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The Hofstadter Consecutive-Sum Sequence Omits Infinitely Many Positive Integers

Quanyu Tang
School of Mathematical Sciences
University of Science and Technology of China
Hefei 230026
P. R. China
tangquanyu827@gmail.com

Abstract

We study Hofstadter's greedy self-generating sequence, which begins with 1 and 2 and repeatedly chooses the least integer larger than the preceding term that is a sum of at least two consecutive earlier terms. We prove that the excess of its nth term over its index is bounded below by a positive constant times $\log\log n$, up to an additive constant, and that the sequence satisfies a polynomial upper bound. In particular, the sequence omits infinitely many positive integers.

1 Introduction

Douglas Hofstadter asked for a description of the asymptotic behavior of the following greedy sequence. The question is recorded by Erdős [7, p. 71], Erdős and Graham [8, p. 83], and Guy [10, E31]; Bloom also lists it as Problem #423 on the *Erdős Problems* website [4]. In the classical case one begins with $(1, 2)$ and obtains a strictly increasing sequence $(a_n)_{n\geq 1}$ as follows.

Definition 1. Set $a_1 = 1$ and $a_2 = 2$. For every $k \geq 3$, define a_k to be the least integer greater than a_{k-1} that is a sum of at least two consecutive earlier terms.

In particular, Definition 1 implies that, for every $k \geq 3$,

$$a_k = \sum_{i=p}^{q} a_i \qquad \text{for some } 1 \leq p < q \leq k-1. \tag{1}$$

Thus (1) records a property implied by the definition, rather than the definition itself. This construction gives sequence A005243 in the OEIS [15]. Erdős reports that Hofstadter's question was inspired by a problem of Ulam about gaps among consecutive sums of a finite increasing sequence [7, p. 71].

Problem 2. What is the asymptotic behavior of the sequence in Definition 1?

Numerically, the sequence appears to contain "most" integers. For orientation, the sequence in Definition 1 begins

$$1, 2, 3, 5, 6, 8, 10, 11, 14, 16, 17, 18, 19, 21, 22, 24, 25, 29, 30, 32, \ldots,$$

and hence it already omits $4, 7, 9, 12, 13, 15, 20, \ldots$. At the same time, the *Comments* section of A005243 records the following conjecture [15].

Conjecture 3. The sequence in Definition 1 omits infinitely many positive integers.

For later use, write

$$b_n := a_n - n \qquad (n \geq 1).$$

Our first result settles Conjecture 3.

Theorem 4. *The sequence $(b_n)_{n\geq 1}$ is nondecreasing and unbounded. Consequently,*

$$a_n = n + \omega(1),$$

and the set $\{a_n : n \geq 1\}$ omits infinitely many positive integers.

In fact, building on Theorem 4, we prove the following quantitative lower bound.

Theorem 5. *We have*

$$a_n \geq n + \frac{\log\log n}{\log 20} - O(1).$$

We also obtain a quantitative upper bound.

Theorem 6. *For every $\varepsilon > 0$ there exists $C_\varepsilon > 0$ such that, for all $n \geq 1$,*

$$a_n \leq C_\varepsilon\, n^{4175/2506+\varepsilon}.$$

The proof of Theorem 6 combines the greedy structure of the sequence with a recent lower bound for difference sets of finite convex sets. Thus, besides settling Conjecture 3, we also obtain a first polynomial upper bound toward Problem 2. For related work on consecutive sums in integer sequences, see Beker [1], Hegyvári [11], and Tao [18].

1.1 Overview of the proofs

The proof of Theorem 4 begins with the observation that $b_n = a_n - n$ is nondecreasing. If (b_n) were bounded, then it would eventually be constant, say $b_n = B$ with $B \geq 0$, so $a_n = n + B$ on a tail. For every sufficiently large r, set $n = 2^r - B$; then $a_n = 2^r$, and this term must be a sum of consecutive earlier terms. For large r, such a representation is too large to lie entirely in the fixed initial prefix, while it cannot lie entirely in the linear tail because a power of 2 is not a sum of at least two consecutive positive integers. It must therefore cross the boundary of the linear tail, and summing its linear part produces an equation

$$v^2 + v + E = 2^k$$

with E taking values in a finite set. By the pigeonhole principle, one value of E would then occur for infinitely many values of r, contradicting the Schinzel–Tijdeman finiteness theorem [13, Corollary 1]. Since $b_n = a_n - n$ counts the omitted positive integers up to a_n, this also gives the final assertion of the theorem.

For Theorem 5, we quantify how long (b_n) can remain constant at a given integer value. By Theorem 4, (b_n) is nondecreasing and unbounded; since it is integer-valued, we have $b_n \to \infty$. For each integer $B \geq 0$, let $N(B)$ be the last index at which $b_n \leq B$, and set

$$M(B) = N(B) + B + 2.$$

These quantities are well defined because $b_1 = 0$ and $b_n \to \infty$.

Now fix an integer $B \geq 1$. Since (b_n) is nondecreasing and integer-valued, the set

$$\{n : b_n = B\}$$

is either empty or a finite interval of consecutive indices. When this set is nonempty, we refer to the corresponding interval as the *level-B plateau*. If b_n never takes the value B, then

$$M(B) = M(B-1) + 1.$$

Otherwise, the level-B plateau is

$$n_1 \leq n \leq n_2, \qquad n_1 = N(B-1) + 1, \quad n_2 = N(B).$$

Setting

$$T = n_1 + B = M(B-1),$$

we have $b_i \leq B - 1$ for $i < n_1$, and hence

$$S = \sum_{i=1}^{n_1 - 1} a_i \leq T^2.$$

If $n_2 + B > 2K_0T^{20}$ for a suitable absolute constant K_0, one can choose a power of two 2^r such that

$$K_0T^{20} < 2^r \leq 2K_0T^{20} < n_2 + B.$$

For such a choice, $n = 2^r - B$ lies in the level-B plateau and $n \geq 3$, so $a_n = 2^r$ has a representation as a sum of at least two consecutive earlier terms. This representation must cross the left endpoint n_1 of the plateau: it cannot end before n_1, since $2^r > S$, and it cannot begin at or after n_1, since a power of 2 is not a sum of at least two consecutive positive integers. Splitting the sum at n_1 and completing the square therefore gives

$$2^m = x^2 + D, \qquad D \neq 0, \qquad |D| \leq 13T^2.$$

Beukers' estimate [2, Corollary 1] then gives $2^r < K_0 T^{20}$, contradicting the choice of r. Consequently,

$$M(B) \leq KM(B-1)^{20}$$

for some absolute constant $K > 0$. Iterating gives $\log M(B) \leq A20^B$ for some absolute constant $A > 0$. Finally, taking $B = b_n$, we have $n \leq N(B) < M(B)$, and hence $\log n \leq A20^B$. Taking logarithms once more yields

$$b_n \geq \frac{\log\log n}{\log 20} - O(1).$$

The proof of Theorem 6 starts from the fact that, from a_3 onward, the greedy sequence lists in increasing order all integers representable as sums of at least two consecutive terms. Let $\mathcal{R}_m$ denote the integers representable as sums of at least two consecutive terms among $a_1, \ldots, a_m$. Recall that a finite ordered set $Y = \{y_0 < y_1 < \cdots < y_m\} \subset \mathbb{R}$ is called *convex* if its consecutive gaps $y_i - y_{i-1}$ are strictly increasing. The prefix-sum set $\widetilde{S}_m = \{s_0, s_1, \ldots, s_m\}$, where $s_0 = 0$ and $s_j = a_1 + \cdots + a_j$, is convex in this sense because its consecutive gaps are $a_1, \ldots, a_m$. Every positive difference of two elements of $\widetilde{S}_m$ is either one of the m one-term differences $a_1, \ldots, a_m$ or belongs to $\mathcal{R}_m$. Bloom's lower bound for difference sets of convex sets [3, Theorem 2] therefore gives

$$|\mathcal{R}_m| \gg_\eta m^\delta, \qquad \delta = \frac{6681}{4175} - \eta,$$

for $0 < \eta < 2506/4175$. Feeding this count back into the greedy enumeration gives, for a suitable constant $C = C_\eta > 0$ and all sufficiently large n, a recursion of the form

$$a_n \leq Cn^{1/\delta} a_{\lceil Cn^{1/\delta} \rceil}.$$

Iterating it gives, for some $B_\eta > 0$,

$$a_n \ll_\eta n^{1/(\delta-1)} (\log n)^{B_\eta}.$$

Choosing η sufficiently small in terms of ε and absorbing the logarithmic factor into n^ε gives the exponent $4175/2506 + \varepsilon$.

1.2 Notation

We use Vinogradov's asymptotic notation. For functions $f = f(n)$ and $g = g(n)$, we write $f = O(g)$, $g = \Omega(f)$, $f \ll g$, or $g \gg f$ to mean that there exists a constant $C > 0$ such that $|f(n)| \leq Cg(n)$ for all sufficiently large n. When we allow the implicit constant to depend on one or more parameters $z_1, \ldots, z_r$, we indicate this by writing $f = O_{z_1,\ldots,z_r}(g)$, $g = \Omega_{z_1,\ldots,z_r}(f)$, $f \ll_{z_1,\ldots,z_r} g$ or $g \gg_{z_1,\ldots,z_r} f$. We write $f \asymp g$ or $f = \Theta(g)$ to mean that $f \ll g$ and $g \ll f$, and write $f = o(g)$ to mean that $f(n)/g(n) \to 0$ as $n \to \infty$. We write $f = \omega(g)$ to mean that $g = o(f)$; in particular, $\omega(1)$ denotes any quantity tending to infinity as $n \to \infty$.

1.3 Paper organization

Section 2 collects the two preliminary ingredients that we use later: the characterization of sums of consecutive positive integers and a Diophantine finiteness consequence of the theorem of Schinzel and Tijdeman. Section 3 proves Theorem 4. Section 4 establishes the quantitative lower bound in Theorem 5. Section 5 proves the polynomial upper bound in Theorem 6. Finally, Section 6 contains concluding remarks on extensions, later improvements, and open questions.

2 Preliminaries

We start with a classical characterization of sums of consecutive positive integers. Sylvester stated the enumeration theorem that the total number of representations of a positive integer N as a sum of one or more consecutive positive integers equals the number of odd divisors of N [17, § 17, p. 265]; the corresponding bijection is established in the following section [17, § 18, p. 266]. Equivalently, after excluding the trivial one-term representation, the number of representations using at least two terms equals the number of odd divisors of N greater than 1; see also Ho, He, and Shiue [12, p. 242]. Gamer, Roeder, and Watkins call integers representable as sums of at least two consecutive positive integers *trapezoidal* and prove that the trapezoidal positive integers are precisely those that are not powers of 2 [9, Proposition 1, p. 108]. For the sake of completeness, we include the short proof.

Lemma 7. *A positive integer N admits a representation as a sum of at least two consecutive positive integers if and only if N is not a power of 2.*

Proof. Suppose $N = x + (x+1) + \cdots + (x+\ell-1)$ with $\ell \geq 2$ and $x \geq 1$. Then

$$2N = \ell(2x + \ell - 1).$$

If ℓ is odd, then $\ell \mid 2N$ implies $\ell \mid N$, so N has an odd divisor > 1. If ℓ is even, then $2x+\ell-1$ is odd and divides $2N$. Hence $2x + \ell - 1$ divides N, so N again has an odd divisor > 1. In either case N cannot be a power of 2.

Conversely, if N is not a power of 2, then N has an odd divisor $d > 1$, say $N = dm$. If $m \geq (d+1)/2$, then

$$N = \Big(m - \frac{d-1}{2}\Big) + \Big(m - \frac{d-1}{2} + 1\Big) + \cdots + \Big(m + \frac{d-1}{2}\Big)$$

is a sum of d consecutive positive integers. If $m < (d+1)/2$, then

$$N = \Big(\frac{d+1}{2} - m\Big) + \Big(\frac{d+1}{2} - m + 1\Big) + \cdots + \Big(\frac{d+1}{2} + m - 1\Big)$$

is a sum of $2m \geq 2$ consecutive positive integers. □

Next we record the Diophantine ingredient. We shall use the following consequence of a result of Schinzel and Tijdeman [13, Corollary 1]. Here a complex zero ξ of a polynomial P is called *simple* if $P'(\xi) \neq 0$, equivalently, if it has multiplicity one. Thus "at least two simple zeros" means two distinct complex zeros, each of multiplicity one.

Lemma 8. *Let $P(x) \in \mathbb{Q}[x]$ be a polynomial with at least two simple zeros. Then the equation*

$$y^k = P(x), \qquad x, y \in \mathbb{Z}, \quad |y| > 1,$$

has only finitely many integer solutions (x, y, k) with $k > 2$.

Corollary 9. *Fix $E \in \mathbb{Z}$. The equation*

$$v^2 + v + E = 2^k$$

has only finitely many integer solutions $(v, k) \in \mathbb{Z}^2$ with $k > 2$.

Proof. Apply Lemma 8 with $y = 2$ and $P(v) = v^2 + v + E$. Since $E \in \mathbb{Z}$, the discriminant $1 - 4E$ is congruent to 1 modulo 4 and hence is nonzero. Therefore P has two distinct zeros, each of multiplicity one, so the lemma applies. □

3 A qualitative lower bound and infinitely many omissions

Throughout this section we write

$$b_n := a_n - n \qquad (n \geq 1).$$

Our goal is to prove Theorem 4. The first step is a simple monotonicity observation.

3.1 Monotonicity and eventual linearity

Lemma 10. *The sequence $(b_n)_{n\geq 1}$ is nondecreasing.*

Proof. For every $n \geq 1$, the greedy construction gives $a_{n+1} > a_n$, and hence $a_{n+1} \geq a_n + 1$. Therefore

$$b_{n+1} - b_n = (a_{n+1} - (n+1)) - (a_n - n) = a_{n+1} - a_n - 1 \geq 0. \qquad \square$$

Lemma 11. *If $(b_n)_{n\geq 1}$ is bounded above, then there exist integers $n_0 \geq 2$ and B such that*

$$a_n = n + B \qquad (n \geq n_0).$$

Proof. By Lemma 10, the sequence $(b_n)_{n\geq 1}$ is a nondecreasing sequence of integers. If it is bounded above, then it is eventually constant: there exist $n_0 \geq 2$ and an integer B such that $b_n = B$ for all $n \geq n_0$. Hence $a_n = n + b_n = n + B$ for all $n \geq n_0$. $\square$

We now assume, for contradiction, that $(b_n)_{n\geq 1}$ is bounded above and hence that

$$a_n = n + B \qquad (n \geq n_0) \tag{2}$$

for some integers $n_0 \geq 2$ and B. Since $b_1 = 0$ and (b_n) is nondecreasing, we have $B \geq 0$.

3.2 Powers of two on a linear plateau

The following lemma contains the part common to the qualitative argument in this section and the quantitative argument in Section 4.

Lemma 12. *Let $B \geq 0$, and suppose that*

$$b_n = B$$

for every integer n with $n_1 \leq n \leq n_2$, where $2 \leq n_1 \leq n_2$. Set

$$T := n_1 + B, \qquad S := \sum_{i=1}^{n_1 - 1} a_i.$$

If $r \in \mathbb{Z}$ satisfies

$$\max\{T, S+1, B+3\} \leq 2^r \leq n_2 + B, \tag{3}$$

then there exist integers C and $v \geq T$ such that

$$2^{r+1} = v^2 + v + 2C - T(T-1), \tag{4}$$

where $0 \leq C \leq S$.

Proof. Set $n = 2^r - B$. The lower bound $2^r \geq T$ and the upper bound in (3) give $n_1 \leq n \leq n_2$, so $a_n = n + B = 2^r$. The bound $2^r \geq B + 3$ gives $n \geq 3$. Hence there are indices $1 \leq p < q \leq n - 1$ such that

$$2^r = \sum_{k=p}^{q} a_k.$$

Since $2^r > S$, we must have $q \geq n_1$. If $p \geq n_1$, then

$$2^r = \sum_{k=p}^{q} (k + B) = \sum_{j=p+B}^{q+B} j.$$

Since $p \geq 1$ and $B \geq 0$, we have $p + B \geq 1$. Thus 2^r would be a sum of at least two consecutive positive integers, contradicting Lemma 7. Therefore $p < n_1 \leq q$. Put

$$C := \sum_{k=p}^{n_1-1} a_k, \qquad v := q + B.$$

Then $0 \leq C \leq S$, $v \geq T$, and

$$2^r = C + \sum_{j=T}^{v} j = C + \frac{v(v+1) - T(T-1)}{2}.$$

Multiplying by 2 gives (4). Here the condition $2^r \geq T$ gives $n \geq n_1$, the condition $2^r \geq S+1$ gives $q \geq n_1$, the condition $2^r \geq B + 3$ ensures that $n \geq 3$ and hence that the defining consecutive-sum representation is available, and the condition $2^r \leq n_2 + B$ gives $n \leq n_2$. □

3.3 Completion of the proof

Proof of Theorem 4. Lemma 10 shows that (b_n) is nondecreasing. It remains to prove that it is unbounded.

Assume, for contradiction, that (b_n) is bounded above. Then (2) holds by Lemma 11. Set

$$T := n_0 + B, \qquad S := \sum_{i=1}^{n_0-1} a_i.$$

For every sufficiently large integer r, we have

$$2^r \geq \max\{T, S + 1, B + 3\}.$$

Moreover, with $n_1 = n_0$ and $n_2 = 2^r - B$, we have

$$n_2 \geq n_0 \qquad \text{and} \qquad n_2 + B = 2^r.$$

Thus Lemma 12 gives integers C and $v \geq T$ such that

$$2^{r+1} = v^2 + v + E_C, \qquad E_C := 2C - T(T-1),$$

where $0 \leq C \leq S$. Since only finitely many integers E_C can occur, the pigeonhole principle gives an integer E for which

$$v^2 + v + E = 2^k$$

has infinitely many integer solutions $(v, k) \in \mathbb{Z}^2$ with $k > 2$. This contradicts Corollary 9. Therefore (b_n) is unbounded.

Since (b_n) is a nondecreasing unbounded sequence of integers, we have $b_n \to \infty$, and hence

$$a_n = n + b_n = n + \omega(1).$$

Finally, because (a_n) is strictly increasing, exactly n elements of $\{a_m : m \geq 1\}$ lie in the interval $[1, a_n]$, namely $a_1, \ldots, a_n$. Thus the sequence omits exactly

$$a_n - n = b_n$$

positive integers in $[1, a_n]$. As $b_n \to \infty$, the set $\{a_n : n \geq 1\}$ omits infinitely many positive integers. □

4 A quantitative lower bound for the classical sequence

We use the following explicit consequence of Beukers' work [2, Corollary 1].

Lemma 13. *Let $D \in \mathbb{Z} \setminus \{0\}$. Every integer solution $(x, m) \in \mathbb{Z}^2$ of*

$$x^2 + D = 2^m$$

satisfies

$$m < 435 + \frac{10 \log |D|}{\log 2}.$$

Lemma 14. *Let $\widehat{B} \geq 1$, and suppose that*

$$b_n = \widehat{B}$$

for every integer n with $n_1 \leq n \leq n_2$, where $2 \leq n_1 \leq n_2$. Set

$$\widehat{T} := n_1 + \widehat{B}, \qquad \widehat{S} := \sum_{i=1}^{n_1 - 1} a_i.$$

If $r \in \mathbb{Z}$ satisfies

$$\max\{\widehat{T}, \widehat{S} + 1, \widehat{B} + 3\} \leq 2^r \leq n_2 + \widehat{B}, \tag{5}$$

then there exist integers x and $D \neq 0$ such that

$$2^{r+3} = x^2 + D \qquad \text{and} \qquad |D| \leq 13\widehat{T}^2.$$

Proof. Since $(b_n)_{n\ge 1}$ is nondecreasing by Lemma 10 and $b_{n_1} = \widehat{B}$, we have

$$b_i \le \widehat{B} \qquad (1 \le i < n_1).$$

Hence, for every $1 \le i < n_1$,

$$a_i = i + b_i \le i + \widehat{B} \le n_1 + \widehat{B} = \widehat{T}.$$

Therefore

$$\widehat{S} \le (n_1 - 1)\widehat{T} \le \widehat{T}^2.$$

By (5), Lemma 12, applied with $B = \widehat{B}$, gives integers C and v such that $0 \le C \le \widehat{S}$ and

$$2^{r+1} = v^2 + v + 2C - \widehat{T}(\widehat{T} - 1).$$

Completing the square gives

$$2^{r+3} = (2v + 1)^2 + D, \qquad D := 8C - 4\widehat{T}(\widehat{T} - 1) - 1.$$

Since $D \equiv -1 \pmod 4$, we have $D \neq 0$. Finally,

$$|D| \le 8C + 4\widehat{T}(\widehat{T} - 1) + 1 \le 12\widehat{T}^2 + 1 \le 13\widehat{T}^2.$$

The interval hypothesis on b_n is used to invoke Lemma 12; monotonicity is used to bound the prefix sum $\widehat{S}$; and the assumption $\widehat{B} \ge 1$ is the range needed in the recursion below. □

We are now ready to prove the quantitative lower bound for the sequence in Definition 1.

Proof of Theorem 5. By Theorem 4, the sequence (b_n) is nondecreasing and unbounded. Hence, for each integer $\widehat{B} \ge 0$,

$$N(\widehat{B}) := \max\{n \ge 1 : \ b_n \le \widehat{B}\}$$

is well defined and finite. Set

$$M(\widehat{B}) := N(\widehat{B}) + \widehat{B} + 2 \qquad (\widehat{B} \ge 0).$$

We claim that there exists an absolute constant $K \ge 2$ such that

$$M(\widehat{B}) \le K\, M(\widehat{B} - 1)^{20} \qquad (\widehat{B} \ge 1). \tag{6}$$

Fix $\widehat{B} \ge 1$. If $N(\widehat{B}) = N(\widehat{B} - 1)$, then

$$M(\widehat{B}) = N(\widehat{B}) + \widehat{B} + 2 = N(\widehat{B} - 1) + \widehat{B} + 2 = M(\widehat{B} - 1) + 1.$$

Since $M(\widehat{B} - 1) \ge 2$, it follows that

$$M(\widehat{B}) \le M(\widehat{B} - 1)^{20}.$$

Suppose now that $N(\widehat{B}) > N(\widehat{B}-1)$. Since (b_n) is integer-valued and nondecreasing, we then have

$$b_n = \widehat{B} \qquad \big(N(\widehat{B}-1)+1 \le n \le N(\widehat{B})\big).$$

Write

$$n_1 := N(\widehat{B}-1)+1, \qquad n_2 := N(\widehat{B}), \qquad \widehat{T} := n_1 + \widehat{B}.$$

Then

$$\widehat{T} = N(\widehat{B}-1)+1+\widehat{B} = N(\widehat{B}-1)+(\widehat{B}-1)+2 = M(\widehat{B}-1).$$

Let

$$K_0 := 2^{432}13^{10}.$$

We shall show that

$$n_2 + \widehat{B} \le 2K_0\widehat{T}^{20}.$$

Assume to the contrary that

$$n_2 + \widehat{B} > 2K_0\widehat{T}^{20}.$$

Then there exists an integer r such that

$$K_0\widehat{T}^{20} < 2^r \le 2K_0\widehat{T}^{20} < n_2 + \widehat{B}.$$

We now verify the hypotheses of Lemma 14. Since $\widehat{T} \ge 2$, we have

$$2^r > K_0\widehat{T}^{20} \ge \widehat{T}^{20} > \widehat{T}^2.$$

Moreover, for every $1 \le i \le n_1 - 1$ we have $b_i \le \widehat{B} - 1$, because $n_1 = N(\widehat{B}-1)+1$. Hence

$$a_i = i + b_i \le i + \widehat{B} - 1 \qquad (1 \le i \le n_1 - 1),$$

and therefore

$$\sum_{i=1}^{n_1-1} a_i \le \sum_{i=1}^{n_1-1} (i + \widehat{B} - 1) = \frac{(n_1-1)(n_1+2\widehat{B}-2)}{2}.$$

On the other hand,

$$\widehat{T}^2 = (n_1 + \widehat{B})^2,$$

and a direct calculation shows that

$$2\widehat{T}^2 - (n_1-1)(n_1+2\widehat{B}-2) = n_1^2 + 2n_1\widehat{B} + 2\widehat{B}^2 + 3n_1 + 2\widehat{B} - 2 > 0.$$

Thus

$$\sum_{i=1}^{n_1-1} a_i \le \widehat{T}^2 < 2^r.$$

Also, from $2^r > \widehat{T}^2$ and $\widehat{T} \ge 2$, we obtain

$$2^r \ge \widehat{T}.$$

Finally, since $n_1 \geq 2$, we have

$$\widehat{T} = n_1 + \widehat{B} \geq \widehat{B} + 2,$$

and therefore

$$2^r > \widehat{T}^2 \geq (\widehat{B}+2)^2 > \widehat{B} + 3,$$

so in particular

$$2^r \geq \widehat{B} + 3.$$

Hence Lemma 14 applies, and yields integers x and $D \neq 0$ such that

$$2^{r+3} = x^2 + D \qquad \text{and} \qquad |D| \leq 13\widehat{T}^2.$$

Applying Lemma 13, we obtain

$$r + 3 < 435 + \frac{10 \log |D|}{\log 2} \leq 435 + \frac{10 \log(13\widehat{T}^2)}{\log 2}.$$

Exponentiating, we get

$$2^r < 2^{432} 13^{10} \widehat{T}^{20} = K_0 \widehat{T}^{20},$$

contrary to the choice of r. This contradiction proves that

$$n_2 + \widehat{B} \leq 2K_0 \widehat{T}^{20}.$$

Since $n_2 = N(\widehat{B})$ and $\widehat{T} = M(\widehat{B}-1)$, it follows that

$$N(\widehat{B}) + \widehat{B} \leq 2K_0\, M(\widehat{B}-1)^{20},$$

and hence

$$M(\widehat{B}) = N(\widehat{B}) + \widehat{B} + 2 \leq (2K_0 + 2)\, M(\widehat{B}-1)^{20}.$$

Thus (6) holds with

$$K := 2K_0 + 2.$$

Taking logarithms in (6) and iterating, we obtain

$$\log M(\widehat{B}) \leq 20^{\widehat{B}} \log M(0) + \left(1 + 20 + \cdots + 20^{\widehat{B}-1}\right) \log K$$

for all $\widehat{B} \geq 0$. Thus

$$\log M(\widehat{B}) \leq 20^{\widehat{B}} \log M(0) + \frac{20^{\widehat{B}} - 1}{19} \log K \leq \widehat{A}\, 20^{\widehat{B}},$$

where

$$\widehat{A} := \log M(0) + \frac{\log K}{19}.$$

Now fix $n \geq 2$ and put $\widehat{B} = b_n$. Then by definition of $N(\widehat{B})$, we have $n \leq N(\widehat{B}) < M(\widehat{B})$, and therefore

$$\log n \leq \log M(\widehat{B}) \leq \widehat{A}\, 20^{\widehat{B}}.$$

Taking logarithms once more, we get

$$\widehat{B} = b_n \geq \frac{\log\log n - \log \widehat{A}}{\log 20}.$$

Finally, since $a_n = n + b_n$, it follows that

$$a_n \geq n + \frac{\log\log n}{\log 20} - \frac{\log \widehat{A}}{\log 20}.$$

This proves the theorem. □

5 A polynomial upper bound for the classical sequence

In this section we prove Theorem 6. The argument combines the greedy ordering of the representable integers with a lower bound for difference sets of finite convex sets.

5.1 Greedy enumeration of the representable integers

We begin by isolating the integers that admit representations as sums of at least two consecutive terms. The key point is that the greedy construction does not merely produce some such integers: it lists all of them in increasing order.

Define the partial sums

$$s_0 := 0, \qquad s_m := \sum_{i=1}^{m} a_i \quad (m \geq 1),$$

and the prefix-sum set

$$\widetilde{S}_m := \{s_0, s_1, \ldots, s_m\}.$$

For $m \geq 2$ let

$$\mathcal{R}_m := \Big\{ \sum_{i=p}^{q} a_i : \ 1 \leq p < q \leq m \Big\} \subset \mathbb{N}, \qquad \mathcal{R} := \bigcup_{m \geq 2} \mathcal{R}_m.$$

Thus $\mathcal{R}$ is the set of all integers representable as a sum of at least two consecutive terms of the sequence.

Lemma 15. *We have $\{a_k : k \geq 3\} = \mathcal{R}$, and $(a_3, a_4, \ldots)$ is the increasing enumeration of $\mathcal{R}$. In particular, for every $X \geq 1$,*

$$\#(\mathcal{R} \cap [1, X]) \ \geq \ n \quad \Longrightarrow \quad a_{n+2} \leq X. \tag{7}$$

Proof. Every a_k with $k \geq 3$ belongs to $\mathcal{R}$ by definition. Conversely, let $x \in \mathcal{R}$. Since $a_k \to \infty$, choose the least $k \geq 3$ such that $x \leq a_k$. Then $a_{k-1} < x \leq a_k$. Suppose that $x < a_k$, and write $x = \sum_{i=p}^{q} a_i$ with $p < q$. If $q \geq k$, then positivity of the summands and monotonicity of (a_i) give $x \geq a_q \geq a_k$, a contradiction. Hence $q \leq k-1$. Since $a_{k-1} < x$ and x is a sum of at least two consecutive terms among $a_1, \ldots, a_{k-1}$, the integer x was one of the candidates in the definition of a_k. By the minimality of a_k, we must therefore have $a_k \leq x$. This contradicts the assumption $x < a_k$. Hence $x = a_k$. It follows that $(a_3, a_4, \ldots)$ is the increasing enumeration of $\mathcal{R}$, and (7) follows immediately. □

The preceding lemma reduces the problem of bounding a_n to that of producing many representable integers below a controlled threshold. The next observation makes this reduction quantitative at a finite stage.

Lemma 16. *For every $m \geq 2$, we have $\mathcal{R}_m \subseteq \mathcal{R} \cap [1, s_m]$. In particular, if $|\mathcal{R}_m| \geq n$ then*

$$a_{n+2} \leq s_m. \tag{8}$$

Proof. Every element of $\mathcal{R}_m$ is a sum of at least two consecutive terms among $a_1, \ldots, a_m$, hence belongs to $\mathcal{R}$. Such a sum is clearly at most $a_1 + \cdots + a_m = s_m$, so $\mathcal{R}_m \subseteq \mathcal{R} \cap [1, s_m]$.

If $|\mathcal{R}_m| \geq n$, then $\#(\mathcal{R} \cap [1, s_m]) \geq n$, and (8) follows from Lemma 15 and (7). □

We now turn to the task of estimating $|\mathcal{R}_m|$. The key idea is to rewrite sums of consecutive terms as differences of prefix sums, thereby bringing in additive-combinatorial results on convex sets.

5.2 From consecutive sums to a convex difference set

We recall the standard additive-combinatorial notion of convexity. We call a finite set $Y = \{y_1 < y_2 < \cdots < y_n\} \subset \mathbb{R}$ *convex* if its consecutive differences form a strictly increasing sequence, that is,

$$y_{i+1} - y_i < y_{i+2} - y_{i+1} \qquad (1 \leq i \leq n-2).$$

We now observe that the prefix-sum set $\widetilde{S}_m$ is convex in this sense.

Lemma 17. *For each $m \geq 2$, the set $\widetilde{S}_m = \{s_0 < s_1 < \cdots < s_m\}$ is convex, i.e.*

$$s_i - s_{i-1} < s_{i+1} - s_i \qquad (1 \leq i \leq m-1).$$

Proof. We have $s_i - s_{i-1} = a_i$ for all $i \geq 1$, and (a_i) is strictly increasing by construction. □

Since every sum of consecutive terms is a difference of two prefix sums, the difference set $\widetilde{S}_m - \widetilde{S}_m$ provides a natural ambient set in which $\mathcal{R}_m$ sits. The next lemma makes this relation explicit.

Lemma 18. *Let* $D_m := \widetilde{S}_m - \widetilde{S}_m = \{x - y : \ x, y \in \widetilde{S}_m\}$. *Then*

$$|\mathcal{R}_m| \geq \frac{|D_m| - 1}{2} - m.$$

Proof. Let

$$D_m^+ := \{d \in D_m : \ d > 0\}.$$

Since D_m is symmetric and contains 0, we have

$$|D_m^+| = \frac{|D_m| - 1}{2}.$$

Every $d \in D_m^+$ has a representation of the form

$$d = s_j - s_i = \sum_{t=i+1}^{j} a_t \qquad (0 \leq i < j \leq m).$$

If $j - i = 1$, then $d = a_j \in \{a_1, \ldots, a_m\}$. If $j - i \geq 2$, then $d \in \mathcal{R}_m$. Therefore

$$D_m^+ \subseteq \{a_1, \ldots, a_m\} \cup \mathcal{R}_m.$$

Hence

$$\frac{|D_m| - 1}{2} = |D_m^+| \leq m + |\mathcal{R}_m|,$$

which rearranges to

$$|\mathcal{R}_m| \geq \frac{|D_m| - 1}{2} - m. \qquad \square$$

5.3 Bloom's convex difference-set bound and the basic recursion

We now combine the convexity of the prefix-sum set with a recent difference-set estimate of Bloom. This gives a lower bound for $|\mathcal{R}_m|$ and, via the greedy enumeration above, a recursive upper bound for a_n.

Elekes, Nathanson, and Ruzsa [6] established foundational additive-growth estimates for convex sets. Schoen and Shkredov [14] later proved the predecessor bound

$$|A - A| \gg |A|^{8/5} (\log |A|)^{-2/5}.$$

We use the following more recent difference-set bound of Bloom [3, Theorem 2].

Theorem 19. *For every* $\eta > 0$ *there exists* $c_\eta > 0$ *such that for every finite convex set* $A \subset \mathbb{R}$,

$$|A - A| \geq c_\eta \, |A|^{6681/4175 - \eta}.$$

We apply this estimate to the convex set $\widetilde{S}_m$ and then use Lemma 18 to obtain the following lower bound for the number of representable integers up to stage m.

Proposition 20. *Fix $0 < \eta < 2506/4175$ and set $\delta = 6681/4175 - \eta$. Then there exist constants $m_0 \in \mathbb{N}$ and $c > 0$ (depending only on η) such that for all $m \geq m_0$,*

$$|\mathcal{R}_m| \geq c\, m^{\delta}.$$

Proof. By Lemma 17, the set $\widetilde{S}_m$ is convex and $|\widetilde{S}_m| = m + 1$. Applying Theorem 19 to $A = \widetilde{S}_m$ gives

$$|D_m| = |\widetilde{S}_m - \widetilde{S}_m| \geq c_\eta\, (m+1)^{\delta}.$$

Insert this into Lemma 18:

$$|\mathcal{R}_m| \geq \frac{c_\eta (m+1)^{\delta} - 1}{2} - m.$$

For all sufficiently large m the right-hand side is $\geq (c_\eta/4)\, m^{\delta}$, which proves this proposition with suitable choices of m_0 and c. □

The proposition shows that $\mathcal{R}_m$ grows polynomially with exponent $\delta > 1$. We now feed this counting information back into Lemma 16 to obtain the basic recursion for (a_n).

Lemma 21. *Fix $0 < \eta < 2506/4175$ and let $\delta = 6681/4175 - \eta > 1$. Then there exist constants $C_1 \geq 2$ and N_0 (depending only on η) such that for all $n \geq N_0$,*

$$a_n \;\leq\; C_1\, n^{1/\delta}\, a_{\lceil C_1\, n^{1/\delta} \rceil}. \tag{9}$$

Proof. Let c, m_0 be as in Proposition 20. For N large, set

$$m_1 := \left\lceil (N/c)^{1/\delta} \right\rceil.$$

Then $m_1 \geq m_0$ and $c m_1^{\delta} \geq N$, so $|\mathcal{R}_{m_1}| \geq N$. By Lemma 16,

$$a_{N+2} \leq s_{m_1}.$$

Since $a_i \leq a_{m_1}$ for $1 \leq i \leq m_1$, we have $s_{m_1} \leq m_1 a_{m_1}$, and hence

$$a_{N+2} \leq m_1 a_{m_1}.$$

Moreover, for N large enough,

$$m_1 = \left\lceil (N/c)^{1/\delta} \right\rceil \leq 2c^{-1/\delta} N^{1/\delta}.$$

Set

$$C_1 := \max\{2,\, 2c^{-1/\delta}\}.$$

Using the monotonicity of (a_n), we obtain

$$a_{N+2} \leq C_1 N^{1/\delta} a_{\lceil C_1 N^{1/\delta} \rceil}$$

for all sufficiently large N.

Now replace N by $n - 2$. For all sufficiently large n,

$$a_n \leq C_1 (n-2)^{1/\delta} a_{\lceil C_1 (n-2)^{1/\delta} \rceil} \leq C_1 n^{1/\delta} a_{\lceil C_1 n^{1/\delta} \rceil}.$$

This is (9) after renaming the lower threshold as N_0. □

5.4 Iteration and the final exponent

We now iterate the basic recursion from Lemma 21. The next lemma shows that any nondecreasing sequence satisfying such a recursion must obey a polynomial upper bound, up to a logarithmic factor.

Lemma 22. *Let (a_n) be nondecreasing and suppose there exist constants $C_1 \geq 2$, $\alpha \in (0,1)$ and N_0 such that*

$$a_n \leq C_1\, n^{\alpha}\, a_{\lceil C_1 n^{\alpha} \rceil} \qquad (n \geq N_0).$$

Then there exist constants $B, C_2 > 0$ such that for all $n \geq 3$,

$$a_n \leq C_2\, n^{\alpha/(1-\alpha)}\, (\log n)^B.$$

Proof. Enlarge N_0 if necessary so that

$$\lceil C_1 x^{\alpha} \rceil \leq 2C_1 x^{\alpha} \leq x - 1 \qquad (x \geq N_0),$$

and also

$$\frac{\log(2C_1)}{1-\alpha} \leq \frac{1}{2} \log N_0.$$

Fix $n > N_0$, and define

$$\widetilde{n}_0 := n, \qquad \widetilde{n}_{j+1} := \lceil C_1 \widetilde{n}_j^{\alpha} \rceil \quad (j \geq 0)$$

for as long as $\widetilde{n}_j > N_0$. Since $\widetilde{n}_{j+1} \leq \widetilde{n}_j - 1$ whenever $\widetilde{n}_j \geq N_0$, the positive integer sequence $(\widetilde{n}_j)$ strictly decreases until it first reaches the interval $[1, N_0]$. Hence there exists a minimal $t = t(n)$ such that $\widetilde{n}_t \leq N_0$.

Repeatedly applying the assumed recursion gives

$$a_n \;\leq\; \Big(\prod_{j=0}^{t-1} C_1 \widetilde{n}_j^{\alpha} \Big)\, a_{\widetilde{n}_t}.$$

Since $\widetilde{n}_t \leq N_0$, we have

$$a_{\widetilde{n}_t} \leq C_0 := \max_{1 \leq r \leq N_0} a_r.$$

Also, from $\widetilde{n}_{j+1} \leq 2C_1 \widetilde{n}_j^{\alpha}$ we obtain by induction

$$\widetilde{n}_j \;\leq\; (2C_1)^{1+\alpha+\cdots+\alpha^{j-1}}\, n^{\alpha^j} \;\leq\; (2C_1)^{1/(1-\alpha)}\, n^{\alpha^j}.$$

Therefore

$$\prod_{j=0}^{t-1} \widetilde{n}_j^{\alpha} \leq (2C_1)^{\alpha t/(1-\alpha)}\, n^{\alpha+\alpha^2+\cdots+\alpha^t} \leq (2C_1)^{\alpha t/(1-\alpha)}\, n^{\alpha/(1-\alpha)}.$$

Combining the previous estimates yields

$$a_n \;\leq\; C_0\, C_1^{\,t}\, (2C_1)^{\alpha t/(1-\alpha)}\, n^{\alpha/(1-\alpha)}.$$

It remains to bound t. Write $x_j := \log \widetilde{n}_j$. Since

$$\widetilde{n}_{j+1} \le 2C_1 \widetilde{n}_j^{\alpha},$$

we have

$$x_{j+1} \le \alpha x_j + \log(2C_1).$$

Iterating this inequality gives

$$x_j \le \alpha^j \log n + \frac{\log(2C_1)}{1-\alpha}.$$

By the choice of N_0,

$$\frac{\log(2C_1)}{1-\alpha} \le \frac{1}{2}\log N_0.$$

Hence, whenever

$$\alpha^j \log n \le \frac{1}{2}\log N_0,$$

we obtain $x_j \le \log N_0$, and therefore $\widetilde{n}_j \le N_0$. Thus t is at most the least integer j such that

$$\alpha^j \log n \le \frac{1}{2}\log N_0.$$

Equivalently, since $0 < \alpha < 1$,

$$j \ge \frac{\log\log n - \log\left(\frac{1}{2}\log N_0\right)}{\log(1/\alpha)}.$$

It follows that

$$t \ll \log\log n.$$

Consequently,

$$C_1^t (2C_1)^{\alpha t/(1-\alpha)} = (\log n)^{O(1)},$$

and the claimed bound follows for all $n > N_0$. Enlarging C_2 covers the finitely many values $3 \le n \le N_0$. □

We are now ready to present the following.

Proof of Theorem 6. Fix $\varepsilon > 0$. Choose $0 < \eta < 2506/4175$ so small that

$$\frac{1}{(6681/4175 - \eta) - 1} \;\le\; \frac{4175}{2506} + \frac{\varepsilon}{2}. \tag{10}$$

Set $\delta = 6681/4175 - \eta$ and $\alpha = 1/\delta \in (0,1)$. By Lemma 21, there exist constants $C_1 \ge 2$ and N_0 such that

$$a_n \le C_1\, n^{\alpha}\, a_{\lceil C_1 n^{\alpha} \rceil} \qquad (n \ge N_0).$$

Applying Lemma 22 yields constants $B, C_2 > 0$ such that

$$a_n \le C_2\, n^{\alpha/(1-\alpha)}\, (\log n)^B \qquad (n \ge 3).$$

Since $\alpha/(1-\alpha) = 1/(\delta - 1)$, the choice (10) gives

$$n^{\alpha/(1-\alpha)} \;\le\; n^{4175/2506+\varepsilon/2}.$$

Finally, $(\log n)^B \le n^{\varepsilon/2}$ for all sufficiently large n, so

$$a_n \;\le\; C_\varepsilon\, n^{4175/2506+\varepsilon}$$

for all large n, and we adjust C_ε to cover the finitely many smaller n. □

6 Concluding remarks

6.1 Generalized seeds of length at least two

Although the body of this paper has focused exclusively on the classical seed $(1, 2)$, one may more generally start with any fixed finite sequence of positive integers $u_1, \dots, u_s$, where $s \ge 2$, and thereafter define each new term by the same greedy rule. The preceding arguments appear to extend to this setting after suitable modifications involving the finite initial segment. If the initial seed is not increasing, the analogue of $u_n - n$ is nondecreasing only from the beginning of the strictly increasing greedy tail, and the convex prefix-sum set used in Section 5 should be formed from that tail rather than from the entire sequence. Additional bookkeeping is also needed for consecutive sums that meet the initial seed. We do not pursue these modifications here and make no formal claim concerning generalized seeds.

6.2 Later upper-bound improvements

After we completed this paper, Sothanaphan pointed out in the comments on Erdős Problem #423 [4] that combining Cushman's recent bound

$$|A - A| \gg_\varepsilon |A|^{8/5+1/3440-\varepsilon}$$

for finite convex sets with the argument of Section 5 yields the improved estimate

$$a_n \ll_\varepsilon n^{688/413+\varepsilon}.$$

See [16, 5]. Since this refinement does not alter the underlying method, we have chosen to keep the original exposition.

6.3 Open questions

A substantial gap remains between Theorems 5 and 6. It seems likely that the true growth of a_n is much closer to linear than our present upper bound indicates. Writing

$$b_n := a_n - n,$$

a natural next step is to determine the true order of magnitude of b_n. Most notably, one may ask whether b_n is sublinear.

Problem 23. Is it true that

$$a_n = n + o(n)?$$

Equivalently, does $b_n = o(n)$ hold?

To complement this problem, we computed b_n for $1 \le n \le 30000$ and plotted the normalized quantities $b_n/n^{1/k}$ for $k = 5, 4, 3, 2$; see Figure 1. The ratio $b_n/n^{1/2}$ appears to decrease with n, whereas $b_n/n^{1/5}$ appears to increase, which is consistent with a power-law heuristic of the form

$$b_n = \Theta\big(n^{\alpha+o(1)}\big) \qquad (n \to \infty)$$

for some exponent α satisfying $1/5 < \alpha < 1/2$. Among these plots, the curve $b_n/n^{1/3}$ appears comparatively flat on the computed range, suggesting that α might be close to $1/3$, although the available data are far from decisive.

7 Acknowledgments

An earlier version of this paper, containing only the qualitative conclusion $a_n = n + \omega(1)$, appeared as a short note in the comments section of the *Erdős Problems* website [4]. Matthew Bolan also independently obtained the qualitative result that the classical sequence omits infinitely many positive integers; see again the comments section of the *Erdős Problems* website [4]. We are grateful to Ingo Althöfer for encouraging us to think about extensions to more general initial seeds. We thank Yann Bugeaud for pointing out that one can strengthen the method in our proof of $a_n = n + \omega(1)$ to obtain the bound of Theorem 5. We thank Norbert Hegyvári for bringing the papers [1, 11, 18] on consecutive sums to our attention, and Boris Alexeev and Wolfram Bernhardt for computing and sharing the numerical data posted in the comments section of the *Erdős Problems* website [4]. Finally, we thank the editor for careful editorial guidance and the anonymous referees for their helpful comments and suggestions, which improved the presentation of the manuscript.

Declaration of AI usage. The author used an AI assistant only during the brainstorming phase related to Theorem 6 and for drafting the Python code that generated Figure 1. Apart from this limited assistance, the author carried out all mathematical arguments, proofs, and writing.

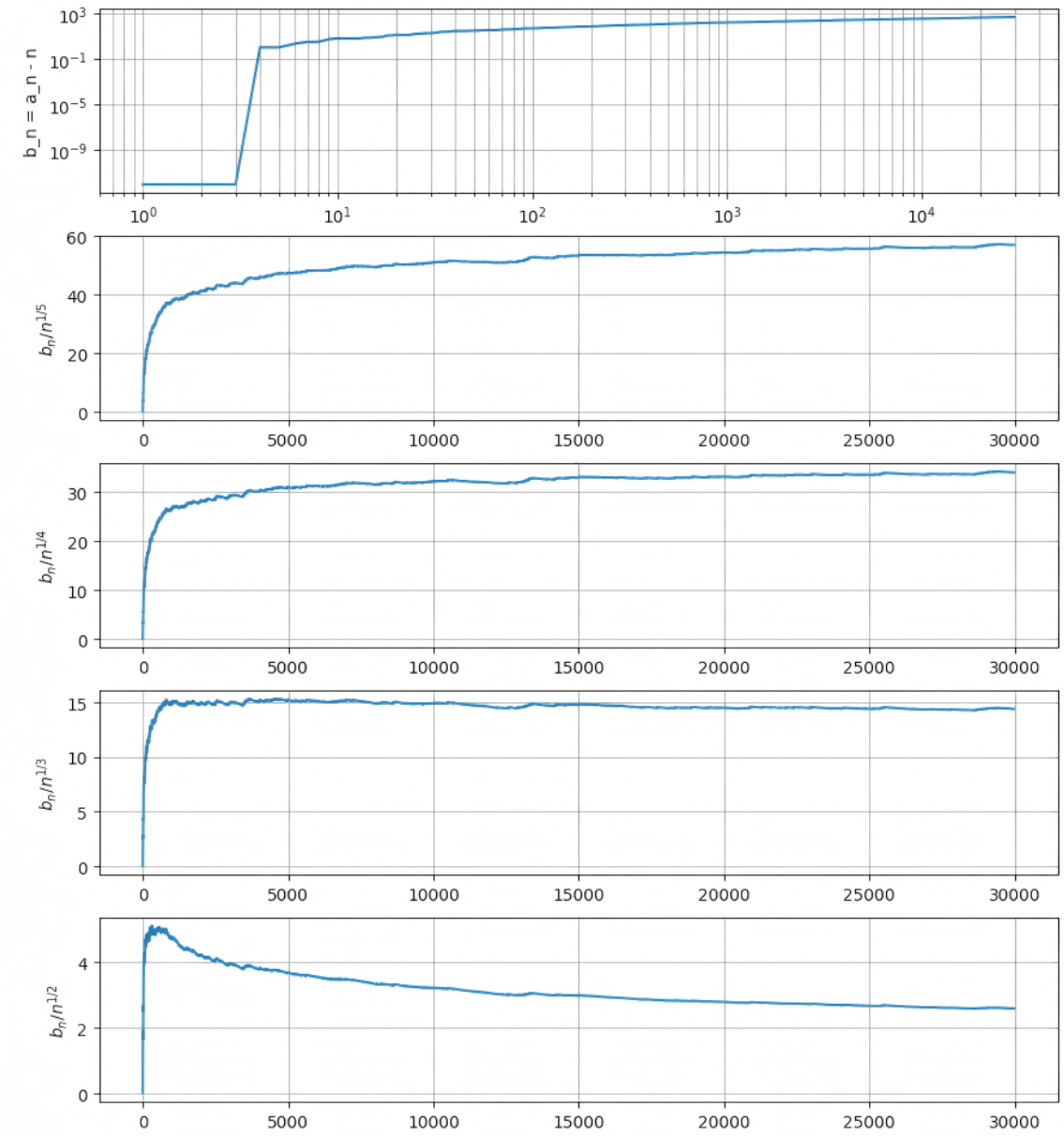


Figure 1: Numerical data for the sequence in Definition 1: the top panel shows $b_n = a_n - n$ (on a logarithmic n-axis), and the remaining panels plot the ratios $b_n/n^{1/k}$ for $k = 5, 4, 3, 2$ over $1 \leq n \leq 30000$.

References

[1] A. Beker, On a problem of Erdős and Graham about consecutive sums in strictly increasing sequences, *Bull. Lond. Math. Soc.* **56** (2024), 2749–2759.

[2] F. Beukers, On the generalized Ramanujan–Nagell equation I, *Acta Arith.* **38** (1981), 389–410.

[3] T. F. Bloom, Control and its applications in additive combinatorics, arxiv preprint arXiv:2501.09470 [math.NT], 2025. Available at https://arxiv.org/abs/2501.09470.

[4] T. F. Bloom, Erdős Problem #423, 2026. Available at https://www.erdosproblems.com/423.

[5] A. Cushman, A note on the sum-product problem and the convex sumset problem, arxiv preprint arXiv:2512.13849 [math.CO], 2025. Available at https://arxiv.org/abs/2512.13849.

[6] G. Elekes, M. B. Nathanson, and I. Z. Ruzsa, Convexity and sumsets, *J. Number Theory* **83** (2000), 194–201.

[7] P. Erdős, Problems and results on combinatorial number theory, III, in M. B. Nathanson, ed., *Number Theory Day*, Lect. Notes in Math., Vol. 626, Springer, 1977, pp. 43–72.

[8] P. Erdős and R. L. Graham, *Old and New Problems and Results in Combinatorial Number Theory*, Monographies de L'Enseignement Mathématique, Vol. 28, Université de Genève, L'Enseignement Mathématique, 1980.

[9] C. Gamer, D. W. Roeder, and J. J. Watkins, Trapezoidal numbers, *Math. Mag.* **58** (1985), 108–110.

[10] R. K. Guy, *Unsolved Problems in Number Theory*, 3rd ed., Problem Books in Mathematics, Springer, 2004.

[11] N. Hegyvári, On consecutive sums in sequences, *Acta Math. Hungar.* **48** (1986), 193–200.

[12] C. Ho, T.-X. He, and P. J.-S. Shiue, Representations of positive integers as sums of arithmetic progressions, I, *Notes Number Theory Discrete Math.* **29** (2023), 241–259.

[13] A. Schinzel and R. Tijdeman, On the equation $y^m = P(x)$, *Acta Arith.* **31** (1976), 199–204.

[14] T. Schoen and I. D. Shkredov, On sumsets of convex sets, *Combin. Probab. Comput.* **20** (2011), 793–798.

[15] N. J. A. Sloane, *The On-Line Encyclopedia of Integer Sequences*, OEIS Foundation Inc., 2026. Available at https://oeis.org.

[16] N. Sothanaphan, *erdos-problem-423-upper-bound-update*, Zenodo, 2026. Available at https://doi.org/10.5281/zenodo.19021757.

[17] J. J. Sylvester and F. Franklin, A constructive theory of partitions, arranged in three acts, an interact and an exodion, *Amer. J. Math.* **5** (1882), 251–330.

[18] T. Tao, A set that represents all large integers multiple times by consecutive elements, preprint, 2026. Available at https://terrytao.wordpress.com/wp-content/uploads/2026/02/erdos-358-2.pdf.

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