Journal of Integer Sequences, Vol. 5 (2002),
Article 02.2.6

Combinatorial Enumeration of Ragas (Scales of Integer Sequences) of Indian Music

K. Balasubramanian

Department of Applied Science
University of California Davis,
L-794 Livermore, CA 94550
Chemistry & Material Science Directorate,
Lawrence Livermore National Laboratory,
Livermore, CA 94550
Glenn T. Seaborg Center,
Lawrence Berkeley National Laboratory,
Berkeley CA 94720
e-mail: kbala@ucdavis.edu

Abstract

It is shown that a combinatorial method based on the principle of inclusion and exclusion (Sieve formula) yields generating functions for the enumeration of integer sequences chosen from 12 musical tones for the ragas (scales) of the Indian music system. Mathematical and computational schemes are presented for the enumeration and construction of integer [*]sequences pertinent to Indian music.

---

[*] Dedicated to Prof V. Krishnamurthy, my mentor and role model of Birla Intstitute of Tech & Sci, India

I. Introduction

Ragas or scales of musical notes with a characteristic pattern of ascent and descent constitute the basic melody of Indian music system [1]. There are two schools of Indian music system, the north Indian Hindustani music system and the south Indian or Carnatic music system. The sequence of notes in all music systems, also known as a chord, can be mapped into integral sequences, and the sequence on the ascent in the Indian music system is called an arohan (or arohanam), while the corresponding sequence on the descent is called the avarohan (or avarohanam). Each raga in the Indian music system is comprised of a unique sequence of notes in the ascent and descent that determines the characteristic of the raga and the musical forms and compositions that originate from the raga. In general, Carnatic music compositions and other forms of musical improvisations must contain the notes defined in the scale of the raga with the exception that for ornamentation and grace, other notes and microtones (notes with frequencies that lie between the frequencies of the 12-tone music system) may be added as in "gamakas"(distantly analogous to vibrato of western music) of the south Indian music system. The intimate connection between music and combinatorics has been a subject of several studies, for example, Babbitt's partition problems in the 12-tone western music compositions [2-4].

There are certain grammatical rules that govern the construction of ragas with the definition of Sa (C in western) and Pa (G in western) that form the basic reference point or "sruthi". The arohan or the avarohan of a raga should generally contain at least 4 notes. Although tertachord ragas are not very common, they do exist, as exemplified by the raga "Mahathi" [1]. The common forms of raga scales are pentatonic, known as the "audava" scales, that is, those containing five notes including the "Sa" (or C in western music), hexatonic, called the "shadava" and heptatonic or the complete (octave completed with static Ŝ included) scale called the "sampurna". A raga's ascent and descent can have a number of combinations of scales chosen from the eleven notes (12 with the upper-octave Ŝ) from the 12-tone system enumerated in Table 1 forming an integer sequence. If the scale is uniform in both ascent and descent without any repetition of a note then the raga is considered "non-kinky" or referred to as a "non-vakra"(vakra is a Sanskrit word meaning kinky) raga [1]. For example, a raga can be pentatonic in ascent and hexatonic in descent. It is then referred to as an audava-shadava raga. We consider here enumeration of only non-vakra (non-kinky) ragas.

Scales of non-vakra ragas in south Indian music system are constructed by choosing the eleven notes (not counting the upper C or Sa, denoted as $\hat{S}$) in Table I so to form the various combinations of scales with uniformly rising frequency in the ascent and decreasing frequency in the descent. If the 11 notes are mapped into integers then in combinatorial terms this corresponds to the enumeration of integer sequences under constraints and equivalences as stipulated by the theory of south Indian music. The objective of this article is to construct the mathematical foundation for such an exhaustive and yet non-repetitive enumeration and generating functions for the ragas of different kinds of scales. This rigorous and exhaustive enumeration scheme provides a basis for the formulation of new ragas that are not known up to now in the south Indian music system or Carnatic music [1]. A computer code is also developed for the construction of such ragas.

2. Combinatorics of integer Sequences of ragas

In mathematical notation the notes in Table I are denoted as S, R_1, R_2, R_3,…..N_3, the last being $\hat{S}$, and has an octavial relation to S. These notes have characteristic rational number relations to the base frequency of S. The arohan or avarohan should contain a combination of notes in Table I according to whether the scale is "pentatonic", "hexatonic" and so on. There are a few restrictions and equivalences. The notes should appear as a sequence of increasing frequencies (in the order shown in table I) with the restriction that only one kind of R or G or M or D or N may appear in a scale, and certain notes are considered equivalent. The notes G_1 and R_2 are equivalent. Likewise the notes G_2 and R_3 are equivalent. The notes N_1 and D_2 are equivalent, while the notes N_2 and D_3 are equivalent. Thus the enumeration of all possible "non-kinky" ragas of Carnatic music system becomes enumerating integral sequences of a prescribed length under equivalence constraints. The Carnatic music system thus uses two different names for the same note. The main advantage is that certain combination becomes allowed when different names are used for the same note, for example, the combination R_1-G_1 or suddha Ri and suddha Ga becomes allowed under this convention [1] in the Carnatic music, but the same combination which becomes komal Ri-Sudh Ri in the north Indian Hindustani music system is forbidden. The use of multiple names for the same note is not unique to Carnatic music, as it is also the case with western music, as can be seen from Table I. For example, E double flat is the same note as D natural, E flat is same as D sharp and so on (Table I).

The enumeration of patterns under equivalences or "equivalence classes" can be formulated by the well-known Polya's theorem and there are many chemical and spectroscopic applications of Polya's theorem [5-7]. However, for the present purpose, since the enumeration often involves integer sequences with certain combinations forbidden due to equivalence restrictions, we find the principle of inclusion and exclusion or the sieve formula [8-10] to be a more convenient choice for the enumeration. This corresponds to enumerating integer sequences with increasing order on the ascent (or decreasing order on the descent) such that certain sequences are forbidden. There are many such applications of enumerative combinatorics [8-12] such as derangements or the problem of Ménage or the Euler function, which generates the number of primes to any integer n and less than n, and also the Reimann-Zeta function [12] related to the prime number distributions rediscovered by Srinivasa Ramanujan [13]. It should also be noted that there is considerable interest in combinatorial problems in western music theory [2-4] known as "Babbitt's partition problems" in 12-tone musical compositions. One of the Babbitt's partition problem asks for an algorithm for determining all mxm matrices with entries drawn from the set {1, 2, ... ,n} for which all rows and columns have the sum n [4].

Let $P_1, P_2, P_3, \ldots P_n$ be a set of n constraints stipulated by the south Indian music theory. Then the generating function F for the enumeration such that none of the constraints $P_1, P_2, P_3, \ldots P_n$ is satisfied is given by the Sieve formula

$$F = f(0) - f(1) + f(2) - f(3) + \ldots\ldots + (-1)^i f(i) + \ldots(-1)^n f(n),$$

where f (i) denotes the generating function for the enumeration that satisfies exactly i of the properties $P_1, P_2, \ldots\ldots P_n$.

The constraints $P_1, P_2 \ldots P_n$ can be constructed for the enumeration of ragas as P_1 being that the sequence of notes R_2 and G_1 occurs (forbidden due to equivalence), P_2: notes R_3 and G_2 are in a sequence (forbidden sequence), P_3: notes R_3 and G_1(forbidden), P_4: D_2 and N_1, P_5: D_3 and N_2, P_6: D_3 and N_1. P_3 and P_6 are forbidden by symmetry in that the R_2-G_2 combination is equivalent to the R_3-G_1 combination. The D_2-N_2 combination is equivalent to D_3-N_1. Thus we are enumerating "patterns" of integer sequences or "equivalence classes".

To illustrate, the number of symmetrical "heptatonic-heptatonic" also known as "sampurna-sampurna" ragas in the Carnatic music system (or the "melakarta (creator) ragas") [1], the numbers *f (0), f (1), f (2)…. f (6)* are obtained as

$$f(0) = \binom{3}{1}\binom{3}{1}\binom{2}{1}\binom{3}{1}\binom{3}{1} = 162$$

$$f(1) = \binom{2}{1}\binom{3}{1}\binom{3}{1} \times 6 = 108$$

$$f(2) = 9 \times \binom{2}{1} = 18$$

$$f(3) = f(4) = f(5) = f(6) = 0$$

Thus F is given by

$$F = f(0) - f(1) + f(2) - f(3) + f(4) - f(5) + f(6)$$

$$= 162 - 108 + 18 = 72$$

The above enumeration is a straight forward application since all notes occur in a heptatonic sequence, that is, S, R, G, M, P, D, and N, occur and the constraint that only one note form a given type such as R or G may be chosen makes it easier to enumerate these sequences.

3. Generating Functions for ragas

The enumerations of different types of scales such as audava (ascent)-audava (descent), audava (ascent)-shadava (descent) etc., can be accomplished utilizing powerful enumerative combinatorial functions. We shall construct a "pattern inventory" in Polya's term [5-7] of all such "non-vakra" or non-kinky ragas of Carnatic music system. We construct a generating function for the ascent and multiply the corresponding generating function for the descent to get the complete pattern inventory of ragas. The ascent GF is constructed by enumerative combinatorics. The maximal chord length allowed is 7 in a sampurna non-vakra (non-kinky heptatonic) type, and that has already been enumerated. The hexatonic (shadava) arohans are enumerated as follows. First the patterns are enumerated for the hexatonic scales as shown in Table II and then the numbers for each pattern. A hexatonic pattern such as S G M P D N Ŝ can be mathematically characterized as $\overline{R}$, since it is missing R (known as rishaba vajra[1]) from a complete heptatonic scale. Thus there are six patterns characterized by $\overline{R}$, $\overline{G}$, $\overline{M}$, $\overline{P}$, $\overline{D}$, and $\overline{N}$. Note that S cannot be missing from a raga as it forms the base (C). The equivalence classes of ragas in each such pattern are enumerated using the Sieve formula [8-10] and thus we have the hexatonic ascent (arohan) generating function as

$$H^a = (12\overline{R} + 36\overline{G} + 36\overline{M} + 72\overline{P} + 12\overline{D} + 36\overline{N}),$$

where, for example, there are 12 hexatonic scales missing R, 36 missing G and so on. In this enumeration scheme the equivalence of notes has been considered, and thus all combinations are allowed in $\overline{G}$, while only non-equivalent ones are considered in $\overline{R}$. The total number of hexatonic arohans is obtained by substituting

$$\overline{R} = 1, \overline{G} = 1, \ldots \overline{N} = 1$$

in the above expression which yields 204 hexatonic arohans. This also corresponds to the number of symmetric hexatonic-hexatonic or the "shadava-shadava" ragas.

The pentatonic scales are those that have two missing notes relative to the heptatonic scales and are thus denoted in mathematical terms by binomials such as $\overline{R}\,\overline{G}$, $\overline{R}\,\overline{M}$, etc., as enumerated in Table III. The patterns are shown in Table III, and the generating function for the pentatonic ascent is given by

$$P^a = (12\overline{R}\overline{G} + 6\overline{R}\overline{M} + 12\overline{R}\overline{P} + 2\overline{R}\overline{D} + 6\overline{R}\overline{N} + 18\overline{G}\overline{M} + 36\overline{G}\overline{P} + 6\overline{G}\overline{D} + 18\overline{G}\overline{N} + 36\overline{M}\,\overline{P} + 6\overline{M}\,\overline{D} + 18\overline{M}\,\overline{N} + 12\overline{P}\overline{D} + 36\overline{P}\overline{N} + 12\overline{D}\overline{N})$$

In the above enumeration scheme the equivalence of notes has been considered and thus the combinations with $\overline{R}\,\overline{M}$ have fewer numbers than $\overline{G}\,\overline{M}$, since they have been already enumerated in the binomial $\overline{G}\,\overline{M}$ they are not duplicated in $\overline{R}\,\overline{M}$, due to symmetry equivalence. Replacing all binomial terms by 1 or equivalently summing the coefficients gives the total number of symmetric pentatonic ragas or pentatonic ascents as 236.

Although ragas with tetratonic (tertachord) scales are rare, they do occur as illustrated before, and thus they are enumerated here for completeness. Such enumerations can also be useful in computer synthesis of musical tertachord compositions, wherein a sequence of four notes is required. The tetratonic GF is given by

$$T^a = (6\overline{RGM} + 12\overline{RGP} + 2\overline{RGD} + 6\overline{RGN} + 6\overline{RMP} + \overline{RMD} + 3\overline{RMN} + 2\overline{RPD} + 6\overline{RPN} + 2\overline{RDN} + 18\overline{GMP} + + 3\overline{GMD} + 9\overline{GMN} + 6\overline{GPD} + 18\overline{GPN} + 6\overline{GDN} + 6\overline{MPD} + 18\overline{MPN} + 6\overline{MDN} + 6\overline{PDN})$$

Again in the above enumeration all possibilities are allowed for R and D, but the ones with G and N, only non-equivalent types are enumerated by way of inclusion-exclusion to eliminate equivalent combinations. Thus the total number of tetratonic scales, also referred to in music theory as tetrachords (sequence of 4 notes), is obtained by substituting all trinomials in T^a by 1 or summing the coefficients in T^a. Thus the number of tertachords or tetratonic ascents is 142.

The trichords (triplets) or sequences of three notes, one of which is S, are enumerated by the expression for Tr^a.

$$Tr^a = (6RG + 6RM + 3RP + 9RD + 3RN + 2GM + GP + 3GD + GN + 2MP + 6MD + 2MN + 3PD + PN + 6DN),$$

where in the above expression instead of complementary notation, the notes themselves are used for the binomials, for example, RG to denote the sequence SRG in the trichord. Thus the total number of trichords is obtained by adding the coefficients in TR^a, which equals 54. The number of dichords or a sequence of 2 notes, one of which has to be S, is simply 11 since that is the number of distinct notes in Table I (note Ŝ is related to S by an octave).

All of the above expressions can be combined into a pattern inventory of arohans of ragas that we refer to as a raga ascent inventory, RI^a, given as a polynomial in x, where x^n denotes the term for n-tonic ascent.

$$RI^a = 1 + x + 11x^2 + 54x^3 + 142x^4 + 236x^5 + 204x^6 + 72x^7 \ ,$$

where the first term is a trivial null set, the second term corresponds to a single note or just S, the x^2 term representing the number of dichords, x^3: the number of trichords, x^4: number of tetrachords etc. For a raga to be stable its scale must have at least a tetrachord, and thus terms with powers more than or equal to 4 are relevant for the scales of ragas.

The Raga inventory for the descent (avarohan) is likewise enumerated by the generating function RI^d given by

$$RI^d = 1 + y + 11y^2 + 54y^3 + 142y^4 + 236y^5 + 204y^6 + 72y^7 \ ,$$

where the symbol y is used to distinguish the descent from the ascent to allow for the possibility of unsymmetrical and bhashanka ragas [1]. The total generating function for all of the ragas is given by the product of the ascent and descent inventories or

$$RI = RI^a \times RI^d = (1 + x + 11x^2 + 54x^3 + 142x^4 + 236x^5 + 204x^6 + 72x^7)\times$$
$$(1 + y + 11y^2 + 54y^3 + 142y^4 + 236y^5 + 204y^6 + 72y^7),$$

The coefficient of $x^m y^n$ in the above generating function enumerates the number of ragas with m-tonic (m-chord) notes in ascent omitting higher octave $\hat{S}$ and n-tonic notes (n-chord) in the descent. For example, the number of shadava-sampurna ragas is given by the coefficient of x^6y^7 in the above expression, which is 14688. The number of symmetrical tetrachords is the coefficient of x^4 which is 142 and the total number of all tetratonic ragas is the coefficient of x^4y^4, which is 20164. All of the symmetrical ragas are enumerated by the terms x^5, x^6 and x^7 for the pentatonic (audava), hexatonic (shadava) and heptatonic (sampurna) scales, respectively. The number of ragas with at least pentatonic scales in the ascent or descent is enumerated in Table IV. It should be mentioned that Pattamal [14] has proposed a scientific naming scheme for some of the ragas, and the numbers obtained before are not rigorously correct [15] as these empirical methods either missed some of the combinations or those methods do not fully consider equivalence restrictions. As mentioned in ref [15] also, earlier counting schemes also suffered from duplication. In the present scheme, we have carefully provided a mathematical framework within combinatorial principles that stipulates equivalence, symmetry and other restrictions and it is yet exhaustive as the polynomial inventory rigorously considers all of the combinations.

More detailed combinatorial generating functions can be constructed by considering the generating functions, H^a, P^a, T^a and Tr^a. For example detailed enumeration for the hexatonic (shadava)-pentatonic (audava) ragas is given by

$$H^aP^d = H^aP^d = [12\overline{R} + 36\overline{G} + 36\overline{M} + 72\overline{P} + 12\overline{D} + 36\overline{N}]$$
$$\times[12\overline{R'}\overline{G'} + 6\overline{R'}\overline{M'} + 12\overline{R'}\overline{P'} + 2\overline{R'}\overline{D'} + 6\overline{R'}\overline{N'} + 18\overline{G'}\overline{M'} + 36\overline{G'}\overline{P'} +$$
$$6\overline{G'}\overline{D'} + 18\overline{G'}\overline{N'} + 36\overline{M'}\overline{P'} + 6\overline{M'}\overline{D'} + 18\overline{M'}\overline{N'} + 12\overline{P'}\overline{D'}$$
$$+ 36\overline{P'}\overline{N'} + 12\overline{D'}\overline{N'}]$$

The above expression enumerates all combinations of hexatonic-pentatonic ragas. For example, the number of ragas missing R in the ascent and missing G and P in the descent is given by the coefficient of $\overline{R}\,\overline{G'}\overline{P'}$, which is 432. Consequently, combining different ascent generating functions with different descent generating functions, all ragas, both symmetrical and unsymmetrical are enumerated. The generating function contains all symmetrical and unsymmetrical tetrachords, trichords, bichords, etc.

Tables II and III contain the detailed enumerations for the most common ragas of different types. Since the number of heptatchords is 72, the number of heptatonics with any combination is obtained by multiplying the corresponding GF by 72.

We have also developed a computer code to construct all ragas (scales) of a given type. Table V illustrates the computer construction of 1296 hexatonic (shadava)-hexatonic (shadava) symmetrical and unsymmetrical ragas. This table was constructed from a computer generation scheme. We show only the first 212 and the last 220 ragas. Full pdf file of all 1296 ragas or any desired combination could be obtained from the author.

Acknowledgement

This research was performed in part under the auspices of the US department of Energy by the University of California, Lawrence Livermore National Laboratory, under contract number W-7405-Eng-48

References

[1] P. Sambamoorthy, *South Indian Music, Vol I-IV*, Indian Music Publishing House, Chennai, India, 1975.

[2] A. R. Bazelow and F. Brickle, A combinatorial problem in music theory---Babbitt's partition problem. With a cassette containing an original composition by Daniel Starr of Yale University. Proc. Second International Conference on Combinatorial Mathematics, *Ann. New York Acad. Sci*., **319**, pp. 47—63, New York Acad. Sci., New York, 1979.

[3] M. Babbitt, Twelve-Tone Invariants as Compositional Determinants, *Musical Quarterly* **46** (1960), 46-59.

[4] R. Kurth, Partition Lattices in Twelve-Tone Music: An Introduction, *Journal of Music Theory*, **43** (1999), 11-20.

[5] K. Balasubramanian, Applications of Combinatorics and Graph Theory to Spectroscopy and Quantum Chemistry, *Chemical Reviews*, **85**, (1985), 599-618.

[6] K. Balasubramanian, Enumeration of Stable Stereo and Position Isomers of Poly-substituted Alcohols, Proc. Second International Conference on Combinatorial Mathematics, *Annals of New York Academy Sciences*, **319**, pp. 33-36 New York Acad. Sci., New York, 1979.

[7] A. T. Balaban, *Chemical Applications of Graph Theory*, Academic Press, New York, NY 1976.

[8] H. J. Ryser, *Combinatorial Mathematics, The Carus Mathematical monographs*, No 14, John Wiley & Sons, New York, NY 1963.

[9] C. Berge, *Principles of Combinatorics. Mathematics in Science and Engineering*, Vol **72**, Academic Press NY and London 1971.

[10] V. Krishnamurhty, *Combinatorics Theory and Applications*, Affiliated East-West press, New Delhi, India 1985.

[11] K. Balasubramanian, Matching Polynomials of Fullerene Cages, *Chemical Physics Letters* **201** (1993), 306-314.

[12] K. Balasubramanian, Laplacians of Fullerene Cages (C_{42}-C_{90}), *Journal of Physical Chemistry* **99** (1995), 6509-6518.

[13] G. H. Hardy, *The Indian Mathematician Ramanujan. Ch. 1 in Ramanujan: Twelve Lectures on Subjects Suggested by his Life and Work,* Chelsea, New York, NY, 1999 pp. 1-21.

[14] D. K. Pattamal, A System of Scientific Names of Ragas, *Sruthi Magazine*, (1985), 16-17.

[15] P. Sriram and V. N. Jambunathan, How many Janya Ragas are there?, *http://www.musicacademymadras.org/article-2.html*, 2002.

Table I Notation of Notes in South Indian (Carnatic), mathematical, western and Hindustani (North Indian) music systems.

South Indian	Math	Western	North Indian
Sa	S(static)	C	Sa
Ra(shudha)	R1	D Flat	Komal Re
Ri(chatusruthi)	R2	D Natural	Shudh Re
Ru(shatsruthi)	R3	D Sharp	Komal Ga
Ga(shudha)	G1	E Double Flat	Shudh Re
Gi(sadharana)	G2	E Flat	Komal Ga
Gu(anthara)	G3	E Natural	Shudh Ga
Ma(shudha)	M1	F natural	Shudh Ma
Mi(prathi)	M2	F Sharp	Tivar Ma
Pa(panchamam)	P(static)	G	Pa
Dha(shudha)	D1	A Flat	Komal Dha
Dhi(chatusruthi)	D2	A Natural	Shudh Dha
Dhu(shatsruthi)	D3	A Sharp	Komal Ni
Na(shudha)	N1	B double flat	Shudh Dha
Ni (kaisiki)	N2	B flat	Komal Ni
Nu (kakali)	N3	B Natural	Shudh Ni
Sa(high)	Ŝ	C(Higher)	Sa (high)

Table II 41616 shadava (hexatonic) ragas(scales) of Carnatic music system. Numbers in parentheses are symmetrical (i.e., same descent and ascent) ragas[a]

Arohan(ascent)	Avarohan(descent)	Polynomial	Number
S R G M P D Ŝ	Ŝ D P M G R S	$\overline{N}^2$	1296 (36)
S R G M P N Ŝ	Ŝ D P M G R S	$\overline{D}\,\overline{N}$	432
S R G M D N Ŝ	Ŝ D P M G R S	$\overline{P}\,\overline{N}$	2592
S R G P D N Ŝ	Ŝ D P M G R S	$\overline{M}\,\overline{N}$	1296
S R M P D N Ŝ	Ŝ D P M G R S	$\overline{G}\,\overline{N}$	1296
S G M P D N Ŝ	Ŝ D P M G R S	$\overline{R}\,\overline{N}$	432
S R G M P D Ŝ	Ŝ N P M G R S	$\overline{N}\,\overline{D}$	432
S R G M P N Ŝ	Ŝ N P M G R S	$\overline{D}^2$	144(12)
S R G M D N Ŝ	Ŝ N P M G R S	$\overline{P}\,\overline{D}$	864
S R G P D N Ŝ	Ŝ N P M G R S	$\overline{M}\,\overline{D}$	432
S R M P D N Ŝ	Ŝ N P M G R S	$\overline{G}\,\overline{D}$	432
S G M P D N Ŝ	Ŝ N P M G R S	$\overline{R}\,\overline{D}$	144
S R G M P D Ŝ	Ŝ N D M G R S	$\overline{N}\,\overline{P}$	2592
S R G M P N Ŝ	Ŝ N D M G R S	$\overline{D}\,\overline{P}$	864
S R G M D N Ŝ	Ŝ N D M G R S	$\overline{P}^2$	5184 (72)
S R G P D N Ŝ	Ŝ N D M G R S	$\overline{M}\,\overline{P}$	2592
S R M P D N Ŝ	Ŝ N D M G R S	$\overline{G}\,\overline{P}$	2592
S G M P D N Ŝ	Ŝ N D M G R S	$\overline{R}\,\overline{P}$	864
S R G M P D Ŝ	Ŝ N D P G R S	$\overline{N}\,\overline{M}$	1296
S R G M P N Ŝ	Ŝ N D P G R S	$\overline{D}\,\overline{M}$	432
S R G M D N Ŝ	Ŝ N D P G R S	$\overline{P}\,\overline{M}$	2592

Table II (continued)

Arohan(ascent)	Avarohan(descent)	Polynomial	Number
S R G P D N Ŝ	Ŝ N D P G R S	$\overline{M}^2$	1296 (36)
S R M P D N Ŝ	Ŝ N D P G R S	$\overline{G}\,\overline{M}$	1296
S G M P D N Ŝ	Ŝ N D P G R S	$\overline{R}\,\overline{M}$	432
S R G M P D Ŝ	Ŝ N D P M R S	$\overline{N}\,\overline{G}$	1296
S R G M P N Ŝ	Ŝ N D P M R S	$\overline{D}\,\overline{G}$	432
S R G M D N Ŝ	Ŝ N D P M R S	$\overline{P}\,\overline{G}$	2592
S R G P D N Ŝ	Ŝ N D P M R S	$\overline{M}\,\overline{G}$	1296
S R M P D N Ŝ	Ŝ N D P M R S	$\overline{G}^2$	1296(36)
S G M P D N Ŝ	Ŝ N D P M R S	$\overline{R}\,\overline{G}$	432
S R G M P D Ŝ	Ŝ N D P M G S	$\overline{N}\,\overline{R}$	432
S R G M P N Ŝ	Ŝ N D P M G S	$\overline{D}\,\overline{R}$	144
S R G M D N Ŝ	Ŝ N D P M G S	$\overline{P}\,\overline{R}$	864
S R G P D N Ŝ	Ŝ N D P M G S	$\overline{M}\,\overline{R}$	432
S R M P D N Ŝ	Ŝ N D P M G S	$\overline{G}\,\overline{R}$	432
S G M P D N Ŝ	Ŝ N D P M G S	$\overline{R}^2$	144 (12)

[a]Note that full enumeration is included for ragas that have R and D, and thus equivalences of $G_1=R_2$, $G_2=R_3$, $N_1=D_2$, $N_2=D_3$ are invoked so that combinations that have G or N will include only unique ragas, that is, those that contain only G_3 and N_3(kakali Nishadha). For example, in ragas with S R G M P N Ŝ, for $N=N_2$ (kaisiki Nishada) are already enumerated in S R G M P D Ŝ with $D=D_3$.

Table III 55696 Pentatonic ragas (scales) of Carnatic Music system. Only Arohans(Ascents) are shown. The complete set is obtained by the combinatorial generating function (equation) in the text.

Polynomial	Arohan (Ascent)	Number
$\overline{R}\,\overline{G}$	SMPDNŜ	12
$\overline{R}\,\overline{M}$	SGPDNŜ	6
$\overline{R}\,\overline{P}$	SGMDNŜ	12
$\overline{R}\,\overline{D}$	SGMPNŜ	2
$\overline{R}\,\overline{N}$	SGMPDŜ	6
$\overline{G}\,\overline{M}$	SRPDNŜ	18
$\overline{G}\,\overline{P}$	SRMDNŜ	36
$\overline{G}\,\overline{D}$	SRMPNŜ	6
$\overline{G}\,\overline{N}$	SRMPDŜ	18
$\overline{M}\,\overline{P}$	SRGDNŜ	36
$\overline{M}\,\overline{D}$	SRGPNŜ	6
$\overline{M}\,\overline{N}$	SRGPDŜ	18
$\overline{P}\,\overline{D}$	SRGMNŜ	12
$\overline{P}\,\overline{N}$	SRGMDŜ	36
$\overline{D}\,\overline{N}$	SRGMPŜ	12

Table IV Enumeration of 262,144 Combinations of Ragas With Pentatonic or higher scales.

Arohan(Ascent)	Avarohan(descent)	Number[a]
Sampurna(complete)	Sampurna(complete)	5184(72)
Sampurna(complete)	Shadava(Hexatonic)	14688
Sampurna(complete)	Audava(pentatonic)	16992
Shadava(Hexatonic)	Sampurna(complete)	14688
Shadava(Hexatonic)	Shadava(hexatatonic)	41616 (204)
Shadava(Hexatonic)	Audava(pentatonic)	48144
Audava(pentatonic))	Sampurna(complete)	16992
Audava(pentatonic)	Shadava(hexatatonic)	48144
Audava(pentatonic)	Audava(pentatonic)	55696(236)
Grand Total:		262,144

[a]Numbers in parentheses are the numbers of symmetrical ragas, wherein the ascents and descents exhibit a mirror symmetry and are thus non-bashanka ragas. There are $512=2^9$ symmetrical ragas of all types containing at least pentatonic scales.

Table V 1296 Shadava(Hexatonic)-Shadava Scales Missing G (Western E)[a]

	Arohan(Ascent)	Avarohan(Descent)		Arohan(Ascent)	Avarohan(Descent)
1	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R1 S	2	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R1 S
3	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R1 S	4	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R1 S
5	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R1 S	6	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R1 S
7	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R2 S	8	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R2 S
9	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R2 S	10	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R2 S
11	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R2 S	12	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R2 S
13	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R3 S	14	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R3 S
15	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R3 S	16	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R3 S
17	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R3 S	18	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R3 S
19	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R1 S	20	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R1 S
21	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R1 S	22	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R1 S
23	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R1 S	24	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R1 S
25	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R2 S	26	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R2 S
27	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R2 S	28	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R2 S
29	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R2 S	30	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R2 S
31	S R1 M1 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R3 S	32	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R3 S
33	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R3 S	34	S R1 M1 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R3 S
35	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R3 S	36	S R1 M1 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R3 S
37	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R1 S	38	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R1 S
39	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R1 S	40	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M1 R1 S
41	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M1 R1 S	42	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R1 S
43	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R2 S	44	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R2 S
45	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R2 S	46	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M1 R2 S
47	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M1 R2 S	48	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R2 S
49	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R3 S	50	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R3 S
51	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R3 S	52	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M1 R3 S
53	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M1 R3 S	54	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R3 S
55	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R1 S	56	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R1 S
57	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R1 S	58	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R1 S
59	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R1 S	60	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R1 S
61	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R2 S	62	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R2 S
63	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R2 S	64	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R2 S
65	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R2 S	66	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R2 S
67	S R1 M1 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R3 S	68	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R3 S
69	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R3 S	70	S R1 M1 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R3 S
71	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R3 S	72	S R1 M1 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R3 S
73	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	74	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
75	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	76	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
77	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	78	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
79	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	80	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
81	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	82	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
83	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	84	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
85	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	86	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
87	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	88	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
89	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	90	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
91	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	92	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
93	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	94	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
95	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	96	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
97	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	98	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
99	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	100	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
101	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	102	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R2 S

	Arohan(Ascent)	Avarohan(Descent)		Arohan(Ascent)	Avarohan(Descent)
103	S R1 M1 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	104	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R3 S
105	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	106	S R1 M1 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
107	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	108	S R1 M1 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R3 S
109	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R1 S	110	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R1 S
111	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R1 S	112	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R1 S
113	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R1 S	114	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R1 S
115	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R2 S	116	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R2 S
117	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R2 S	118	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R2 S
119	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R2 S	120	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R2 S
121	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R3 S	122	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R3 S
123	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R3 S	124	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R3 S
125	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R3 S	126	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R3 S
127	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R1 S	128	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R1 S
129	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R1 S	130	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R1 S
131	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R1 S	132	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R1 S
133	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R2 S	134	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R2 S
135	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R2 S	136	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R2 S
137	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R2 S	138	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R2 S
139	S R1 M1 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R3 S	140	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R3 S
141	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R3 S	142	S R1 M1 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R3 S
143	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R3 S	144	S R1 M1 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R3 S
145	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	146	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
147	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	148	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
149	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	150	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
151	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	152	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
153	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	154	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
155	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	156	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
157	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	158	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
159	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	160	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
161	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	162	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
163	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	164	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
165	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	166	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
167	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	168	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
169	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	170	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
171	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	172	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
173	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	174	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R2 S
175	S R1 M1 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	176	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R3 S
177	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	178	S R1 M1 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
179	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	180	S R1 M1 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R3 S
181	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	182	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
183	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	184	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
185	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	186	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
187	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	188	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
189	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	190	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
191	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	192	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
193	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	194	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
195	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	196	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
197	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	198	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
199	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	200	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
201	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	202	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
203	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	204	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
205	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	206	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
207	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	208	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
209	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	210	S R1 M1 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R2 S
211	S R1 M1 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	212	S R1 M1 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R3 S

	Arohan(Ascent)	Avarohan(Descent)		Arohan(Ascent)	Avarohan(Descent)
1077	S R2 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	1078	S R2 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
1079	S R2 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	1080	S R2 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R3 S
1081	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R1 S	1082	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R1 S
1083	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R1 S	1084	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R1 S
1085	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R1 S	1086	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R1 S
1087	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R2 S	1088	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R2 S
1089	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R2 S	1090	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R2 S
1091	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R2 S	1092	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R2 S
1093	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M1 R3 S	1094	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M1 R3 S
1095	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M1 R3 S	1096	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M1 R3 S
1097	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M1 R3 S	1098	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M1 R3 S
1099	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R1 S	1100	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R1 S
1101	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R1 S	1102	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R1 S
1103	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R1 S	1104	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R1 S
1105	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R2 S	1106	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R2 S
1107	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R2 S	1108	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R2 S
1109	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R2 S	1110	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R2 S
1111	S R3 M2 P D1 N1 Ŝ	Ŝ N1 D1 P M2 R3 S	1112	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D1 P M2 R3 S
1113	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D1 P M2 R3 S	1114	S R3 M2 P D1 N1 Ŝ	Ŝ N2 D2 P M2 R3 S
1115	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D2 P M2 R3 S	1116	S R3 M2 P D1 N1 Ŝ	Ŝ N3 D3 P M2 R3 S
1117	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R1 S	1118	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R1 S
1119	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R1 S	1120	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D2 P M1 R1 S
1121	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D2 P M1 R1 S	1122	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R1 S
1123	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R2 S	1124	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R2 S
1125	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R2 S	1126	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D2 P M1 R2 S
1127	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D2 P M1 R2 S	1128	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R2 S
1129	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M1 R3 S	1130	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M1 R3 S
1131	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M1 R3 S	1134	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M1 R3 S
1135	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R1 S	1136	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R1 S
1137	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R1 S	1138	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R1 S
1139	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R1 S	1140	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R1 S
1141	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R2 S	1142	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R2 S
1143	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R2 S	1144	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R2 S
1145	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R2 S	1146	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R2 S
1147	S R3 M2 P D1 N2 Ŝ	Ŝ N1 D1 P M2 R3 S	1148	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D1 P M2 R3 S
1149	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D1 P M2 R3 S	1150	S R3 M2 P D1 N2 Ŝ	Ŝ N2 D2 P M2 R3 S
1151	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D2 P M2 R3 S	1152	S R3 M2 P D1 N2 Ŝ	Ŝ N3 D3 P M2 R3 S
1153	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	1154	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
1155	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	1156	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
1157	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	1158	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
1159	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	1160	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
1161	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	1162	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
1163	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	1164	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
1165	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	1166	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
1167	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	1168	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
1169	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	1170	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
1171	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	1172	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
1173	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	1174	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
1175	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	1176	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
1177	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	1178	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
1179	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	1180	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
1181	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	1182	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R2 S
1183	S R3 M2 P D1 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	1184	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D1 P M2 R3 S
1185	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	1186	S R3 M2 P D1 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
1187	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	1188	S R3 M2 P D1 N3 Ŝ	Ŝ N3 D3 P M2 R3 S

	Arohan(Ascent)	Avarohan(Descent)		Arohan(Ascent)	Avarohan(Descent)
1189	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R1 S	1190	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R1 S
1191	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R1 S	1192	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R1 S
1193	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R1 S	1194	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R1 S
1195	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R2 S	1196	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R2 S
1197	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R2 S	1198	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R2 S
1199	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R2 S	1200	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R2 S
1201	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M1 R3 S	1202	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M1 R3 S
1203	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M1 R3 S	1204	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M1 R3 S
1205	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M1 R3 S	1206	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M1 R3 S
1207	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R1 S	1208	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R1 S
1209	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R1 S	1210	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R1 S
1211	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R1 S	1212	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R1 S
1213	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R2 S	1214	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R2 S
1215	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R2 S	1216	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R2 S
1217	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R2 S	1218	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R2 S
1219	S R3 M2 P D2 N2 Ŝ	Ŝ N1 D1 P M2 R3 S	1220	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D1 P M2 R3 S
1221	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D1 P M2 R3 S	1222	S R3 M2 P D2 N2 Ŝ	Ŝ N2 D2 P M2 R3 S
1223	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D2 P M2 R3 S	1224	S R3 M2 P D2 N2 Ŝ	Ŝ N3 D3 P M2 R3 S
1225	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	1226	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
1227	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	1228	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
1229	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	1230	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
1231	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	1232	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
1233	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	1234	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
1235	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	1236	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
1237	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	1238	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
1239	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	1240	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
1241	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	1242	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
1243	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	1244	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
1245	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	1246	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
1247	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	1248	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
1249	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	1250	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
1251	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	1252	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
1253	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	1254	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R2 S
1255	S R3 M2 P D2 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	1256	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D1 P M2 R3 S
1257	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	1258	S R3 M2 P D2 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
1259	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	1260	S R3 M2 P D2 N3 Ŝ	Ŝ N3 D3 P M2 R3 S
1261	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R1 S	1262	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R1 S
1263	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R1 S	1264	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R1 S
1265	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R1 S	1266	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R1 S
1267	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R2 S	1268	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R2 S
1269	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R2 S	1270	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R2 S
1271	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R2 S	1272	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R2 S
1273	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M1 R3 S	1274	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M1 R3 S
1275	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M1 R3 S	1276	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M1 R3 S
1277	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M1 R3 S	1278	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M1 R3 S
1279	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R1 S	1280	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R1 S
1281	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R1 S	1282	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R1 S
1283	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R1 S	1284	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R1 S
1285	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R2 S	1286	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R2 S
1287	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R2 S	1288	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R2 S
1289	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R2 S	1290	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R2 S
1291	S R3 M2 P D3 N3 Ŝ	Ŝ N1 D1 P M2 R3 S	1292	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D1 P M2 R3 S
1293	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D1 P M2 R3 S	1294	S R3 M2 P D3 N3 Ŝ	Ŝ N2 D2 P M2 R3 S
1295	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D2 P M2 R3 S	1296	S R3 M2 P D3 N3 Ŝ	Ŝ N3 D3 P M2 R3 S

[a]The first 212 and the last 220 ragas of a total of 1296 combinations for the polynomial $\overline{G}^2$ are shown.

2000 Mathematical Subject Classification: 05A05, 05A15
***Keywords**: Indian Carnatic music, combinatorics, ragas, music scales*

Received August 28, 2002; revised Version received December 9, 2002. Published in the Journal of Integer Sequences December 10, 2002.