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Sequences realized as Parker vectors of oligomorphic permutation groups

Daniele A. Gewurz and Francesca Merola

Dipartimento di Matematica
Università di Roma "La Sapienza"
Piazzale Aldo Moro, 2 – 00185 Roma
Italy
gewurz@mat.uniroma1.it
merola@mat.uniroma1.it

Abstract

The purpose of this paper is to study the Parker vectors (in fact, sequences) of several known classes of oligomorphic groups. The Parker sequence of a group G is the sequence that counts the number of G-orbits on cycles appearing in elements of G. This work was inspired by Cameron's paper on the sequences realized by counting orbits on k-sets and k-tuples.

1 Introduction

In a recent paper [6], P. J. Cameron describes several "classical" sequences (in the sense of appearing in the *Encyclopedia of Integer Sequences* [12]) obtainable as U- or L-sequences of oligomorphic groups, that is as sequences of numbers counting the orbits of such groups on k-subsets and on ordered k-tuples, respectively.

Oligomorphic permutation groups [5] constitute a class of infinite groups to which it is meaningful to extend the concept of Parker vector, originally defined for finite groups (see

[8]). So it is natural to study which integer sequences are obtained as Parker sequence, that is, by counting orbits on k-cycles.

Recall that the *Parker sequence*, or *Parker vector*, of an oligomorphic permutation group G is the sequence $\mathbf{p}(G) = (p_1, p_2, p_3, \dots)$, where p_k is the number of orbits of G on the set of k-cycles appearing in elements of G, with G acting by conjugation. For instance, for the symmetric group S acting on a countable set, the Parker sequence is just $(1, 1, 1, \dots)$. A less trivial example is the group C preserving a circular order on a countable set; for the Parker sequence one has $p_k = \varphi(k)$

Let us fix the notation for some sequences needed in this paper: $\varphi(k)$ is the Euler (totient) function (A000010 in Sloane's *Encyclopedia* [12]), $d(k)$ is the number of divisors of k (A000005), and $\sigma(k)$ is the sum of the divisors of k (A000203).

2 Operators on sequences

Cameron [6] describes how obtaining "new groups from old" (mainly by taking direct and wreath product, and by taking the stabilizer) corresponds to operators on and transforms of their U- and L-sequences (in the sense of Sloane [13]).

Analogously, it is possible to study how the Parker sequences of "new" groups are related to those of "old" ones. The general effect on Parker sequences of taking direct and wreath products of groups is studied in the authors' papers [7] and [8].

Let G and H be permutation groups acting on the sets X and Y, respectively. Recall that, if we consider the direct product $G \times H$ acting on the disjoint union of X and Y, the U-sequence for $G \times H$ is obtained as CONV of the U-sequences of the factors (we are multiplying the ordinary generating functions of the sequences); on the other hand, the L-sequence of the direct product is obtained as EXPCONV (here one considers the exponential generating functions).

For the Parker sequences the corresponding operation is simply the sum (element by element):

$$p_k(G \times H) = p_k(G) + p_k(H).$$

Forming the direct product of G with the countable symmetric group S gives, as U-sequence, PSUM of the L-sequence of G; as L-sequence, BINOMIAL of its L-sequence. For the Parker sequence, it simply yields

$$p_k(G \times S) = p_k(G) + 1.$$

One may also consider the product action of $G \times H$ on the cartesian product $X \times Y$. For this action one has:

$$p_k(G \times H) = \sum_{\substack{i,j \\ \operatorname{lcm}(i,j)=k}} p_i(G) p_j(H).$$

What happens for wreath products is more interesting. Recall [7, 8] that for the Parker sequences of the wreath product of G and H the following holds:

$$p_k(G \wr H) = \sum_{d|k} p_d(G) p_{k/d}(G).$$

This is the Dirichlet convolution, which in the terminology of Sloane [13] is the DIRICHLET transform of the two sequences.

We may now study, for a given oligomorphic group H, the operator mapping the Parker sequence of any group G to that of $G \wr H$. For U-sequences, this procedure gives rise to the operators EULER, INVERT, and CIK, respectively for $H = S$, $H = A$, and $H = C$. For Parker sequences we get, for $H = S$, the MOBIUSi operator

$$p_k(G \wr S) = \sum_{d|k} p_d(G);$$

and, for $H = A$, the identity operator

$$p_k(G \wr A) = p_k(G).$$

For $H = C$ we get

$$p_k(G \wr C) = \sum_{d|k} p_d(G)\varphi(k/d);$$

in particular note that for square-free k's (that is, the values of k such that $\mu(k) \neq 0$) one has $p_k(G \wr C) = \varphi(k) \sum_{d|k} p_d(G)/\varphi(d)$.

Notice that, while in general $G \wr H$ and $H \wr G$ may be different groups, they have the same Parker sequence; so these operators are also those mapping $\mathbf{p}(G)$ to $\mathbf{p}(H \wr G)$.

3 Parker sequences and circulant relational structures

Recall [8] that, if we are dealing with a group G defined as the automorphism group of the limit of a Fraïssé class $\mathcal{F}$ of relational structures, the Parker sequence of G has an alternative interpretation as the sequence enumerating the finite circulant structures in that class. More precisely, the kth component of the Parker sequence counts the relational structures in (the age of) $\mathcal{F}$ on the set $\{1, 2, \ldots, k\}$ admitting as an automorphism the permutation $(1\ 2\ \ldots\ k)$ (note that this is different than just requesting that the structure admits a circular symmetry). In what follows we shall use "circulant [structure]" to mean "circulant [structure] on the set $\{1, 2, \ldots, k\}$ admitting the automorphism $(1\ 2\ \ldots\ k)$". All of the Parker sequences listed in the "Fraïssé class" table were obtained by counting these circulant structures.

This mirrors what happens with the L-sequence (F_k) of the same group, which is defined as the number of orbits on k-tuples of distinct elements, and is equal to the number of labelled structures on k points. The same holds for the U-sequence f_k of the number of orbits on k-sets, giving the number of unlabelled structures. The theory behind this can be found in Cameron's book [5].

In order to give an idea of the techniques involved in deriving Parker sequences, let us first briefly recall [8] what happens for graphs.

To describe a circulant graph Γ on the vertex set $\{0, 1, 2, \ldots, k-1\}$, it is sufficient to give the neighbours of a fixed vertex (say 0); this subset, which has the property that it contains a vertex i if and only if it contains $k - i$, is called *symbol* of Γ. On the other hand any subset

S of $\{1, 2, \dots, k-1\}$ such that $i \in S$ implies $k - i \in S$ is a possible symbol for a graph. So the kth entry of the Parker sequence of the automorphism group of the limit of the Fraïssé class of graphs (that is the well-known random, or Erdős-Rényi, or Rado, graph) is $2^{\lfloor k/2 \rfloor}$.

Several variations to this method yield the Parker sequences for other relational structures.

For instance, if we consider the symbol for a digraph (a structure with a relation $\rightarrow$ in which for each pair of distinct vertices a, b, any of $a \rightarrow b$, $b \rightarrow a$, both, or none may hold) we choose whether or not to join, by putting a directed edge, 0 with any other vertex. So we get $p_k = 4^{(k-1)/2} = 2^{k-1}$. Similarly, if we do not allow a double orientation on an edge, we get the class of oriented graphs, for which $p_k = 3^{\lfloor k/2 \rfloor}$.

Of course, this kind of argument holds also for the class of n-ary relations, for $n \geq 2$. The symbol for a circulant n-relation on k points can be any possible set of $(n-1)$-tuples (admitting repetitions) of the points. For instance, for a ternary relation, we may have $(0, 0)$ (meaning that $(0, 0, 0)$ holds), $(0, 1)$, $(1, 0)$, $(1, 1)$, ... So we have k^{n-1} such $(n-1)$-tuples, and $2^{k^{n-1}}$ possible symbols (sets of such tuples).

More examples in same vein appear in the tables.

The same techniques can be applied to the class of two-graphs; this case, however, requires some care.

Recall that a *two-graph* is defined as a pair (X, T), where X is a set of points, and T a set of 3-subsets of X with the property that any 4-subset of X contains an even number of members of T.

Two-graphs on k vertices are in bijection with switching classes of graphs on k vertices. Recall that switching a graph $\Gamma = (V, E)$ with respect to $S \subseteq V$ gives a graph (V, E') such that $\{v, w\} \in E'$ if and only if either v and w are both in S or both in $V \setminus S$ and $\{v, w\} \in E$, or one is in S and the other is in $V \setminus S$, and $\{v, w\} \notin E$ (see [11], also for the description of the correspondence between two-graphs and switching classes).

Note that a two-graph (X, T) is circulant if and only if at least one graph in the corresponding switching class is. In fact, assume that α is a permutation of X inducing an automorphism of (X, T); then α induces an automorphism of at least one graph in the corresponding switching class (as proved by Mallows and Sloane [10]; see also Cameron [2]).

The following result relates circulant two-graphs to circulant graphs.

Theorem 3.1 *Let Γ be a circulant k-vertex graph. If k is odd, then Γ is the only circulant graph in its switching class; if k is even, there are exactly two circulant graphs in its switching class.*

In order to prove this, let us first show in some detail what happens switching circulant and regular graphs.

Proposition 3.2 *For k odd, in each switching class of graphs on k vertices there is at most one regular graph.*

Proof. Let Γ be a regular graph of valency r on k vertices. Let us switch it with respect to the set $S \subseteq V(\Gamma)$, $0 < |S| = m < k$. Then, for each $t \notin S$, call n_t the number of neighbours of t included in S (before switching). Then the valency of t in the switched graph is $r - n_t + (m - n_t)$. Analogously, if $s \in S$ and n_s is the number of neighbours of s not in S, the valency of s in the switched graph is $r - n_s + (k - m - n_s)$.

Therefore, if the switched graph is regular, given two vertices s and t as above, their new valencies must be equal:

$$r - n_t + (m - n_t) = r - n_s + (k - m - n_s),$$

or,

$$k = 2(m - n_t + n_s).$$

That is, the number of vertices must be even for a non-trivial switching equivalence to hold between Γ and another regular graph. $\diamondsuit$

We have now the first part of the theorem (because any circulant graph must be, *a fortiori*, regular). For the second part, the following proposition describes explicitely when switching a circulant graph yields another circulant graph.

Proposition 3.3 *If Γ is a circulant graph on the vertices $\{1, 2, \ldots, k\}$, k even, the only non-trivial switching yielding a circulant graph is with respect to the set of vertices $S = \{1, 3, 5, \ldots, k-1\}$ (or its complement).*

Proof. For Γ to be circulant, it must be possible to decompose it in cycles $(i, i+l, i+2l, \ldots, i-l)$ (all additions modulo k). In each such cycle the vertices either have all the same parity, or an odd and an even vertex alternate. So, switching with respect to S either preserves the whole cycle, or causes all its edges to vanish. In either case, the graph remains circulant.

On the other hand, if switching is performed with respect to any other non-trivial set S', this set or its complement must include two consecutive vertices i, $i+1$ (mod k) and of course there exists j such that $j \in S'$, $j+1 \notin S'$. In a circulant graph either $1 \sim 2 \sim \cdots \sim k \sim 1$ or $1 \not\sim 2 \not\sim \cdots \not\sim k \not\sim 1$; assume, up to complementing, the former. Then in the switched graph $i \sim i+1$ while $j \not\sim j+1$; so the new graph is not circulant. $\diamondsuit$

A variation of the previous argument shows that the same holds for oriented two-graphs.

4 Groups and their sequences

In this section we consider the tables included in Cameron's paper [6] and add, as far as possible, the data concerning Parker sequences.

For the five closed highly homogeneous groups of Cameron's theorem (i.e., the groups admitting only one orbit on k-sets for all k; see [1]) the Parker sequences are readily obtained. Recall that S is the infinite symmetric group, A (or ∂C) is the subgroup of S of the permutations preserving the ordering on the rational numbers, B (or ∂C^*) of those preserving or reversing it, C of those preserving a cyclic order on a countable set (say, the complex roots of unity), and D (or C^*) of those preserving or reversing such a cyclic order.

The Parker sequence for S is clearly the all-1 sequence; while in the finite case this property characterises (with a single exception) the symmetric groups, in the infinite case this sequence is shared by other, not highly transitive groups. An instance of this fact is the group of the Fraïssé class of trees with the action on edges.

The Parker sequence for A is unremarkable, but for its being the neutral element for the Dirichlet convolution. So, for each group G, $\mathbf{p}(A \wr G) = \mathbf{p}(G \wr A) = \mathbf{p}(G)$.

The sequences for C and D can be obtained by noting that these groups induce on k-sets the groups C_k and D_k (dihedral of degree k), respectively; see also [8].

Highly Homogeneous Groups

Group	**Parker sequence**	**EIS entry**	**Notes**
S	1, 1, 1, ...	A000012	
A	1, 0, 0, ...	A000007	
B	1, 1, 0, 0, ...	A019590	
C	$\varphi(k)$	A000010	
D	1, 1, 1, $\varphi(k)/2$	∼A023022	

Direct Products

Group	**Parker sequence**	**EIS entry**	**Notes**
$S \times S$	2, 2, 2, ...	A007395	
$S \times A$	2, 1, 1, ...	A054977	
$A \times A$	2, 0, 0, ...	A000038	
S^3	3, 3, 3, ...	A010701	
S^k	k, k, k, ...		

In the following table, S_n denotes the (finite) symmetric group of degree n, and E is the trivial group acting on two points.

Note also that A00005 = MOBIUSi(A000012), A007425 = MOBIUSi(A000005).

Wreath Products

Group	**Parker sequence**	**EIS entry**	**Notes**
$S \wr S$	$d(k)$	A000005	
$A \wr S$	1, 1, 1, ...	A000012	
$C \wr S$	k $(= \sum_{d\mid k} \varphi(d))$	A000027	
$(C \wr S) \wr S$	$\sum_{d\mid k} d = \sigma(k)$	A000203	
$S \wr A$	1, 1, 1, ...	A000012	
$S \wr S_2$, $S_2 \wr S$	1, 2, 1, 2, ...	A000034	
$S \wr S_3$, $S_3 \wr S$	1, 2, 2, 2, 1, 3, 1, 2, 2, 2, 1, 3, ...	A083039	See $S \wr S_n$
$S \wr S_4$, $S_4 \wr S$	1, 2, 2, 3, 1, 3, 1, 3, 2, 2, 1, 4, ...	A083040	See $S \wr S_n$
$S \wr S_n$, $S_n \wr S$	$p_k = \lvert\{d : d\mid k,\ d \le n\}\rvert$		See remark 1
$S \wr S \wr S$	$\sum_{d_0\mid k} d(d_0) = 1, 3, 3, 6, 3, 9, 3, 10, 6, 9, 3, \ldots$	A007425	
$A \wr A$	1, 0, 0, ...	A000007	
$S_k \wr A$	1, ... (k times) ..., 1, 0, 0, ...		
$E \wr S$	2, 2, 2, ...	A007395	
$E \wr A$	2, 0, 0, ...	A000038	
$S^{\wr n}$	$\sum_{d_0\mid k} \sum_{d_1\mid d_0} \sum_{d_2\mid d_1} \cdots \sum_{d_{n-3}\mid d_{n-4}} d(d_{n-3})$	MOBIUSi^n(A000005)	See remark 2
$C \wr C$	$\sum_{d\mid k} \varphi(d)\varphi(k/d) = 1, 2, 4, 5, 8, 8, 12, 12, 16, 16, \ldots$	A029935	See remark 3

Remark 1 The sequence is periodic of period $\mathrm{lcm}(1,\dots,n)$.

Remark 2 The sequence associated with $S^{\wr n}$ (i.e., the iterated wreath product of S with itself with n factors) can be expressed as follows. Let $\delta_0(k) := 1$ for each k, and for $i > 0$ let

$$\delta_i(k) := \sum_{d|k} \delta_{i-1}(d),$$

that is, δ_i is the Dirichlet convolution $\delta_{i-1} * \delta_0$. Thus, $\delta_i(k) = p_k(S^{\wr i+1})$.

All the functions δ_i are multiplicative, because δ_0 is, and the Dirichlet convolution preserves multiplicativity. Thus, it suffices to compute the value of δ_i on prime powers.

We claim that

$$\delta_i(p^j) = \binom{i+j}{i}.$$

To obtain a different description of the δ_is, note that $\delta_1(k)$ gives the number of divisors of k, including 1 and k; so it is equal to $d(k)$. Next, $\delta_2(k)$ is the sum over the divisors of k of the number of their divisors; in other words, it gives the number of pairs (h, d) with $h|d$ and $d|k$ (observe that h and d may well coincide). In general, we see that $\delta_i(k)$ gives the number of i-ples $(d_1, d_2, \dots, d_i)$ with $d_1|d_2$, $\dots$, $d_{i-1}|d_i$, $d_i|k$. We call such a sequence a *generalised gozinta chain*, recalling that a gozinta ("goes into") chain for k is a sequence of divisors of k each of which strictly divides the next one.

When $k = p^j$, a sequence of divisors of k each of which divides the next one corresponds to a nondecreasing sequence of exponents of p, that is to a nondecreasing sequence of numbers in $[j] = \{0, 1, \dots, j\}$, which in turn can be seen as a multiset of elements of $[j]$.

So, it is enough to enumerate the multisubsets of $\{0, 1, \dots, j\}$ of size i. It is well known (see for instance [14]) that their number is given by $\binom{i+j}{i}$, as claimed.

Remark 3 If k is square-free, p_k is equal to $\sum_{d|k} \varphi(k) = d(k)\varphi(k)$.

* * *

The following groups arise as automorphism groups of Fraïssé classes (see section 3).

The calculation of Parker sequences for "treelike objects" and related structures is carried out in detail in the forthcoming paper [9].

The letters R and L mean "shifted right" and "shifted left" respectively.

Automorphism Groups of Homogeneous Structures

Fraïssé class	**Parker sequence**	**EIS entry**	**Notes**
Graphs	$2^{\lfloor k/2\rfloor}$	A016116	See [8]
Graphs up to complement	$p_1 = 1$, $p_k = 2^{\lfloor k/2\rfloor -1}$ for $k > 1$	A016116RR	See rem. 4
K_3-free graphs	1,2,1,3,3,4,4,8,4,14,11,14,...	A083041	See rem. 5
Graphs with bipartite block	2,2,2,...	A007395	See rem. 6
Graphs with loops	$2^{\lfloor k/2\rfloor +1}$	A016116LL	See rem. 7
Digraphs	$2^{k-1} (= 4^{(k-1)/2})$	A000079R	
Digraphs with loops (or binary relations)	2^k	A000079	
Oriented graphs	$3^{\lfloor k/2\rfloor}$	[missing]	
Topologies	$d(k)$	A000005	See rem. 8
Posets	1,1,1,...	A000012	See rem. 9
Tournaments	k odd: $2^{\lfloor k/2\rfloor}$, k even: 0	[missing]	See [8]
Local orders	k odd: $\varphi(k)$, k even: 0	[missing]	See [9]
Two-graphs	$2^{\lceil k/2\rceil}$	A016116L	See Thm. 3.1
Oriented two-graphs	$2^{\lceil k/2\rceil}$	A016116L	See Thm. 3.1
Total orders with subset	2,0,0,...	A000038	
Total orders with 2-partition	1,0,0,...	A000007	
C-structures with subset	$2\varphi(k)$	[missing]	See rem. 10
D-structures with subset	$\varphi(k)$	A000010	See rem. 10
2 total orders (distinguished)	1,0,0,...	A000007	
2 total orders (not distinguished)	1,1,0,0,...	A019590	
2 betweennesses (not distinguished)	1,1,0,0,...	A019590	
Boron trees (leaves) (or T_3)	characteristic fn. of $\{3^a 2^b\}_{a\in\{0,1\},b\geq 0}$	[missing]	See [9]
HI trees (leaves) (or T)	nr. of ordered factorisations of k	A002033R	See [9]
R(Boron trees (leaves)) (or ∂T_3)	characteristic fn. of powers of 2	A036987	See [9]
R(HI trees (leaves)) (or ∂T)	nr. of ordered factorisations of k	A002033R	See [9]
Trees (edges)	1,1,1,...	A000012	See [9]
Covington structures (or $\partial T_3(2)$)	$p_{2^i} = 2^i$, 0 otherwise	A048298	See [9]
Binary trees (or ∂PT_3)	1,0,0,0,...	A000007	See [9]
Binary trees up to reflection (or $\partial P^* T_3$)	1,1,0,0,...	A019590	See [9]
Plane trees (or PT)†	$\varphi(k)$	A000010	See [9]
Plane trees up to reflection (or P^*T)†	$\sim \varphi(k)/2$	A023022	See [9]
Plane boron trees (or PT_3)	1,1,2,0,0,...	[missing]	See [9]
Plane boron trees up to reflection (or P^*T_3)	1,1,1,0,0,...	[missing]	See [9]
3-hypergraphs	$2^{f(k,3)}$, where $f(k,3) = 0,0,1,1,4,4,5,7,10,12,15,19,\ldots$	[missing]	See [8]
t-hypergraphs†	$2^{f(k,t)}$		See [8]
Ternary relations	2^{k^2}	A002416	
Quaternary relations	2^{k^3}	[missing]	

† Not in [6].

Remark 4 Each (symbol for a) circulant graph represents also its complement, so (for $k > 1$) each term is one half of the corresponding term for graphs. For instance, $p_2 = 1$ because the graphs K_2 and N_2 are now identified.

Remark 5 This is the number of symmetric sum-free subsets of $\mathbf{Z}/(k)^*$ (see [3]): if the symbol contains a and b, it cannot contain $a + b$, and (as for generic graphs) if it contains a, it must contain $k - a$.

Remark 6 We cannot exchange "black" and "white" vertices, so a circulant structure is an all-black or all-white null graph.

Remark 7 Reason as in section 3, but take in addition to "basic" circulant graphs (those with symbol of the form $\{i, k - i\}$) also that with k vertices, each with a loop attached, and no other edges. In other words, in the symbol (set of "neighbours" of 0) for a circulant graph with loops, also 0 may appear.

Remark 8 The "basic" graphs do not work as they are; the request for the relation to be transitive forces any k-gon to "become" a complete directed graph (that is, K_k where all edges are bidirected): by transitivity, connect vertices at distance 2, then at distance 3 and so on. The superposition of d copies of $K_{k/d}$ and l copies of $K_{k/l}$ becomes by transitivity the superposition of $\mathrm{GCD}(d, l)$ copies of $K_{\mathrm{lcm}(k/d,k/l)}$. So the lattice of divisors of k describes all the possible circulant transitive digraphs, that is topologies.

In other words, a topology is the transitive closure of a union of cyclic graphs; its incidence matrix can be seen as the kth power of the incidence matrix of the starting graph with, as its entries, boolean variables 0 and 1 (so that $1 + 1 = 1$).

Remark 9 By acyclicity, for each n the only circulant poset is the one with n incomparable elements.

Remark 10 The only possible distinguished sets are the empty and the full ones.

One Last Example

Group	**Parker sequence**	**EIS entry**	**Notes**
S^2 (product action)	$d(k^2)$	A048691	See rem. 11

Remark 11 The result follows from Section 2, keeping in mind that $d(k^2)$ is equal to the number of pairs (i, j) such that $\mathrm{lcm}(i, j) = k$.

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