know how to differentiate any function; all its knowledge resides in the Maple library at pathnames `diff/<function name>`). This is a fundamental feature in the design of Maple as it permits flexibility (changing the library), personal tailoring (defining your own handling functions), readability (the source is in Maple code and available to all users), and allows the system to remain small by unloading unnecessary functions from the basic system.

6.2. Internal Representation of Data Types

The parser and some basic internal functions are responsible for building all of the data structures used internally by Maple. All of the internal data structures have the same general format:



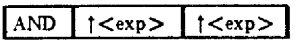
The *header* field encodes the length $(n+1)$ of the structure, the type, one bit to indicate simplification status, and two bits to indicate garbage collection status. The data items are normally pointers to similar data structures; the few exceptions to this rule are the terminal symbols.

Every data structure is created with its own length, and this length will not change during its entire existence. Furthermore, data structures should *not* be changed during execution since it is not predictable how many other data structures are pointing to a given structure. The normal procedure to modify structures is to create a copy and

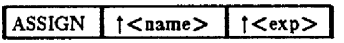
modify the copy, hence returning a new data structure. The only safe modifications are those done by the basic simplifier which produces the same value, albeit simpler. It is the task of the garbage collector to identify unused structures.

In the following figures we will describe the individual structures and the constraints on their data items. We will use the symbolic names of the structures since the actual numerical values used internally are of little interest. The symbol ↑<xxx> will indicate a pointer to a structure of type xxx. In particular we will use, whenever possible, the same notation as in the formal syntax (section 4.2).

Logical and



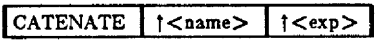
Assignment statement



The <name> entry should evaluate to a valid name, which is one of the following data structures: NAME, CATENATE, LOCAL, or PARAM.

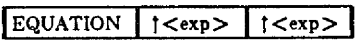
Break statement

Concatenation of a name



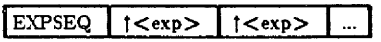
The <name> entry is treated as in ASSIGN. The <exp> entry must evaluate to a nonnegative integer or to a name to be successful. There are two exceptions: if <exp> is an EXPSEQ the entry is taken to be an array reference (the content of the EXPSEQ being the indices), and if <exp> is a RANGE the entry is a generator of an EXPSEQ (e.g. a.(1..2) generates a1,a2).

Equation or test for equality



This structure, together with all of the relational operators, has a double interpretation: as an equation and as a comparison.

Expression sequence

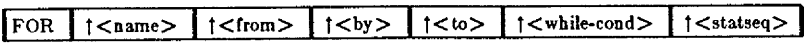


An EXPSEQ may be of length 1 (no entries); this empty structure is called NULL.

Factorial

This data structure will be replaced by a function call (identical to factorial(...)) in version 3.0 of Maple. The parser and the printing routine will still handle the mathematical notation (with the ! symbol).

For-while loop statement

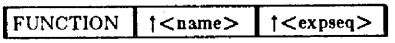


The entries for <from>, <by>, <to>, and <while> are general expressions which are filled with their default values, if necessary, by the parser. The <name> entry follows the same rules as in ASSIGN except that a NULL value indicates its absence. A NULL value in the <to> expression indicates that there is no upper limit on the loop.

Fortran function

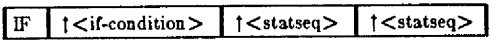
Not implemented for production yet.

Function call



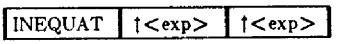
This structure represents a function invocation (as distinct from a procedure definition which uses the PROC data structure). The <name> entry follows the same rules as in ASSIGN, or it may be a PROC definition. (The parser will not generate this structure with a PROC definition for the <name> entry, but this may happen internally). The <expseq> contains the list of parameters.

If statement



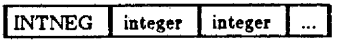
The parser generates a NULL third entry for the if-then-fi statement, and generates an IF entry for the if-then-elif... statement.

Not equal or test for inequality



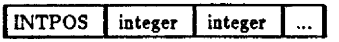
Same comments as for EQUATION.

Negative integer



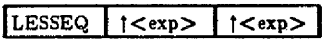
Integers are represented in base BASE (BASE=10000 for 32-bit machines and BASE=100000 for 36-bit machines). Each entry contains one "digit". A normalized integer contains no additional zeros. The integers are represented in reverse order; i.e., the first entry is the lowest order "digit", the last is the highest order "digit". BASE is the largest power of 10 such that $BASE^2$ can be represented in the host-machine integer arithmetic.

Positive integer



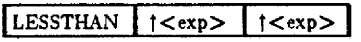
Similar to INTNEG.

Less or equal relation



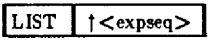
Similar to EQUATION. The parser also translates a "greater or equal" into a structure of this type, interchanging the order of its arguments.

Less than relation



Similar to EQUATION. The parser also translates "greater than" into a structure of this type, interchanging the order of its arguments.

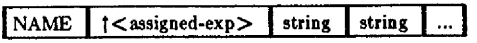
List



Occurrence of a local variable

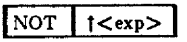
This entry indicates the usage of the <integer>th local variable. This structure is only generated by the simplifier when it processes a function definition. LOCAL entries cannot exist outside functions.

Identifier

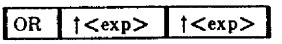


The first entry contains a pointer to the assigned value (if this identifier has been assigned a value) or 0. The string entries contain the name of the variable.

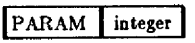
Logical not



Logical or

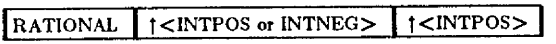


Occurrence of a parameter variable



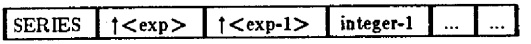
Similar to LOCAL, but using the parameters of the function.

Rational number



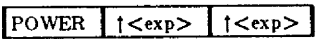
The second integer is always positive and different from 0 or 1. The two integers are relatively prime.

Series



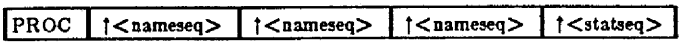
The first expression is the "taylor" variable of the series, the variable used to do the series expansion. The remaining entries have to be interpreted as pairs of coefficient and exponent. The exponents are integers (not pointers to integers) and appear in increasing order. A coefficient O(1) (function call to the function "O" with parameter 1) is interpreted specially by Maple as an "order" term.

Power



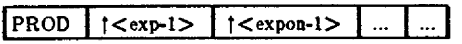
If the second entry is a rational constant, this structure is changed to a PROD structure by the simplifier.

Procedure definition



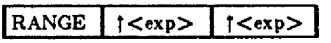
The first <nameseq> is an EXPSEQ of the names specified for the formal parameters. The second corresponds to an EXPSEQ of the names specified for the local variables and the third to the options specified. The <statseq> points to the body of the function.

Product/quotient/power

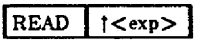


This structure should be interpreted as pairs of expressions and their (rational) exponents. Rational or integer expressions to an integer power are expanded. If there is a rational constant in the product, this constant will be moved to the first entry by the simplifier.

Range

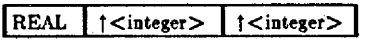


Read statement



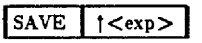
The expression should evaluate to a name (string).

Real number



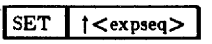
The real number is interpreted as the first integer times 10 powered to the second.

Save statement



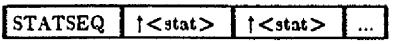
The expression should evaluate to a name (string).

Set



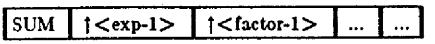
The entries in the <expseq> are sorted in increasing address order. This is an arbitrary order, but is necessary for sets. (Any other arbitrary, but consistent, order could serve.)

Statement sequence



End execution

Sum of several terms



This structure should be interpreted as pairs of expressions and their (rational) factors. The simplifier lifts as many constant factors as possible from each expression and places them in the <factor> entries. A rational constant is multiplied by its factor and represented with factor 1.

Unevaluated expression

6.3. Portability of the Maple system

One of the design goals of Maple is to be portable. The level of portability that we envision is one for which the scope of machines includes personal computers as well as present-day time-sharing systems. It was a very early decision that the language to be chosen should belong to the BCPL family. The reasons behind this decision are: efficiency, suitability, and availability. On the other hand, no single language in the BCPL family is sufficiently widely available to satisfy our needs. In view of this, we decided to write our system in a language which closely resembles B and C. This language is processed by the Margay macro-processor into either B or C, and in the near future into Port and WSL (which are two systems implementation languages developed at the University of Waterloo). Margay is a straightforward macro-processor which resembles closely, although is more powerful than, C's macro-processing. The most important difference is that Margay is written in its own macros and hence is portable across several systems.

The level of portability for the Maple user should be total. That is to say, a user should not be able to recognize in which hardware he is running. This is an easy consequence of the fact that there is a single source for Maple; the macro processing is done only before compilation and the intermediate code is never kept. It is important to realize that the entire basic system is only about 4500 lines of code. Being so small, we can afford minor changes in the code to improve portability across systems. In many instances we add redundant information to be used by Margay, which may be ignored by some systems and used by others.

The Margay macros which aid in portability can be classified in various groups:

- name changes; e.g. concat(..) in B is equivalent to strcat(..) in C.
- declaration information; Margay recognizes EXF (external function definitions), FUN (function definition), PAR (definition of parameters), LOC (definition of local variables), EXT (definition of external variables), and three types: ALGEB (algebraic),

LONGINT (multiple precision integers), and INT.

- system constants; EOF (end-of-file), TRUE, FALSE, MAXINT, MAXADDR, QUOTE, CHNL (new line character), etc.

- casting: I(...), forcing an expression to be of type integer.

- input/output: The input and output is one of the most delicate aspects of portability. The Maple system requires a very simple type of sequential input/output. Maple knows of only one sequential input stream (possibly stacked) and one sequential output stream (unique). The input and output are done either in words or in characters.

The macros used for input are:

- Ropen("filename") Opens a file for input, stacks the present input file, and returns false if it failed to open the file.

- Readch() Reads one character from the input stream.

- Readws(vect,nws) Reads nws binary words into vect. Returns the number of words read; 0 if EOF.

- Rclose() Closes file and unstacks previous file for input.

The output macros are:

- Wopen("filename") Opens a file for output (there is only one at a given time), and returns false if it failed.

- printf("format",...) Outputs characters.

- Writews(vect,nws) Outputs nws binary words from the vector vect.

- Wclose() Closes the output file.

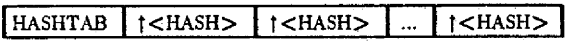
6.4. Searching Tables in Maple

Maple handles all table searching in a uniform way. All of the searching is done by an algorithm which is a slight modification of direct-chaining hashing. Although it is not obvious, the internal tables play a crucial role; they are used for: locating variable names (nametab); keeping track of simplified expressions (simpltab); keeping track of partial computations (pctable); mapping expression trees into sequential files for internal input/output (puttab); and for storing arrays and tables. It is immediately obvious that the searching in these tables has to be fast enough to guarantee overall efficiency.

The algorithm used for these tables can be understood as an implementation of direct-chaining where instead of storing a linked list for each table entry, we store a variable-length array. This requires a versatile and efficient storage manager, but without one symbolic computation would not be feasible.

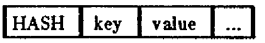
The two data structures used to implement tables are:

Table entry



Each entry points to a HASH entry or it is 0 if no entry was created. The size of HASHTAB is constant for the implementation. For best efficiency, the number of entries should be prime.

Hash-chain entry



Each entry in the table consists of a consecutive pair, the first one being the hashing key and the second the stored value. A key cannot have the value 0 as this is the indicator for the end of a chain. For efficiency reasons, the HASH entries are incremented by 5 entries at a time and consequently some entries may not be filled. Keys may be any integer or pointer which is representable in one word. In many cases the key is itself a hashing value (two step hashing).

6.4.1. The Simplification Table

All simplified expressions and subexpressions are stored in the simplification table. The main purpose of this table is to ensure that expressions appear internally only once. Every expression which is entered to Maple or which is internally generated is checked against this table, and if found, the new expression is discarded and the old one is used. This task is done by the simplifier which recursively simplifies (applies all the basic simplification rules) and checks against the table.

The task of checking for equivalent expressions within thousands of sub-expressions would not be possible if it was not done with the aid of a "hashing" concept. Every expression is entered in the simplification table using its *signature* as a key. The signature of an expression is a hashing function itself, with one very important attribute: it is order independent. For example, the signatures of the expressions a+b+c and c+a+b are identical; the signatures of a**b and b**a are also identical. Searching for an expression in the simplification table is done by:

- Simplifying recursively all of its components;
- Applying the basic simplification rules.
- Computing its signature and searching this signature in the table. If the signature is found then we perform a full comparison (taking into account that additions and products are commutative, etc.) to verify that it is the same expression. If the expression is found, the one in the table is used and the searched one is discarded.

The number of times that we have to do a full comparison on expressions is minimal; it is only when we have a "collision" of signatures. Some experiments have indicated that signatures coincide once every 50000 comparisons for 32-bit signatures. (Notice that the signatures are still far from uniform random numbers). The resulting expected time spent doing full comparisons is absolutely negligible. Of course, if the signatures disagree then the expressions cannot be equal at the basic level of simplification.

6.4.2. The Partial-Computation Table

The partial-computation table is responsible for handling the option remember in function definitions in its explicit and implicit forms. Basically, the table stores function calls as keys and their results as values. Since both these objects are data structures

already created, the only cost (in terms of storage) to place them in the table is a pair of entries (pointers). Searching these hashing tables is extremely efficient and even for simple functions it is orders of magnitude faster than the actual computation of the function.

The change in efficiency due to the use of the remembering facility may be dramatic. For example, the Fibonacci numbers computed with

```
f := proc(n)
        if n<2 then n else f(n-1)+ f(n-2) fi end;
```

take exponential time to compute, while

```
f := proc(n) option remember;
        if n<2 then n else f(n-1)+ f(n-2) fi end;
```

requires linear time.

Besides the facility provided to users, the internal system uses the partial-computation table for diff, taylor, expand, and evalr. The internal handling of expand is straightforward. There are some exceptions with the others, namely:

- diff will store not only its result but also its inverse; in other words, if you integrate the result of a differentiation the result will be "table-looked up" rather than computed. In this sense, integration "learns" from differentiation.

- taylor and evalr need to store some additional, environment, information (Degree for taylor and Digits for evalr). Consequently the entries in these cases are extended with the precision information. If a result is requested with less precision than what it is stored in the table, it is retrieved anyway and "rounded". If a result is produced with more precision than what it is stored, it is replaced in the table.

- evalr only remembers function calls (this includes constants); it does not remember the results of arithmetic operations.

Both the simplification table and the partial-computation table are cleared of all unreferenced entries at garbage collection time.

6.4.3. Arrays

Arrays and tables are implemented with internal tables. In this case the address of the simplified EXPSEQ of indices is used as a key for the searching. (Note that since simplified expressions appear only once, we can use their addresses as keys.) Arrays and tables are treated very similarly at the internal level. This implementation permits efficient use of sparse arrays of any kind without overhead. (Note: Arrays and tables will be implemented in version 3.0 of Maple).

6.5. Style Recommendations for Library Contributions

In this section we include several recommendations (or a checklist) which should be useful in preparing Maple software intended to be part of the library. The main motivation for this document is to provide uniformity and ease in porting and maintaining the library. We expect that contributors will find the recommendations sound and that these will be followed as closely as possible.

General Recommendations.

6.5.1. Nothing, absolutely nothing, replaces good algorithms and good programming techniques. No matter how closely it follows the recommendations or how much it is embellished, a bad algorithm will always be a disgrace to the library.

6.5.2. Each function should have a precise objective. In this respect we think that functions that can be trivially implemented with other commands or functions, are a disservice to the user community. They take space in the libraries, manual, and minds without giving a substantial service.

6.5.3. Each function should have comments in its heading which, without much verbosity, explain the usage, purpose, author, level, algorithm and possibly some other useful information. Pages of comments where it is difficult to find the above information may be worse than no comments at all.

6.5.4. Each function should be accompanied by a test file which tests its correctness. Test files should not be tedious repetitions of the same situation, but instead the shortest and quickest program that explores all of the code in the function. Test files will normally grow with the examples that detected errors previously undetected. Such "errors" are pieces of code which run through a sensitive path and are, in general, excellent tests. Long and slow tests tend not to be run, and are self-defeating.

6.5.5. The code in the library is likely to be taken as an example for users and future implementors of Maple. Consequently we are doubly motivated to produce high quality code.

More specific points.

6.5.6. If a function resides in <any-directory>/xxx.m then its source, that is the Maple source code that generates it, will be placed in <any-directory>/src/xxx . The only exceptions to this rule are the functions that, for being thematically related and very short, are included in a single file. System library functions should be saved with a statement like

```
save `` . libname . `xxx.m`;
```

so that their ".m" files can be created in a portable way.

6.5.7. File names (without the ".m"), and consequently function names, should be 9 characters long or shorter. This is not counting directories. This is caused by system limitations and the need to load with readlib(...). Internal functions defined entirely within another function body are not restricted by this limitation. Upper-lower case distinctions are not respected by some systems; consequently, different function names should not rely on case differences alone.

6.5.8. Local variables should be reasonably economized. Also, excessive use of local variables tends to reduce readability of the programs. E.g.,

```
for i to nops(exp) do ... od;
```

is more efficient and readable than

```
limit := nops(exp);
for i to limit do ... od;
```

The use of the RETURN(...) function typically eases the understanding of the flow of control and saves local variables. Simple operations on parameters may not be worth the assignment of local variables; for example, if op(1,param1) is used only twice, then assigning temp := op(1,param1) is not a real saving.

6.5.9. Global variables should be avoided. If unavoidable, global variables should be named starting with the at-sign (@). The Maple library convention for returning a "fail" condition (in cases where a direct FAIL return is inappropriate) is to return the global name @FAIL.

6.5.10. Data types should be used properly where needed. For example: a pair of 2 elements where order is important should be accommodated in a list; an indicator should only take the values true or false; etc.

6.5.11. Packages (collections of functions for a given purpose) should be structured according to the following example. Suppose users want to call X(...) directly. If X is nontrivial, it may optionally call sub-functions A, B, or C. Furthermore let us assume that any of X, A, B or C can call the lowest level functions E and F. Then:

(a) X should be the only name known to Maple, or the only name subject to be read with readlib(X).

(b) All of the functions that are likely to be loaded within the execution of X should be included in the module of X. That is to say that the number of loading operations should be minimized.

(c) The remaining functions in the package, which may or may not be loaded, should be defined in the module X as B:='readlib('B')', etc. This will cause the loading of B to be delayed until B(...) is used.

(d) When there are two or more possible entry points which share most of the package, then all of the definitions should be included in a single module. If X and Y are two entry points for the same package, all of the code for X and Y will be stored together (say in X.m). The initial definitions of X and Y will now be:

```
X := 'readlib('X')';
Y := 'readlib( 'Y', `` . libname . `X.m` )';
```

(e) Finally, a sub-function may be used directly by the user independently of the package. For example, if C could be used independently of X and Y then we need an entry for C. Within the package X we will define C as before, namely:

```
C := 'readlib('C')';
```

For the direct use of C we need to load its accompanying E and F; consequently, the definition for direct usage of C (not through X) will be:

```
C := 'readlib( 'C', ``.libname.`C.m`, ``.libname.`E.m`, ``.libname.`F.m` )';
```

6.5.12. The option 'remember' may be crucial for efficiency. It should be used when it is reasonably effective: whenever recomputation is likely. It should not be unnecessarily nested. Functions which produce side effects (printing of values, returning values through parameters, etc.) cannot use the option remember since this option reproduces the function result, not its side effects.

6.5.13. Atomized programming (splitting all steps of a computation) is not very efficient and, frequently, is unreadable (the "Assembler syndrome"). At the other extreme, "one-liners" are also unreadable (the "APL syndrome"). Both extremes should be avoided.

6.5.14. In complicated packages, it may be desirable to inform the user about the progress of a computation. Such printing should be regulated by printlevel. The values 2 and 3 are reserved for this purpose. For example:

```
if printlevel>2 then print(`Risch method applied`) fi;
```

6.5.15. It has proved to be valuable to have a "benchmark" for each function. A benchmark is a test file that not only tests for correctness but also for timing. When changes are done, it can be precisely measured if more/less time/space is used. Sometimes naive-looking modifications produce significant changes in performance.

6.5.16. Reminder: Don't forget to use the *load* option "-l" when loading Maple library functions. (See section 8.3).

7. LIBRARY FUNCTIONS

Maple's library functions fall into three categories: functions internal to the Maple system, automatically-loaded library functions, and miscellaneous library functions that are not automatically loaded. Functions in the first category are coded internally in the basic Maple system. Functions in the second category are specified by Maple code in the Maple system library, and their names are initially assigned as unevaluated *readlib* functions (see section 5.7). The functions in the first two categories will be grouped together in this section since the user will not generally make any distinction between these two categories. In fact, the grouping of functions into these two categories may be different on different host systems. For a specific function 'f', the user can easily determine which of the first two categories it belongs to by entering Maple and displaying the value op('f'); the result will be the name 'f' for functions in the first category and the result will be 'readlib('f')' for functions in the second category. Functions in the third category will be listed separately at the end of this section because they cannot be used without being explicitly loaded by the user.

The general rule for function invocations in Maple is that all arguments are fully evaluated. Two exceptions are the functions *assigned* and *evaln* where the argument is evaluated to a name, a third exception is the function *evalb* where the argument is evaluated by the Boolean evaluator rather than by the general expression evaluator, and a fourth exception is the function *remember* where the argument involves a procedure invocation which will not be invoked. The names of the library functions are not reserved words in Maple. A user may define his own function using the same name as one of the system-supplied functions.

7.1. abs (expr)

If expr is of type rational (or integer) then the absolute value of expr is returned, otherwise the function invocation remains unevaluated.

7.2. analyze (expr)

The purpose of this function is to analyze an expression in the following sense. The expression expr is viewed as a sum of products of the form:

const * f1**e1 * f2**e2 * . . . * fn**en .

If expr is a product (including the case of a single factor) then the value returned is the list

[const, f1, e1, . . . , fn, en]

(where const will be 1 if there is no explicit constant in the product). If expr is a sum or an equation or a range then the function *analyze* is mapped onto expr. (See the function *map*).

7.3. anames ( )

This function takes no arguments. It returns an expression sequence consisting of all of the active names in the current Maple session which are *assigned names*, meaning names which have been assigned values other than their own names. (See also the function *unames*).

7.4. assigned (name)

This function returns the value *true* if name is active in the current session and it has a value other than its own name, and returns the value *false* otherwise. The argument to this function must be a valid <name>. The argument is not fully evaluated but is evaluated to a name.

7.5. asympt (expr, x) or asympt (expr, x, n)

The purpose of this function is to compute the asymptotic expansion of expr with respect to the variable x (as x approaches infinity). If there is a third argument 'n' then it must evaluate to an integer which specifies the 'truncation degree' to be used. If there is no third argument then the 'truncation degree' is specified by the current value of the global variable *Degree* (which initially has the value 5 in the Maple system). Specifically, this function is defined in terms of the *taylor* function as follows:

```
subs( x=1/x, taylor( subs(x=1/x, expr), x=0, n ) )
```

(where the third argument 'n' to the *taylor* function will be omitted if it was omitted in the call to *asympt*).

7.6. cat (a, b, c, . . .)

This function takes an arbitrary number of arguments which must evaluate to valid strings, and it concatenates these strings into a new string. The result of this function can be specified in terms of Maple's concatenation operator '.' as follows:

```
`` . a . b . c
```

(for the case of only three arguments, for example).

7.7. coeff (expr, x, n)

For this function the expression expr must be in expanded form (see the function *expand*). The value of this function is the coefficient in expr of the term involving x**n .

Examples: If

```
p := 70*y*x**4 - 70*x**4 - 177*x**2 + 19*y**5*x - 35*y**2 + 105
```

then

```
coeff(p, x, 0);          yields    105-35*y**2
coeff(p, x, 1);          yields    19*y**5
coeff(p, x, 2);          yields    -177
coeff(p, x, 3);          yields    0
coeff(p, x, 4);          yields    70*y-70
```

7.8. commonden (expr)

This function computes the common denominator of an expression. Specifically, it first applies the *analyze* function to expr. Then it extracts from each term the denominator of the constant factor and all factors whose exponents have negative sign, and forms the least common multiple of the denominators thus extracted from each term.

7.9. convert (expr, typename)

The purpose of this function is to explicitly convert an expression from one data type to another. Two kinds of conversion are currently supported, corresponding to the two values for 'typename': rational and polynom.

For the case where 'typename' is 'rational', if expr is not of type 'rational' then it must be of type 'real' and a rational number is generated which approximates the given real number. The accuracy of the approximation depends on the number of significant digits in the input real number.

For the case where 'typename' is 'polynom', if expr is not of type 'polynom' then it must be of type 'series' and the result is the polynomial obtained by removing the order term (if any) from the series and converting from the series data structure to the ordinary sum-of-products data structure.

Examples:

```
convert(3.14, rational);          yields    22/7
convert(3.1415, rational);        yields    311/99
convert(0.30, rational);          yields    1/3
convert(0.300, rational);         yields    3/10
s := taylor(sin(x), x=0);         yields    s := 1*x+ (-1/6)*x**3+ 1/120*x**5+ O(x**6)
convert(s, polynom);              yields    x-1/6*x**3+ 1/120*x**5
```

User Interface: New conversion procedures can be made known to the *convert* function by the following mechanism. If the user assigns a procedure to the name `conv/newtype` (where 'newtype' is any name chosen by the user) as in

```
`conv/newtype` := proc ( expr, <extra parameters> ) ... end
```

then the function invocation

```
convert ( expr, newtype, <extra parameters> )
```

will cause the function invocation

`conv/newtype` (expr, <extra parameters>) .

If `conv/newtype` is not assigned then Maple looks for it in the Maple system library at the pathname

`` . libname . `conv/newtype.m`

and if it is not found then an error occurs.

7.10. degree (expr, x)

If expr is a polynomial in x (allowing both positive and negative exponents) then this function returns the degree of expr in x. It is not necessary that expr be in expanded form. This function may be applied as well to the series data structure. If expr is neither a series in the indeterminate x nor a polynomial in the indeterminate x then the value returned is the largest word-size negative integer. (See also the function *ldegree*).

7.11. diff (expr, x1, x2, . . ., xn)

This function computes the partial derivative of expr with respect to x1, x2, . . . , xn, respectively. The latter n expressions must evaluate to <name>s. In the case where n is greater than one, the syntax is simply a shorthand notation for nested applications of the *diff* function.

Examples: Assuming that x and y are names which stand for themselves, if the following statements are executed:

```
p := -30*x**3*y + 90*x**2*y**2 + 5*x**2 - 6*x*y;
diff(p, x, y);
```

then the result of the function invocation of 'diff' is:

$$-90*x**2+360*x*y+(-6)$$

This is equivalent to executing the statement diff(diff(p,x),y) .

User Interface: New functions can be made known to Maple's *diff* function by the following mechanism. If the user assigns a procedure to the name `diff/newfcn` (where 'newfcn' is any name chosen by the user) as in

`diff/newfcn` := proc (expr,x) newfcn1(expr) * diff(expr,x) end

(where the name 'newfcn1' is being used as the name of the derivative function) then the function invocation

diff (newfcn(expr), x)

will cause the function invocation

`diff/newfcn` (expr, x) .

If `diff/newfcn` is not assigned then Maple looks for it in the Maple system library at the pathname

```
`` . libname . `diff/newfcn.m`
```

and if it is not found then a FAIL return occurs from the *diff* function.

Functions whose derivatives are currently defined in the Maple system library include the elementary functions (all of the circular, inverse circular, hyperbolic, and inverse hyperbolic functions, as well as the functions exp and ln), abs, GAMMA, Psi (which satisfies the relationship

Psi(x) = diff(GAMMA(x),x) / GAMMA(x)),

and the first four derivatives of Psi (represented by the names Psi1, Psi2, Psi3, and Psi4). The derivative of an unevaluated 'int' function is also defined in the Maple system library.

7.12. divide (a, b, 'q')

The purpose of this function is to attempt to perform exact polynomial division of expression 'a' by expression 'b'. The division is considered successful only if the resulting quotient is a 'true polynomial' in its indeterminates – i.e., negative exponents are not acceptable in the result of a polynomial division. The value of the *divide* function is 'true' if the division was successful, 'false' otherwise. Furthermore, if there is a third argument 'q' (which must evaluate to a name) and if the division was successful then the value of the quotient is assigned to q. In the case of an unsuccessful division the value of q will remain unaffected.

Examples:

```
a := 7*y**3*x**4 - 2*y*x**3 - (y**4 - 21*y**3)*x**2 - 6*y*x - 3*y**4;
b := y*x**2 + 3*y;
divide(a, b, 'q');                          yields    true
q;                                          yields    7*y**2*x**2-2*x-y**3

r := expand( (2*x-5)**3 * (x+1) );
while divide(r, 2*x-5, 'r') do od;
r;                                          yields    x+1

f := expand((c-1)/c);                       yields    1-c**(-1)
g := c-1;                                   yields    c-1
divide(f, g);                               yields    false
```

In the latter example, note that it is possible to simplify the expression f/g to the value c**(-1) but this cannot be accomplished by the *divide* function because the result is not a 'true polynomial'. For this purpose, the *normal* function should be used, as in:

```
f/g;                                        yields    (1-c**(-1))/(c-1)
normal(");                                  yields    c**(-1)
```

7.13. ERROR (message)

This function is a special function whose purpose is to cause an immediate exit from a procedure, and 'message' is a <string> which is the error message to be printed out. (See section 5.5). Upon execution of this function, control returns to the top level of the Maple system and the following error message is printed out:

ERROR: message

where 'message' (a <string>) is the argument to the ERROR function.

7.14. evalb (expr)

This function invokes the Boolean expression evaluator on expr. For example, the expression a = b will be considered an algebraic equation if it does not appear in an explicit Boolean context, but evalb(a = b) will evaluate the equation as a Boolean (i.e., it will evaluate the equation to the value 'true' or to the value 'false').

7.15. evaln (name)

The purpose of this function is to apply to the argument 'name' Maple's name evaluator, which is the evaluator that is always applied to left-hand-sides of assignments, for example. The argument must be a syntactically valid <name> and, of course, it is not fully evaluated. One of the uses for this function is to *unassign* names formed with the concatenation operator. For example,

```
for i to 5 do a.i := evaln(a.i) od
```

will unassign the names a1, a2, a3, a4, and a5. Note that in this case the *evaln* function cannot be replaced by the use of the unevaluated expression construct 'a.i' because then the concatenation on the right-hand-side will remain unevaluated (and the names a1, . . . , a5 will remain *assigned*).

7.16. evalr (expr) or evalr (expr, n)

This is the 'evaluate to a real' function, which evaluates the argument 'expr' to a real number (if possible). If there is no second argument then the number of significant digits appearing in the result is controlled by Maple's global variable *Digits*. (The initial value of the global variable *Digits* is 10, but the user may assign any integer value to this global variable). If there is a second argument 'n' to the *evalr* function then it must evaluate to an integer, and the number of significant digits appearing in the result is determined by the value of n.

Examples:

```
a := (5**40 + 3**50) / 2**90;   yields
        a := 45478324578584871158695804 37/6189700196426901374495621 12
evalr(a);              yields    7.3474196060
evalr(a, 40);          yields    7.3474196060154781089322604350878881161323
```

```
Digits := 25;
evalr( 5/3 * exp(-2) * sin(Pi/4) ); yields      .1594941608506848732679800
```

User Interface: New functions, and also new constants, can be made known to Maple's *evalr* function by the user.

For the case of new functions, if the user assigns a procedure to the name `real/newfcn` (where 'newfcn' is any name chosen by the user) as in

```
`real/newfcn` := proc (x)
                    local t;
                    t := evalr(x);
                    evalr( exp(t**2) * sin(Pi/2 * t) )
                    end
```

then the function invocation

```
evalr ( newfcn(x) )
```

will cause the function invocation

```
`real/newfcn` ( x ) .
```

If `real/newfcn` is not assigned then Maple looks for it in the Maple system library at the pathname

```
`` . libname . `real/newfcn.m`
```

and if it is not found then an error occurs.

For the case of new constants, if the user assigns a procedure to the name `real/constant/newconst` (where 'newconst' is any name chosen by the user) as in

```
`real/constant/newconst` := proc () evalr( (5**(1/2) - 1) / 2 ) end;
```

then the function invocation

```
evalr ( newconst )
```

will cause the function invocation

```
`real/constant/newconst` ( ) .
```

If `real/constant/newconst` is not assigned then Maple looks for it in the Maple system library at the pathname

```
`` . libname . `real/constant/newconst.m`
```

and if it is not found then an error occurs.

Functions for which 'evalr' procedures are currently defined in the Maple system library include the elementary functions (all of the circular, inverse circular, hyperbolic, and inverse hyperbolic functions, as well as the functions exp and ln), and the Psi function. Constants for which 'evalr' procedures are currently defined in the Maple system library include:

Pi, e (exp(1)), gamma (Euler's constant), and C (Catalan's constant) .

7.17. expand (expr) or expand (expr, e1, e2, . . ., en)

The purpose of this function is to expand expr by distributing products over sums. If the number of arguments is greater than one then the additional arguments e1, e2, . . ., en are expressions which will be 'frozen' (i.e., the effect is to replace every occurrence of ei by a <name> before performing the *expand* operation and then to restore the original expression ei unchanged). If the expression is an equation then *expand* is applied to the operands of the equation.

Examples:

```
p := (2*x - 5) * (35*x**2 - x + 7);
expand(p);                          yields     70*x**3-177*x**2+ 19*x+ (-35)

q := 3*sin(x) * (x*sin(x) - y*z) * (2*x**2 - 3);
expand(q);                          yields
        6*sin(x)**2*x**3-9*sin(x)**2*x-6*sin(x)*y*z*x**2+ 9*sin(x)*y*z

r := 3*(x+ 1)**3 - 5*(x+ 1)**2;
expand(r, x+ 1);                    yields     3*(x+ 1)**3-5*(x+ 1)**2
expand(r);                          yields     3*x**3+ 4*x**2-x+ (-2)
```

7.18. factor (expr)

This function computes a complete factorization over the integers of the multivariate polynomial expr. (Work on this function has not been completed at the time of writing).

7.19. frac (a)

This function computes the *fractional part* of a rational number. It is the complement of the *trunc* function; the value of frac(x) is specified by

x - trunc(x) .

7.20. gcd (a, b, 'result1', 'result2')

This function computes the *greatest common divisor* of the multivariate polynomials 'a' and 'b'. It is an error if 'a' and 'b' are not polynomials in their indeterminates. The gcd is computed in the domain of polynomials with integer coefficients, but the input polynomials may have rational coefficients in which case the common denominator is simply removed. If the third argument 'result1' is present then it must evaluate to a <name> and upon return its value will be a / gcd(a,b) . Similarly, if the fourth argument 'result2' is present then it must evaluate to a <name> and upon return its value will be b / gcd(a,b) .

7.21. has (expr1, expr2)

The value of this function is *true* if expr1 contains expr2 as an explicit subexpression, *false* otherwise. The concept of 'explicit subexpression' corresponds to the semantics of the *op* function: if op(expr1), or recursive application of op to each operand of expr1, yields expr2 as an operand then expr1 contains expr2 as an 'explicit subexpression'; otherwise it does not.

Examples:

has ((a+ b)**(4/3), a+ b);	yields	true
has ((a+ b)**(4/3), a);	yields	true
has (a+ b+ c, a+ b);	yields	false

7.22. lcontent (expr)

This function computes the integer content of expr – i.e., the greatest common divisor of the integer coefficients in the case of an expanded polynomial. If expr is not in expanded form then the *icontent* function is mapped onto its components to obtain the result. For the common case of an expanded polynomial with integer coefficients, this function has a concise definition in terms of the functions *lcoeff*, *map*, *igcd*, and *op* as follows:

igcd(op(map(lcoeff, [op(expr)])))

7.23. ifactor (n)

This function returns an integer factor of n; if n is prime then the value returned is n. The integer factor returned by this function is not necessarily a prime.

7.24. igcd (i, j, k, . . .)

This function takes an arbitrary number of arguments which must evaluate to integers, and it computes the nonnegative *greatest common divisor* of these integers. If *igcd* is called with no arguments then the value 0 is returned.

Examples:

igcd();	yields	0
igcd(3);	yields	3
igcd(–10, 6, –8);	yields	2

7.25. ilcm (i, j, k, . . .)

This function takes an arbitrary number of arguments which must evaluate to integers, and it computes the nonnegative *least common multiple* of these integers. If *ilcm* is called with no arguments then the value 0 is returned.

Examples:

ilcm();	yields	0
ilcm(–5);	yields	5
ilcm(7, –6, 14);	yields	42

7.26. lmodp (n, p)

The functions *imodp* and *imods* are two functions for computing the integer modular operation

n mod p .

The final letter 'p' or 's' in the function name stands for 'positive range' or 'symmetric range'. If n and p are integers then the function *imodp*(n,p) returns an integer r lying in the 'positive range':

$$0 \leq r < abs(p),$$

where $n=pq + r$ for some integer q. If p is zero then an error occurs. Note that the *imodp* function satisfies the property:

imodp(n, p) = imodp(n, –p) .

Examples:

imodp(7, 5);	yields	2
imodp(8, 5);	yields	3
imodp(–8, –5);	yields	2
imodp(7, –6);	yields	1
imodp(–7, 6);	yields	5

7.27. lmods (n, p)

The functions *imods* and *imodp* are two functions for computing the integer modular operation

n mod p .

The final letter 's' or 'p' in the function name stands for 'symmetric range' or 'positive range'. If n and p are integers then the function *imods*(n,p) returns an integer r lying in the 'symmetric range':

$$-abs(p)/2 < r \leq abs(p)/2,$$

where $n=pq + r$ for some integer q. If p is zero then an error occurs. Note that the *imods* function satisfies the property:

imods(n, p) = imods(n, –p) .

Examples:

imods(7, 5);	yields	2
imods(8, 5);	yields	–2
imods(–8, –5);	yields	2
imods(9, –6);	yields	3
imods(–9, 6);	yields	3

7.28. indets (expr)

The purpose of this function is to determine the indeterminates which appear in expr. The value of the function is a set whose elements are the indeterminates. The concept of 'indeterminate' is that expr is viewed as a rational expression (i.e. an expression formed by applying only the operations +, –, *, / to some given symbols) and therefore unevaluated functions such as sin(x), exp(x**2), f(x,y), and x**(1/2) are treated as indeterminates. When an indeterminate which is not a <name> appears in the set then so will all of its component indeterminates. Expressions of type 'constant' such as sin(1), f(3,5), and 2**(1/2) are not considered to be indeterminates. Note that if expr is a sum or product of terms t1, t2, . . . , tn then the result of applying indets(expr) will be identical to the result of applying the set union:

indets(t1) + indets(t2) + . . . + indets(tn) .

Examples: If the following statements are executed:

```
p := 3*x**3*y**4*z - 2*x**2*z**2 + y**3*z - 7*y + 5;
r := (2*x**2 - 5) * (x - 2)**(1/3) / (x*exp(x**2));
```

then

indets(p);	yields	{z,y,x}
indets(r);	yields	{exp(x**2),(x+(–2))**(1/3),x}

Furthermore,

indets(exp(x**2));	yields	{exp(x**2),x}
indets(x**(1/2));	yields	{x**(1/2),x}
indets(2**(1/2)*f(9));	yields	{}

7.29. int (expr, x) or int (expr, x = a..b)

If the second argument is not an equation then this function attempts to compute the indefinite integral of expr with respect to the second argument 'x' which must evaluate to a <name>. If the second argument is an equation then its left-hand-side must evaluate to a <name> 'x' and its right-hand-side must evaluate to a <range> 'a..b', and this function attempts to compute the definite integral of expr with respect to 'x' over the interval specified by the range 'a..b'. If Maple is not successful in performing the integration then a FAIL return occurs, meaning that the value of the function invocation is the unevaluated function invocation.

Examples: If

```
f := 1/2*x**(-2) + 3/2*x**(-1) + 2 - 5/2*x + 7/2*x**2;
```

then

```
int(f,x);                yields   -1/2*x**(-1)+3/2*ln(x)+2*x-5/4*x**2+7/6*x**3
int(f, x = 1..2);        yields   20/3 + 3/2*ln(2)
evalr(");                yields   7.706387438

int( (x-1)/(x+1), x );   yields   (x-1)*ln(x+1)-(x+1)ln(x+1)+x+1
expand(");               yields   -2*ln(x+1)+x+1

int(tan(x), x);          yields   -ln(cos(x)
int(sin(t)*cos(t), t);   yields   1/2*sin(t)**2
int(x*cos(x), x);        yields   x*sin(x)+cos(x)

int(exp(x**2), x);       yields   int(exp(x**2),x)
```

7.30. iquo (m, n)

This function computes the *integer quotient* of 'm' divided by 'n'. The result of this function is identical with the result of applying trunc(m/n). The *iquo* function will be more efficient than the latter when 'm' and 'n' are long integers because it avoids first simplifying the rational number m/n to lowest terms. Specifically, if m and n are integers then the function *iquo*(m,n) returns an integer q satisfying

$$m = nq + r$$

for some integer r such that

$$abs(r) < abs(n) \quad and \quad mr \geq 0.$$

If n is zero then an error occurs.

Examples:

```
iquo(7, 5);      yields   1
iquo(-7, 5);     yields   -1
iquo(7, -5);     yields   -1
iquo(-7, -5);    yields   1
```

7.31. lcm (a, b)

This function computes the *least common multiple* of the multivariate polynomials 'a' and 'b'. It is an error if 'a' and 'b' are not polynomials in their indeterminates. This function invokes the *gcd* function, using the definition

```
lcm(a,b) = a * b / gcd(a,b)
```

(with an adjustment of the *sign* of the result to make the leading coefficient positive).

Restrictions on the input expressions are therefore the restrictions of the *gcd* function.

7.32. lcoeff (expr)

This function computes the constant *leading coefficient* in the multivariate polynomial expr, with respect to the set of indeterminates indets(expr). If expr is in expanded form and if indets(expr) yields the set {x.1, x.2, . . . , x.n} then the result of the *lcoeff* function can be expressed as follows:

```
u := expr;
for i to n while not type(u, constant) do
  u := coeff( u, x.i, degree(u, x.i) )
od;
u
```

It is an error if expr is not a polynomial in its indeterminates.

Examples:

```
p := 31*x**4*y**4 + 2*x**3*y**3*z - y**2*z**5;
indets(p);                          yields    {z,y,x}
lcoeff(p);                          yields    -1

q := 17*x**5 + x**3 - 5*x**2 + 111;
indets(q);                          yields    {x}
lcoeff(q);                          yields    17

r := 3/2*y**3 - 5/2*ln(2)*x**5*y + x**4*y - 1;
indets(r);                          yields    {x,y}
lcoeff(r);                          yields    -5/2*ln(2)
```

7.33. ldegree (expr, x)

This function is a companion to the *degree* function. If expr is a polynomial in x (allowing both positive and negative exponents) then this function returns the *low degree* of expr in x, which is the least exponent of x in expr. It is not necessary that expr be in expanded form. This function may be applied as well to the series data structure. If expr is neither a series in the indeterminate x nor a polynomial in the indeterminate x then the value returned is the largest word-size negative integer.

7.34. length (n)

For this function, the argument 'n' must evaluate to an integer and the value returned is the length of the integer (i.e., the number of digits in its base-10 representation).

7.35. lexorder (name1, name2)

This function tests to determine whether **name1** and **name2** are in lexicographical order. It returns *true* if **name1** occurs before **name2** in lexicographical order, or if **name1** is equal to **name2**. Otherwise, it returns *false*. The lexicographical order depends in part upon the collation sequence of the underlying character set, which is system-dependent. For names consisting of ordinary letters, lexicographical order is the standard alphabetical order.

Examples: For a typical implementation of the ASCII character set the following results are obtained:

lexorder(a, b);	yields	true
lexorder(A, a);	yields	true
lexorder(` a`, a);	yields	true
lexorder(John, Harry);	yields	false
lexorder(determinant, determinate);	yields	true
lexorder(greatest, great);	yields	false
lexorder(`*`, `**`);	yields	true
lexorder(`**`, `+ `);	yields	true

7.36. limit (expr, x = a)

This function attempts to compute the limiting value of expr as x approaches a. The second argument must be an <equation> and the left-hand-side of the equation must evaluate to a <name>. This function applies the *taylor* function and deduces the limit from the form of the taylor series. The limit point 'a' may take the special value 'infinity' in which case the limiting value is determined by applying the change of variable x = 1/x in expr and then computing the taylor series about x = 0.

Examples:

limit(sin(x)/x, x=0);	yields	1
limit((tan(x)-x)/x**3, x=0);	yields	1/3
limit((5*x-3)/(x**2+ x+ 1), x=infinity);	yields	0
r := (x**2 - 1) / (11*x**2 - 2*x - 9);		
limit(r, x=0);	yields	1/9
limit(r, x=infinity);	yields	1/11
limit(r, x=1);	yields	1/10
limit(1/x, x=0);	yields	infinity
limit(1/x, x=infinity);	yields	0

7.37. map (f, expr, arg2, arg3, . . ., argn)

For this function, f must evaluate to a name or to a procedure definition. The purpose of this function is to map f (as a function name or as a procedure invocation) onto the components of expr. The result of the *map* function is a new expression which can be defined as follows: replace the ith operand in expr by the result of applying f to the ith operand, for i = 1, 2, . . . , nops(expr). If f takes more than one argument then there must be additional arguments to the *map* function: arg2, arg3, . . . , argn which are simply passed through as the 2nd, 3rd, . . . , nth arguments to f.

Examples:

```
map(f, x + y*z);                          yields   f(x) + f(y*z)
map(f, y*z);                              yields   f(y)*f(z)
map(f, {a,b,c});                          yields   {f(a),f(b),f(c)}

map( proc (x) x**2 end, x + y );          yields   x**2 + y**2
map( proc (x) x**2 end, [1,2,3,4] );      yields   [1,4,9,16]

map(int, [exp(t),ln(t),tan(t)], t);       yields   [exp(t),t*ln(t)-t,-ln(cos(t))]

expr := 2/3 * x/sin(x) - 1/x + sin(x);
den := 3*x*sin(x);
map( proc (e,m) m*e end, expr, den );     yields   2*x**2-3*sin(x)+ 3*x*sin(x)**2

mult := proc (e,m) m*e end;
map(mult, h(u,v,w), 10);                  yields   h(10*u,10*v,10*w)
```

7.38. max (a, b, c, . . .)

This function takes an arbitrary number of arguments. If each of the arguments evaluates to an integer, a rational number, or a real number, then the value of this function is the maximum of these numbers. If one or more of the arguments does not evaluate to such a constant then a FAIL return occurs. It is an error if *max* is called with no arguments.

Examples:

```
max(3/2, 1.49);                           yields   3/2
max(3/5, evalr(ln(2)), 9/13);             yields   .6931471805
max(5);                                   yields   5
max(-1001, 1/2, -1/2, -9);                yields   1/2
max(x, y);                                yields   max(x,y)
```

7.39. member (expr, set_or_list, 'position')

The purpose of this function is to test for set membership or to test for list membership and (optionally) to locate the position of expr in a list. The second argument 'set_or_list' must be either a set or a list, and this function returns **true** if expr is one of the elements in set_or_list, **false** otherwise. If the third argument is present then it must evaluate to a <name>, the second argument must be a list, and, in the case where the value of this function is **true**, the position of expr in the list will be assigned to the third argument.

Examples:

member(y, {x,y,z});	yields	true
member(y, {x*y, y*z});	yields	false
member(x, {});	yields	false
member(3*exp(x/2), {sin(x), 3*exp(x/2)});	yields	true
member(w, [x,y,w,u]);	yields	true
member(w, [x,y,w,u], 'k');	yields	true
k;	yields	3
member(x, [x+y, x-y, x*y, x/y], 'k');	yields	false
member(x+y, [x+y, x-y, x*y, x/y], 'k');	yields	true
k;	yields	1

7.40. min (a, b, c, . . .)

This function takes an arbitrary number of arguments. If each of the arguments evaluates to an integer, a rational number, or a real number, then the value of this function is the minimum of these numbers. If one or more of the arguments does not evaluate to such a constant then a FAIL return occurs. It is an error if *min* is called with no arguments.

Examples:

min(3/2, 1.49);	yields	1.49
min(3/5, evalr(ln(2)), 9/13);	yields	3/5
min(evalr(ln(2)), evalr((5**(1/2)-1)/2));	yields	.6180339890
min(-1001, 1/2, -1/2, 9);	yields	-1001
min(x, y);	yields	min(x,y)

7.41. nops (expr)

The purpose of this function is to determine the number of operands appearing in expr. The manner in which expr is viewed by this function corresponds to the manner in which an expression is viewed by the function *op*. In the most common case, expr has operands indexed from 1 to n (such as in a general algebraic expression, a set, or a list) and nops(expr) is n. If expr is a function invocation with operands indexed from 0 to n then nops(expr) is n. If expr is a series with operands indexed from 0 to n then

nops(expr) is n.

Examples: Assuming that f, x, y, and z are names which stand for themselves, if the following statements are executed:

```
g := f(x, y, z);
a := (3*sin(x**3) - (2/3)*x + y) / (2*x**2 - 1);
```

then

```
nops(g);                    yields    3
nops(a);                    yields    2
nops(op(1,a));              yields    3
nops(op(2,a));              yields    2 .
```

Note that the latter result of 2 is not because the denominator of 'a' is the expression

```
2*x**2+(-1)
```

which is an addition of two terms but rather op(2,a) is the expression

```
(2*x**2+(-1))**(-1)
```

which is a power (and a power necessarily consists of exactly two operands).

7.42. normal (expr)

This function normalizes expr into the *factored normal form* for rational expressions – i.e., into the form

```
numerator / denominator
```

where numerator and denominator are relatively prime. In the general case, each of 'numerator' and 'denominator' will be left in factored form as far as possible (without actually performing any factorization) subject to the condition that sums of factors will be expanded whenever this is necessary to guarantee that zero will be recognized. In the special univariate case (i.e., when there is only one indeterminate in expr) each of 'numerator' and 'denominator' will be in expanded form.

Examples:

```
normal( 2/x + y );                      yields    (2+y*x)/x
normal( 3*y*(x-5)**2 / (x**2-25) );     yields    3*y*(x-5)/(x+5)

num := (3*x**2 - 5*x*y)**2 * (x**2 - 2*x*y + y**2);
den := x * (y-x)**3;
normal( num/den );                      yields    x*(3*x-5*y)**2/(y-x)

normal( (sin(x)**3 - 27) / (sin(x) - 3) );   yields    sin(x)**2+3*sin(x)+9
normal( x**2/(1-x) - x/(1-x) );              yields    -x
```

7.43. numerator (expr, 'denom')

This function computes the numerator of expr that results from first forming a common denominator for the terms in expr. It calls the *commonden* function. If the second argument is present then it must evaluate to a <name> and it will be assigned the value commonden(expr).

Examples:

numerator(2/x + y);	yields	2+y*x
numerator(3*y*(x–5)**2 / (x**2–25));	yields	3*y*(x–5)**2
numerator(x**2/(1–x) – x/(1–x), 'den');	yields	x**2–x
den;	yields	1–x

7.44. op (i, expr) or op (i..j, expr) or op (expr)

The purpose of this function is to extract one or more operands from the expression expr. If *op* is called with two arguments and if the first argument evaluates to a nonnegative integer, say i, then the value of the function is the i-th operand in expr. General algebraic expressions have operands indexed from 1 to n (for some positive integer n). A function invocation

<name> (<expression sequence>)

is considered to have as its 0-th operand <name> and the arguments are operands 1 through n (for some integer n). If expr is a <series> formed by expansion about the point x=a (where x is the name of the indeterminate) then the 0-th operand of expr is x–a, the 1-st, 3-rd, . . . operands are the coefficients (which may be arbitrary expressions), and the 2-nd, 4-th, . . . operands are the corresponding exponents (with the exponents ordered from least to greatest). For a more detailed description of the operands corresponding to each of Maple's data types, see section 4.1.

If *op* is called with two arguments and if the first argument evaluates to a <range> then the value returned is an expression sequence (i.e., a sequence of expressions separated by commas) consisting of the operands specified by the <range>.

If *op* is called with only one argument, say expr, then the result is equivalent to the result of the invocation

op (1..nops(expr), expr) .

For general algebraic expressions, this value is an expression sequence consisting of all of the operands in expr. Note, however, that if expr is one of the structures for which operand 0 is defined (e.g., a series or a function invocation) then the 0-th operand will be missing from the expression sequence op(expr).

The special case where expr evaluates to a <name> must be noted. A <name> is defined to have exactly one operand, which is the value assigned to <name>. If no value has been explicitly assigned to <name> then its value is its own name. Note that in the case where a value has been assigned to <name>, say x, the *op* function must be called in the form

```
op( 'x' )
```

(or equivalently, op(1, 'x')) if it is desired to see what value was assigned to x; otherwise, if the argument is not quoted then it will be the *value* of x which is passed to the *op* function.

Examples:

```
g := f(x, y, z);
op(0, g);                                   yields   f
op(2, g);                                   yields   y
op(0..2, g);                                yields   f,x,y
op(g);                                      yields   x,y,z

e := [2*x, y+1];
[op(e), z];                                 yields   [2*x,y+1,z]

a := (3*sin(x**3) - 2/3*x + y) / (2*x**2 - 1);
op(2, a);                                   yields   (2*x**2-1)**(-1)
op(1, a);                                   yields   3*sin(x**3)-2/3*x+y
op(2, op(1,a));                             yields   sin(x**3)

w := 3*x**2 - 2*x*y + y**2;
x := 1/2;
op('w');                                    yields   3*x**2-2*x*y+y**2
op('x');                                    yields   1/2
op(w);                                      yields   3/4,-y,y**2
op(x);                                      yields   1,2
```

7.45. param (i)

This is a special function which is only valid within the body of a procedure. The argument i must evaluate to a positive integer not greater than *nargs*, the number of actual parameters with which the procedure was invoked, and the value of this function is the i-th actual parameter. (See section 5.2.)

7.46. prem (a, b, x, 'm')

This function computes the *pseudo-remainder* of 'a' divided by 'b' with respect to the variable x, where 'a' and 'b' must be polynomials with integer coefficients. Specifically, the value of this function is the unique polynomial r with integer coefficients such that

$$m\,a = b\,q + r$$

for some polynomial q with integer coefficients, with $r = 0$ or $\text{degree}(r,x) < \text{degree}(b,x)$, where the *multiplier* m is defined by

m = c ** (degree(a,x) - degree(b,x) + 1)

where c = coeff(b, x, degree(b,x)) -- i.e., c is the leading coefficient in b with respect to x. If the fourth argument is present then it must evaluate to a <name> and it will be assigned the value of the *multiplier* m defined above.

7.47. print (expr1, expr2, . . .)

The effect of this function is to print the values of the expressions appearing as arguments. Three spaces are printed between each of the output expressions. If this function is called with no arguments then the effect is to create a blank line in the output stream.

7.48. product (expr, i = m..n)

This function forms the product of the factors obtained by substituting for i in expr the values m, m+ 1, . . ., n. The second argument must be an equation and its left-hand-side must evaluate to a <name>, its right-hand-side must evaluate to a <range>. It is an error if n - m does not evaluate to an integer and it is an error if m > n+ 1. If m = n+ 1 then the value of the product is 1.

7.49. readlib ('f') or readlib ('f', file1, file2, . . ., filen)

Each argument to this function must evaluate to a <name>. If there is only one argument then the following **read** statement is executed:

read `` . libname . f . `.m`;

and the value returned is the value of the argument 'f' after the **read** statement has been executed. If there is more than one argument then the following **read** statements are executed:

read file1; read file2; . . . ; read filen;

and the value returned is the value of the first argument 'f' after the the **read** statements have been executed. It is an error if 'f' is not assigned a value in the file (or one of the files) being read. For further details, including a complete definition in Maple code, see section 5.7.

7.50. Real (m, n)

This is a special function used to specify a *real number*. The arguments to this function must evaluate to integers, and the value of Real(m, n) is the real number

m * 10**n .

This function is particularly useful for specifying a real number with a very large or a very small magnitude, as in Real(173, 21) or Real(1952135, -30).

7.51. remember (f (x, y, . . .) = result)

This is a special function to be used in conjunction with the *option remember* facility in procedures (see section 5.3). The argument to this function must be an <equation> and its left-hand-side must take the form of a procedure invocation. The name 'f' must evaluate to a procedure definition in which *option remember* has been specified. The effect of this function is to place an entry in the system table known as the *partial computation table* which associates the specified procedure invocation with the specified 'result'. If there is ever another invocation of this procedure with actual parameters that have the same values as those specified here then the Maple system will immediately retrieve the 'result' from the partial computation table without performing any computation. This function generalizes the *option remember* facility since it may be invoked either from within the body of the procedure 'f' or externally.

Note that this function has special rules for the evaluation of its arguments. The name 'f' will be evaluated to a procedure definition and each of the specified arguments x, y, . . . will be evaluated, but the procedure will not be invoked. The right-hand-side of the equation, 'result', will be evaluated.

7.52. RETURN (expr1, expr2, . . .)

This function is a special function whose purpose is to cause an immediate return from a procedure (see section 5.5). Upon execution of this function, control returns to the point where the current procedure was invoked and the value of the procedure invocation is the expression sequence expr1, expr2, ... It is an error if a call to the function RETURN occurs at a point which is not within a procedure definition.

7.53. sign (expr)

This function computes the *sign* of expr in the sense of the sign of the constant lcoeff(expr). (See the function *lcoeff*). Specifically, the definition of the *sign* function is:

if lcoeff(expr) < 0 then -1 else 1 fi

7.54. solve (eqn, var) or solve ({eqn1,...,eqnk}, {var1,...,vark})

This function takes two arguments. The first argument is either a single equation or a set of equations, and correspondingly, the second argument is either a single name which is the variable to be solved for or a set of names which are the variables to be solved for. Whenever an equation is expected in the input arguments, if it is instead an ordinary algebraic expression e then the equation e = 0 is understood. In the case of a single equation and a single variable, it is valid to specify one of the arguments as a set or both arguments as sets. The value of this function is an expression sequence of the solutions, and in the case where the second argument is a set the value is a sequence of solution sets.

As of this writing, the *solve* function is able to solve single equations involving elementary transcendental functions, systems of linear equations, single polynomial equations, and equations requiring the inversion of taylor series.

Examples:

```
solve( cos(x) + y = 9, x );                    yields    arccos(9-y)
solve( 2**a + G, a );                          yields    ln(-G)/ln(2)
solve( taylor(arcsin(x)-y, x), x );            yields
                    1*y+(-1/6)*y**3+1/120*y**5+O(y**6)
solve( x**2 - 46*x + 529, x );                 yields    23,23
solve( 1/2*a*x**2 + b*x + c, x );              yields
                    (-b+(b**2-2*a*c)**(1/2))/a,(-b-(b**2-2*a*c)**(1/2))/a

eqn1 := x + 2*y + 3*z + 4*t + 5*u = 6;
eqn2 := 5*x + 5*y + 4*z + 3*t + 2*u = 1;
eqn3 := 3*y + 4*z - 8*t + 2*u = 1;
eqn4 := x + y + z + t + u = 9;
eqn5 := 8*x + 4*z + 3*t + 2*u = 1;
solve( {eqn.(1..5)}, {x,y,z,t,u} );            yields
                    {u=8589/110,x=56,z=-13983/110,y=168/5,t=-1736/55}
```

7.55. subs (old1 = new1, . . ., oldk = newk, expr)

This function takes an arbitrary number of arguments and each argument except the last one must be an equation. The value of this function is the expression resulting from applying the substitutions specified by the equations to the last argument, expr. The substitutions are performed sequentially starting with the first argument old1 = new1. Thus, the following two statements are equivalent:

```
subs( old1 = new1, old2 = new2, expr )
subs( old2 = new2, subs(old1 = new1, expr) )
```

More specifically, the semantics of the function subs(old = new, expr) are that every occurrence in 'expr' of the subexpression 'old' is replaced by the expression 'new'.

Examples:

```
subs( x=1, 3*x*ln(x**3) );                     yields    0
subs( a+b = y, (a+b)**(4/3) );                 yields    y**(4/3)
subs( a=b+1, b=3, a+b );                       yields    7
```

7.56. subsop (i = newexpr, expr)

The value of this function is the expression resulting from replacing op(i, expr) by newexpr in expr. The first argument must be an equation and the left-hand-side of the equation must evaluate to a nonnegative integer not greater than nops(expr). In the special case where op(i, expr) does not occur anywhere in expr except as the i-th operand, the result of this function will be equivalent to the result of

```
subs( op(i,expr) = newexpr, expr ) .
```

7.57. sum (expr, i) or sum (expr, i = m..n)

If the second argument is not an equation then this function attempts to compute the 'indefinite summation' of expr with respect to the second argument 'i' which must evaluate to a <name>. Specifically, if we denote the functional dependency of expr on the variable 'i' by the notation expr(i) then the *indefinite summation* is defined to be an expression g(i), containing the variable 'i', such that

$$g(i+1) - g(i) = expr(i) .$$

In other words, 'indefinite summation' is the inverse of the forward difference operator.

If the second argument is an equation then its left-hand-side must evaluate to a <name> 'i' and its right-hand-side must evaluate to a <range> 'm..n', and this function attempts to compute the definite summation of expr with respect to 'i' with lower limit i = m and upper limit i = n. The limits 'm' and 'n' may evaluate to arbitrary expressions. Note that the definite summation over the range m..n can be obtained from the 'indefinite summation' g(i) as the value:

$$g(n+1) - g(m)$$

and this method is used whenever m - n does not evaluate to an integer or m - n is a very large integer; otherwise, direct summation is performed. Note that the value of the definite summation is zero whenever m = n+1.

If Maple is not successful in performing the summation then a FAIL return occurs, meaning that the value of the function invocation is the unevaluated function invocation.

Examples:

```
sum(i**2, i);                          yields    1/3*i**3-1/2*i**2+1/6*i
expand( subs(i=i+1, ") - " );          yields    i**2

e := (5*i - 3)*(2*i + 9);
sum(e, i = 1..5000);        yields     417279137500
sum(e, i = 1..n);           yields     10/3*(n+1)**3+29/2*(n+1)**2-269/6*n+(-107/6)
expand(");                  yields     10/3*n**3+49/2*n**2-35/6*n

sum(i**2 - 2*a*i, i = a..5);           yields
                            55-181/6*a-1/3*a**3+1/2*a**2+2*a*(1/2*a**2-1/2*a)
expand(");                  yields     55-181/6*a-1/2*a**2+2/3*a**3

sum(x**i, i = 0..n);        yields     x**(n+1)/(x+(-1))-(x+(-1))**(-1)
```

7.58. taylor (expr, x = a) or taylor (expr, x = a, n)

The purpose of this function is to compute a Taylor series (more generally, Laurent series) expansion of expr. If the second argument evaluates to an equation then its left-hand-side must evaluate to a name 'x' which will be the variable of expansion and its right-hand-side 'a' will be the point about which the expansion is taken. If the second

argument is not an equation then it must be a name 'x', and the effect is the same as if the second argument had been the equation x = 0 . If there is a third argument 'n' then it must evaluate to an integer which specifies the 'truncation degree' to be used. If there is no third argument then the 'truncation degree' is specified by the current value of the global variable *Degree* (which initially has the value 5 in the Maple system). An 'order term' appears in the result of the *taylor* function whenever the result is not known to be exact. (See section 4.1.8 for a description of the series data structure).

Examples:

```
f := (3*x**2 - 5*x) / (x**3 - x + 7);
taylor(exp(f), x=0);                    yields
  1+ (-5/7)*x+ 57/98*x**2+ (-509/2058)*x**3+ 12841/57624*x**4+
                                              (-18971/134456)*x**5+ O(x**6)
taylor(f, x=1, 2);                      yields
  (-2/7)+ 11/49*(x+ (-1))+ 167/343*(x+ (-1))**2+ O((x+ (-1))**3)

e := (x**2 + a*x - 1) / (a+ 1-x);
taylor(e, x=a, 2);                      yields
  (2*a**2+ (-1))+ (3*a+ 2*a**2+ (-1))*(x-a)+ (3*a+ 2*a**2)*(x-a)**2+ O((x-a)**3)

h := y*exp(y)*sin(x)/x**3 + y*ln(sin(x));
taylor(h, x=0);                         yields
  y*exp(y)*x**(-2)+ (-1/6*y*exp(y)+ y*ln(x))+ (-1/6*y+ 1/120*y*exp(y))*x**2+ O(x**3)

taylor( 1/x + y + x**3, x );            yields    1*x**(-1)+ y+ 1*x**3
taylor( x + x**3 + O(x**2), x );        yields    1*x+ O(x**2)

`diff/g` := proc (a,x) `g'`(a) * diff(a,x) end;
`diff/g'` := proc (a,x) `g''`(a) * diff(a,x) end;
Degree := 2;
taylor( sin(g(x)), x=0 );               yields
  sin(g(0))+ cos(g(0))*g'(0)*x+ (-1/2*sin(g(0))*g'(0)**2+ 1/2*cos(g(0))*g''(0))*x**2+ O(x**3)
```

User Interface: New functions can be made known to Maple's *taylor* function by the following mechanism. If the user assigns a procedure to the name `tayl/newfcn` (where 'newfcn' is any name chosen by the user) as in

```
`tayl/newfcn` := proc (expr, x)
                    taylor(expr, x);
                    # Code to compute taylor series for
                    #          newfcn(expr)
                    # from the taylor expansion of expr
                    # about x = 0 using global variable Degree
                    # to specify the 'truncation degree'.
                    . . .
                    end;
```

then the function invocation

```
taylor ( newfcn(expr), x )
```

will cause the function invocation

```
`tayl/newfcn` ( expr, x ) .
```

In the case of a more general invocation of the *taylor* function:

```
taylor ( newfcn(expr), x = a, n )
```

the internal *taylor* function will perform a transformation of the variable x, and will set the global variable *Degree*, before invoking the `tayl/newfcn` procedure. If `tayl/newfcn` is not assigned then Maple looks for it in the Maple system library at the pathname

```
`` . libname . `tayl/newfcn.m`
```

and if it is not found then the mechanism described below comes into effect.

A second mechanism for making a new function known to Maple's *taylor* function is to define the derivatives of the function via the user interface for the *diff* function. If Maple is not able to find a definition for the name `tayl/newfcn` then it looks for a definition of the name `diff/newfcn` and, if it is found, then the taylor expansion is generated via differentiation and substitution.

Functions whose series expansions are currently defined in the Maple system library include the elementary functions (all of the circular, inverse circular, hyperbolic, and inverse hyperbolic functions, as well as the functions exp and ln), and the factorial function (which interfaces to the GAMMA function known to *diff*).

7.59. trunc (expr)

The value of this function when expr evaluates to a rational number (or an integer) is the 'integer part' of expr which would be obtained if expr was expanded in a decimal expansion. For example, trunc(8/3) is 2 and trunc(–8/3) is –2.

7.60. type (expr, typename)

This is Maple's type-checking function. The value returned is *true* if expr is of type typename and the value returned is *false* otherwise. The following typenames known to the *type* function correspond to Maple's data types which are described in section 4.1:

`.`, `<`, `<=`, `<>`, `+`, `*`, `**`, `!`, `and`, `not`, `or`, equation (alternatively `=`), range (alternatively `..`), function, integer, list, name, procedure, rational, real, series, set.

Additionally, the following typenames are known to the *type* function and they are defined in terms of the basic data types as indicated:

algebraic (any of the following types: `.`, `+`, `*`, `**`, `!`, function, integer, name, rational, real, series)

constant (any of the following types: integer, rational, real, or any expression whose operands are all of type constant)

User Interface: New type-checking procedures can be made known to the *type* function by the following mechanism. If the user assigns a procedure to the name `type/newtype` (where 'newtype' is any name chosen by the user) as in

`type/newtype` := proc (expr, <extra parameters>) . . . end

then the function invocation

type (expr, newtype, <extra parameters>)

will cause the function invocation

`type/newtype` (expr, <extra parameters>) .

If `type/newtype` is not assigned then Maple looks for it in the Maple system library at the pathname

`` . libname . `type/newtype.m`

and if it is not found then an error occurs.

One additional typename is currently defined in the Maple system library: polynom. It can be used in either of the two forms:

type(expr, polynom)
type(expr, polynom, domain)

where in the latter case, the extra parameter 'domain' can take any one of five possible forms:

typename [x, y, . . .]
typename [{ x, y, . . . }]
[x, y, . . .]
{ x, y, . . . }
x

The expression expr is checked as a polynomial in the indeterminate(s) specified by 'domain'. In the case of the first two forms of 'domain', there is an additional check that expr, as a polynomial in the specified indeterminates, has coefficients of type 'typename'. In the case where the argument 'domain' is omitted, the implied value of 'domain' is indets(expr). In all cases, the concept of a 'polynomial' is that expr is not of type 'series'

and the *degree* of expr in each of the indeterminates is finite (i.e., not equal to the largest word-size negative integer).

7.61. unames ( )

This function takes no arguments. It returns an expression sequence consisting of all of the active names in the current Maple session which are *unassigned names*, meaning names which have no value other than their own name. Note that in Maple every 'string' is equivalent to a 'name', so the result of the *unames* function will include every 'string' that has been defined in the session (including file names and error messages). (See also the function *anames*).

7.62. Miscellaneous Library Functions

The following functions reside in the Maple system library but they are not automatically loaded. In other words, the names of these functions are not initially defined in the Maple system. If the user wishes to load one of these functions, named 'fname', then he may use the **read** statement:

```
read `` . libname . `fname.m`
```

or he may use the *readlib* function to initiate an 'automatic loading' facility by specifying the assignment:

```
fname := 'readlib('fname')' .
```

7.62.1. cfrac (f) or cfrac (f, maxit)

```
#
#       Computation of partial fraction convergents.
#       Use:    cfrac( f [,maxit] )
#               Prints the sequence of (maximum maxit) quotients
#               and convergents to f.  f can be rational or real.
#
```

7.62.2. convergents (a, b, n)

```
#
#       convergents: Find (and print) the convergents of a continued
#               fraction.
#
#       The continued fraction
#
#                          b2      b3      b4
#                a1 +     ----    ----    ----    ...
#                         a2 +    a3 +    a4 +
#
#               is entered as:  convergents( a, b, n );
#               where a and b are either lists ([a1,a2,..],[1,b2,..]) or
#               functions which compute the respective coefficients.
#               If all the b coefficients are 1, b can be omitted.
#               n is an optional parameter which indicates the number
#               of convergents to compute.
#
```

7.62.3. E_ML (f, x, n)

```
#
#       Compute an n-th degree Euler-Maclaurin summation formula
#               of f (an expression in x).
#       In general, E_ML(f,x,n) is an asymptotic approximation of
#               sum( f, x ).
#
```

7.62.4. isqrt (n)

```
#
#       isqrt: integer square root of an integer.
#
#               Calling sequence:
#               isqrt(n,x)  if an approximation to the sqrt is known (x);
#               isqrt(n)            otherwise.
#
#                       It returns the closest integer to the
#                               square root of n.
#
```

7.62.5. orthog.p

```
#
#               All the orthogonal polynomials in this package are
#               generated by their recurrences. This seems to be
#               the most efficient procedure.  It is better
#               than the generating function, even if you want
#               all of the polynomials from 1 to n.

#
#       Generate the nth orthogonal Hermite polynomial.
#
#         Calling sequence: H(n,x)

#
#       Generate the nth orthogonal Legendre polynomial.
#
#         Calling sequence: P(n,x)
```

```
#
#       Generate the nth orthogonal Tschebysheff polynomial
#               of the first kind.
#
#         Calling sequence: T(n,x)

#
#       Generate the nth orthogonal Tschebysheff polynomial
#               of the second kind.
#
#         Calling sequence: U(n,x)
```

7.62.6. primtest (n) or primtest (n, iter)

```
#
#       Heuristic primality testing.
#
#       Use:    primtest ( n ) or
#               primtest ( n, iter )
#                       Returns 'false' if it can prove in 10 (or iter)
#                                       iterations or less that the number is not
#                                       prime; returns 'true' otherwise.
#
```

8. MISCELLANEOUS FACILITIES

8.1. Debugging Facilities

The current version of Maple does not have the sophisticated syntax error messages that we envision for Maple in the future. The best mode of operation for detecting syntax errors in procedure definitions is to develop the procedure definition into a file (using a text editor external to Maple) and then to use the **read** statement to read the file into Maple. In this mode, when a syntax error is encountered the corresponding line number in the file is displayed with the syntax error message.

One name whose value determines the amount of information displayed to the user during execution of a Maple session is *yydebug*. The default value for yydebug is 0. If the user assigns the value 1 to yydebug as in the statement

```
yydebug := 1;
```

then the system displays a very large amount of information which is a trace of the Maple session from the basic system viewpoint.

A more useful facility from the user viewpoint is the *printlevel* facility. The default value for printlevel is 1. Any integer may be assigned to the name printlevel and, in general, higher values of printlevel cause more information to be displayed. Negative values indicate that no information is to be displayed. More specifically, there are *levels* of statements recognized within a particular procedure (or in the main session) determined by the nesting of selection and/or repetition statements. If the user assigns

```
printlevel := 0;
```

then the following statements within the main session

```
b :=2;
for i to 5 do a.i := b**i od;
```

will cause the printout b := 2 after execution of the first statement and there will be no printout caused by the for-statement (the value of the for-statement is null). If the user assigns

```
printlevel := 1;
```

before the above statements are executed (or equivalently, if no assignment to printlevel has been made) then each statement within the for-statement will be displayed as it is executed (in the same manner as if these statements appeared sequentially in the 'mainstream'), yielding the following printouts for the above statements:

```
b := 2
a1 := 2
a2 := 4
a3 := 8
a4 := 16
a5 := 32
```

The statement b := 2 is considered to be at level 0 while the other assignment statements in this example are at level 1 because they are nested to one level in a repetition statement. If statements are nested to level i then the value of printlevel must be i if the user wishes to see the results of these statements displayed.

More generally, statements are nested to various levels by the nesting of procedures. The Maple system decrements the value of printlevel by 2 upon each entry into a procedure and increments it by 2 upon exit, so that normally (with printlevel = 1) there is no information displayed from statements within a procedure. If the user assigns

```
printlevel := 2;
```

in the main session then statements within procedures called directly from the main session (but not nested statements) will be displayed as they are executed, because the effective value of printlevel within the procedure is 0. If the user assigns

```
printlevel := 3;
```

in the main session then, in addition, statements nested to one level of selection and/or repetition statements in the procedure will be displayed because the effective value of printlevel within the procedure is 1. Alternatively, the user may explicitly set the value of printlevel within the procedure for which the information is desired.

It is often useful for debugging purposes to set a high value of printlevel in the main session if information is desired from within procedures to various levels of nesting. When the effective value of printlevel upon entry to a procedure is 3 or greater, the printout will display the entry point and exit point for that procedure as well as the values of the arguments at the entry point. It is not uncommon to use a debug setting such as

```
printlevel := 1000;
```

in which case entry and exit points and statements will be displayed for procedures up to 500 levels deep. For more selective debugging information, the value of printlevel should be assigned within specific procedures.

A program called *profile* is available for processing the output produced by Maple with a high setting of printlevel. This program is separate from the Maple system and is available under the same directory where the Maple system resides. It is used in the form:

```
profile <outfile
```

where 'outfile' is a file containing Maple output produced with a high setting of printlevel

(in particular, entry and exit points of Maple functions must be displayed). The output from the *profile* program is a table showing the name of each Maple procedure (including the Main Routine) that was entered, the number of entries to the procedure, and the number of lines (also the number of characters) of output in 'outfile' originating from the procedure. This information can be useful to pinpoint 'bottleneck' procedures which should be candidates for efficiency improvements.

8.2. Monitoring Space and Time

As execution proceeds in a Maple session the user will see lines displayed in the form "words used n" for integer values n. This information indicates the number of *words* of memory that have been requested up to that point in the execution of the session. This information is also displayed at the end of a session when the **quit** statement is executed, where the phrase "Final 'words used'=n" is displayed. It should be noted that this measure of memory usage is not directly related to the actual memory requirements of the Maple session at any point, but rather is a cumulative count of all memory requests made to the internal Maple memory manager during execution of the session. Typically, a significant proportion of the 'words used' at any point may have been re-allocations of actual memory that was previously used and then released to Maple's memory manager.

A second measure of memory requirements is displayed at the end of a session when the **quit** statement is executed, in the form of the phrase "storage=n" for some integer n. This measures the memory space actually occupied by the Maple system plus the data area, and the unit of measurement is the 'natural' unit of memory for the particular host system (e.g., *bytes* on the Vax machine and *words* on the Honeywell machine). Note that Maple's internal memory manager requests 'storage' from the host system in large chunks and then allocates it as needed, so that the final "storage=n" measure typically includes a significant number of memory units that were never actually required by the Maple session.

Monitoring timing information for a Maple session can be accomplished by using the timing facilities of the host system. Typically there is a **time** command on the host system and it is often convenient to use this command along with the host system's facilities for re-direction of input and output. For example, if *infile* denotes the pathname of a file containing the Maple session to be timed and if *outfile* denotes the pathname of the file where the Maple output is to be directed then the UNIX command

```
time /u/maple/bin/maple <infile >outfile
```

or the Honeywell TSS command

```
time : maple/maple <infile >outfile
```

will cause the Maple session in *infile* to be executed, with output into *outfile*, and after completion the host system will display the timing information for that session. Of course, the **time** command may also be used without necessarily using re-direction of input and/or output.

8.3. Other Facilities

Escape Character

The character ! when it appears as the first character in a line is treated as an 'escape to host' operator. This allows one to execute any command in the host system from within a Maple session.

Garbage Collection

Maple's automatic garbage collection facility has not been implemented at the time of this writing. It can be useful to effect a manual garbage collection from the interactive level of Maple by using the sequence

```
save `temp.m`;
quit
```

followed by re-entering the Maple system and then

```
read `temp.m`;
```

This will restore the Maple environment but with all 'garbage' having disappeared.

Wrap Program

There is a program called *wrap* which will insert <newline> characters at appropriate intervals in files containing very long lines of output. (Note that it is not uncommon for Maple to produce very long expressions in its output). This program is necessary on some host systems as a pre-processor before the host system's editor will accept the file for editing. The *wrap* program is separate from the Maple system and is available under the same directory where the Maple system resides (on those host systems where it is required). It is used in one of the following two forms:

```
wrap  <file1  >file2
wrap n  <file1  >file2
```

where 'file1' is the original Maple output file, 'file2' is the file into which the 'wrapped' output will be deposited, and 'n' (if present) specifies the maximum number of characters to be allowed in a line before a <newline> character. (The default value of 'n' is 240). The *wrap* program uses some knowledge about mathematical expressions in attempting to insert the <newline> characters at 'natural' break points (when possible), rather than breaking after exactly 'n' characters.

Load Option

Maple has a *load* option which must be used whenever functions are being loaded into the Maple system library, and which should be used whenever an internal-format ('.m') file is created by a user. This option is activated by specifying '-l' immediately following the 'maple' command. For example, on the Vax UNIX system a typical command for loading a library function named 'f' would be

```
/u/maple/bin/maple -l </u/maple/lib/src/f
```

where the source file for 'f' should end with the statements

```
save `` , libname . `f.m`;
quit
```

In Maple's normal mode of operation (without the *load* option), when an internal-format **save** is done the Maple <name>s which correspond to automatically-loaded library functions (*readlib*-defined functions) are not saved. Therefore, in this normal mode it is impossible to update the '.m' files which define the Maple library functions. The effect of the *load* option is to initiate a Maple system in which none of the library function names is initially defined (and the global variable names *printlevel*, *Digits*, and *Degree* are also undefined). It follows that such a Maple system is of limited value for ordinary use; its sole purpose is for loading '.m' files.

It is recommended that every user should use the *load* option when creating a '.m' file. Otherwise, the *readlib* definition of each Maple library function which is referenced, and also the current values of any of the above-mentioned global variables which are referenced, will be stored in the user's file. This may lead to several undesirable effects: the value of the global variables will be 'mysteriously' redefined when the user's '.m' file is loaded; there may be unwanted re-loading of library functions (which not only is costly but also destroys previously-remembered values for functions with option remember); and, even more seriously, there may arise a circular loop loading and re-loading files!

9. REFERENCES

References

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