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Proofs of Several Conjectures From the OEIS

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Abstract

We prove several conjectures from the On-Line Encyclopedia of Integer Sequences, concerned with constrained lattice point enumeration, pattern-avoiding words, rounding-related formulas, and greedily defined sequences. Our methods are mostly elementary and include generating functions, recurrence analysis, and symbolic combinatorics.

1 Introduction

The On-Line Encyclopedia of Integer Sequences (OEIS) [7] is an indispensable tool for mathematicians and computer scientists. It catalogs over 385,000 sequences, many of which are accompanied by conjectural closed-form formulas, recurrence relations, generating functions, or asymptotic estimates. For example, Stephan [8] listed 100 such conjectures. These conjectures provide fertile ground for mathematical exploration, and their resolution can yield elegant combinatorial or analytic insights.

In this work, we prove several conjectures from the OEIS, covering a diverse range of topics and techniques. Specifically, we address problems from the following areas:

1. Lattice point enumeration. We compute the generating function corresponding to a certain constrained lattice points enumeration problem and extract exact formulas (cf. A371835).

2. Pattern-avoiding words. We compute the generating functions corresponding to the enumeration of words over finite alphabets that avoid the patterns $z, z+1, z$ and $z, z, z+1$ (cf. A005251 and A206790).

3. Restricted repeating letters. We compute the generating function corresponding to the enumeration of words over finite alphabets with a certain restriction regarding repeating letters (cf. A269467).

4. Rounding-related identities. We verify conjectured closed-form formulas for integer sequences involving central binomial coefficients and sums of cube roots (cf. A112884 and A136269).

5. Median absolute deviation. We determine the asymptotic behavior of the median absolute deviation of a certain set, confirming the conjecture in A345318.

6. Greedily defined sequences. Not directly addressing a conjecture, we prove that A128135 is a greedily defined sequence. We also propose our own conjecture related to the Fibonacci numbers (cf. A248982).

Let us introduce some notation: The set of natural numbers $\{1, 2, \ldots\}$ is denoted by $\mathbb{N}$. For any real number x, the floor of x, denoted by $\lfloor x \rfloor$, is the greatest integer less than or equal to x.

2 Lattice point enumeration

Let k and n be two nonnegative integers and let $m \in \mathbb{N}$. We let $a_{n,k}^{(m)}$ denote the number of integer points $(x_1, \ldots, x_m)$, such that $\max_{1\le i\le m} |x_i| \le k$ and $\sum_{i=1}^{m} |x_i| \le n$. Sequence A371835 is concerned with the array $\left(a_{n,k}^{(3)}\right)_{\substack{n\ge 0\\ 0\le k\le n}}$.

We let $b_{n,k}^{(m)}$ denote the number of integer points $(x_1, \ldots, x_m)$, such that $\max_{1\le i\le m} |x_i| \le k$ and $\sum_{i=1}^{m} |x_i| = n$. For fixed k and m, we let $B_k^{(m)}(x)$ denote the generating function for the numbers $b_{n,k}^{(m)}$. In the following theorem we calculate $B_k^{(m)}(x)$. We then use this result to resolve and extend a conjecture stated in A371835, regarding a closed-form formula for $a_{n,k}^{(3)}$.

Theorem 1. *We have*

$$B_k^{(m)}(x) = \left(1 + \frac{2x(1-x^k)}{1-x}\right)^m. \tag{1}$$

Proof. Clearly, $b_{n,k}^{(m)}$ is also the number of compositions of n into m parts such that the absolute value of each part is $\le k$. We use the symbolic method (e.g., [2, I.3]). To this end, let $\mathcal{I}_k = \{0, \pm 1, \pm 2, \ldots, \pm k\}$ and let the size of an element in $\mathcal{I}_k$ be its absolute value. Then the corresponding generating function, for one part, is given by

$$I_k(x) = 1 + 2\sum_{i=1}^{k} x^i = 1 + \frac{2x(1-x^k)}{1-x}.$$

Consequently, the generating function for m parts is given by $(I_k(x))^m$ and the assertion follows. ∎

Corollary 2. *We have*

$$a_{n,k}^{(3)} = \frac{1}{3}\begin{cases} 4n^3+6n^2+8n+3, & \text{if } 0 \le n < k;\\ 12k^3-36k^2n+36kn^2-8n^3+6n^2+6k+2n+3, & \text{if } k \le n < 2k;\\ -84k^3+108k^2n-36kn^2+4n^3-72k^2+72nk-12n^2-6k+8n+3, & \text{if } 2k \le n < 3k;\\ 24k^3+36k^2+18k+3, & \text{otherwise.}\end{cases}$$

Proof. For fixed k and m, we let $A_k^{(m)}(x)$ denote the generating function for the numbers $a_{n,k}^{(m)}$. Clearly, $A_k^{(m)}(x) = B_k^{(m)}(x)/(1-x)$. Thus

$$A_k^{(3)}(x) = \frac{-8x^{3k+3}+12x^{2k+3}+12x^{2k+2}-6x^{k+3}-12x^{k+2}-6x^{k+1}+x^3+3x^2+3x+1}{(1-x)^4}.$$

Extracting coefficients, which is routine (e.g., [11, (2.5.7) on p. 53]), proves the stated formulas. □

3 Pattern-avoiding words

3.1 Words avoiding the patterns $z, z+1, z$ and $z, z, z+1$

The systematic study of words avoiding specific patterns was initiated by Guibas and Odlyzko [3]. Although the two cases considered here fall within the scope of their general results, we present direct and self-contained arguments. Let $k \ge 2$ be an integer. We let $[k]$ denote the set $\{1, 2, \ldots, k\}$. For a nonnegative integer n, a word over k of length n is an element of $[k]^n$. A word $w_1 \cdots w_n \in [k]^n$ is said to avoid the pattern $z, z+1, z$ if no $1 \le i \le n-2$ and $z \in [k]$ exist such that $w_i = w_{i+2} = z$ and $w_{i+1} = z+1$. Avoidance of the other pattern $z, z, z+1$ is defined similarly. We refer to a word avoiding the pattern in question as legal. The first part of the following theorem corresponds to A005251 by taking $k = 2$, to A098182 by taking $k = 3$ (giving the sequence a combinatorial interpretation), and to A206790 by taking $k = 4$ (proving the conjectures stated therein). The second part corresponds to A000071 by taking $k = 2$, to A206727 by taking $k = 3$ (proving the conjectures stated therein), and to A206570 by taking $k = 4$ (proving the conjectures stated therein).

Theorem 3.

(a) *Let $f_k(n)$ denote the number of words over k of length n that avoid the pattern $z, z+1, z$. Then, for $n \ge 3$, the numbers $f_k(n)$ satisfy the recursion*

$$f_k(n) = kf_k(n-1) - f_k(n-2) + f_k(n-3),$$

with initial values $f_k(0) = 1, f_k(1) = k$, and $f_k(2) = k^2$. In particular, the corresponding generating function is given by

$$\frac{1+x^2}{1-kx+x^2-x^3}.$$

(b) Let $f_k(n)$ denote the number of words over k of length n that avoid the pattern $z, z, z+1$. Then, for $n \geq 2k-1$, the numbers $f_k(n)$ satisfy the recursion

$$f_k(n) = \sum_{i=0}^{k-1} (-1)^i (k-i) f_k(n-2i-1).$$

Furthermore, the corresponding generating function is given by

$$\frac{1}{1 - \sum_{i=0}^{k-1} (-1)^i (k-i) x^{2i+1}}.$$

Proof.

(a) We derive the recurrence for $f_k(n)$ as follows: To construct a legal word of length n, consider all legal words of length $n-1$ and append one of the k letters. This gives $kf_k(n-1)$ possibilities. However, some of these extensions may introduce the forbidden pattern at the end of the word. Such a pattern is created only when we append $z+1, z$ to a legal word of length $n-2$ ending with z. Thus, we must subtract $f_k(n-2)$ to remove these cases. This subtraction, however, erroneously includes cases where $z = k$ (since $k+1$ is not in the alphabet and hence the pattern $k, k+1, k$ cannot occur). Thus, we must add back the number of legal words of length $n-2$ ending with k. These are precisely the words obtained by appending the letter k to a legal word of length $n-3$. This gives the term $f_k(n-3)$.

(b) We derive the recurrence for $f_k(n)$ using inclusion-exclusion as follows: To construct a legal word of length n, consider all legal words of length $n-1$ and append one of the k letters. This gives $kf_k(n-1)$ possibilities. This extension however may introduce words having $z, z, z+1$ at the end. Such words arise from legal words of length $n-3$ by appending $z, z, z+1$ for some $z \leq k-1$. Hence, we subtract $(k-1)f_k(n-3)$. Now we need to add back words that end with $z, z, z+1, z+1, z+2$ for some $z \leq k-2$, since they have an earlier occurrence of the pattern. Thus we add $(k-2)f_k(n-5)$. Now we need to subtract words that end with $z, z, z+1, z+1, z+2, z+2, z+3$ for some $z \leq k-3$ for the same reason. Thus, we subtract $(k-3)f_k(n-7)$. Proceeding in this manner, the recursion follows.

$\square$

3.2 Restricted repeating letters

In the following theorem we resolve some of the conjectures stated in A269467. The sequence is concerned with the number of legal words over k of length n, denoted by $f_k(n)$, where legal here means that no repeating letter is equal to the previous repeating letter (in particular, it is not allowed to have three consecutive equal letters). More precisely, a word $w_1 \cdots w_n \in [k]^n$

is legal if for every $1 \leq i < j \leq n-1$ and $s \in [k]$, such that $w_i = w_{i+1} = w_j = w_{j+1} = s$, there exist $i < m < j$ and $t \in [k]$ such that $t \neq s$ and $w_m = w_{m+1} = t$. For example, the word 11233 is legal while the words 111233 and 11211 are not.

Theorem 4. *Let $F_k(x)$ denote the generating function for the sequence $(f_k(n))_{n\geq 1}$. Then*

$$F_k(x) = -\frac{(k-2)x^3 + 2(k-2)x^2 + (k-3)x - 1}{2(k-1)^2x^3 + (k-1)(k-4)x^2 + (3-2k)x + 1}. \tag{2}$$

Proof. Let $0 \leq u \leq k$ and $1 \leq v \leq k$. We let $f_{k,u,v}(n)$ denote the number of legal words over k of length n whose last letter is v and whose last repeating letter is u. Notice that $u = 0$ encodes legal words having no repeating letters at all and therefore $\sum_{v=1}^{k} f_{k,0,v}(n) = k(k-1)^{n-1}$. For $n \geq 3$ we have

$$f_{k,u,v}(n) = \begin{cases} \sum_{1\leq t\leq k, t\neq v} f_{k,u,t}(n-1) + \sum_{0\leq t\leq k, t\neq v} f_{k,t,u}(n-1), & \text{if } v = u; \\ \sum_{1\leq t\leq k, t\neq v} f_{k,u,t}(n-1), & \text{if } v \neq u. \end{cases}$$

Then

$$\begin{aligned} f_k(n) &= \sum_{u=0}^{k}\sum_{v=1}^{k} f_{k,u,v}(n) \\ &= \sum_{u=0}^{k}\sum_{v=1}^{k}\sum_{1\leq t\leq k, t\neq v} f_{k,u,t}(n-1) + \sum_{v=1}^{k}\sum_{0\leq t\leq k, t\neq v} f_{k,t,v}(n-1) \\ &= \sum_{v=1}^{k}\sum_{u=0}^{k}\left(\sum_{t=1}^{k} f_{k,u,t}(n-1) - f_{k,u,v}(n-1)\right) + \sum_{v=1}^{k}\left(\sum_{t=0}^{k} f_{k,t,v}(n-1) - f_{k,v,v}(n-1)\right) \\ &= kf_k(n-1) - \sum_{v=1}^{k} f_{k,v,v}(n-1). \end{aligned} \tag{3}$$

Now

$$\begin{aligned} \sum_{v=1}^{k} f_{k,v,v}(n-1) &= \sum_{v=1}^{k}\left(\sum_{t=1}^{k} f_{k,v,t}(n-2) + \sum_{t=0}^{k} f_{k,t,v}(n-2) - 2f_{k,v,v}(n-2)\right) \\ &= 2f_k(n-2) - \sum_{t=1}^{k} f_{k,0,t}(n-2) - 2\sum_{v=1}^{k} f_{k,v,v}(n-2) \\ &= 2f_k(n-2) - k(k-1)^{n-3} - 2\sum_{v=1}^{k} f_{k,v,v}(n-2). \end{aligned} \tag{4}$$

Iterating (4) until we reach $\sum_{v=1}^{k} f_{k,v,v}(2)$, which is equal to k, and substituting the result into (3), we obtain the equation

$$f_k(n) = kf_k(n-1) + \sum_{i=1}^{n-3}\left((-2)^i f_k(n-1-i) + (-2)^{i-1}k(k-1)^{n-2-i}\right) - (-2)^{n-3}k.$$

Multiplying both sides of the equation by x^n and summing over $n \geq 4$, we may (eventually) solve for $F_k(x)$ and obtain (2). $\square$

Corollary 5. *The denominator of $F_k(x)$ confirms the conjectured recurrences stated in A269467.*

4 Rounding-related identities

4.1 The number of bits required to represent $\binom{2^n}{2^{n-1}}$

The statement of the following theorem was conjectured in A112884.

Theorem 6. *Let $n \in \mathbb{N}$. The number of binary bits required to represent $\binom{2^n}{2^{n-1}}$ is $2^n - \lfloor n/2 \rfloor$.*

Proof. The number of binary bits required to represent a nonnegative integer m is $\lfloor \log_2 m \rfloor + 1$. Now the number $\binom{2^n}{2^{n-1}}$ is actually a central binomial coefficient, for which many bounds exist. We use the following elementary bound stated in [9, (21)]:

$$\frac{4^m}{\sqrt{4m}} < \binom{2m}{m} < \frac{4^m}{\sqrt{3m+1}}, \quad m \in \mathbb{N}.$$

Plugging $m = 2^n$ and applying the logarithm, we then have

$$2^n - \frac{n}{2} - \frac{1}{2} < \log_2 \binom{2^n}{2^{n-1}} < 2^n - \frac{n}{2} + \frac{1}{2} - \frac{1}{2}\log_2 3 \approx 2^n - \frac{n}{2} - 0.292,$$

from which the assertion immediately follows. $\square$

4.2 The floor of the sum of the first 10^n cube roots

The statement of the following theorem was conjectured in A136269.

Theorem 7. *Let $n \in \mathbb{N}$ be divisible by 3. Then*

$$\left\lfloor \sum_{i=1}^{10^n} \sqrt[3]{i} \right\rfloor = \frac{3 \cdot 10^{4n/3}}{4} + 5 \cdot 10^{n/3-1} - 1.$$

Proof. For every $m \in \mathbb{N}$ and any real $r \geq 1$, we have by [6],

$$\sum_{i=1}^{m} i^{\frac{1}{r}} = \frac{r}{r+1}(m+1)^{\frac{1+r}{r}} - \frac{1}{2}(m+1)^{\frac{1}{r}} - \phi_m(r),$$

where $0 \leq \phi_m(r) \leq \frac{1}{2}$. Taking $r = 3$ and $m = 10^n - 1$, we obtain

$$\frac{3 \cdot 10^{4n/3}}{4} - \frac{10^{n/3}}{2} - \frac{1}{2} \leq \sum_{i=1}^{10^n-1} \sqrt[3]{i} \leq \frac{3 \cdot 10^{4n/3}}{4} - \frac{10^{n/3}}{2}. \tag{5}$$

Adding $\sqrt[3]{10^n}$ to (5) proves the assertion. $\square$

5 Median absolute deviation

Sequence A345318 is concerned with a statistical measure of spread, called the median absolute deviation (e.g., [4, p. 291]), of the set $\{2k^2 \ : \ k = 1, \ldots, n\}$. Notice that the purpose of the factor 2 in $2k^2$ is merely to ensure that the resulting sequence is an integer sequence. We omit this factor in our analysis.

Theorem 8. *Let $n \in \mathbb{N}$ and set $A_n = \{k^2 \ : \ k = 1, 2, \ldots, n\}$. Let a_n denote the median absolute deviation of A_n, i.e.,*

$$a_n = \text{median}(\{|x - \text{median}(A_n)| \ : \ x \in A_n\}).$$

Then $\lim_{n\to\infty} a_n/n^2 = \sqrt{3}/8$.

Proof. Let $m_n = \text{median}(A_n)$. Then $m_n = n^2/4 + O(n)$. Let $\delta \in (0, 1/4)$ and let $N(n, \delta)$ denote the number of values of k such that $|k^2 - m_n| \le \delta n^2$. Then

$$N(n, \delta) = \sqrt{m_n + \delta n^2} - \sqrt{m_n - \delta n^2} + O(1).$$

Thus,

$$\frac{N(n,\delta)}{n} = \sqrt{\frac{1}{4} + \delta + O\left(\frac{1}{n}\right)} - \sqrt{\frac{1}{4} - \delta + O\left(\frac{1}{n}\right)} + O\left(\frac{1}{n}\right) \underset{n\to\infty}{\longrightarrow} \sqrt{\frac{1}{4} + \delta} - \sqrt{\frac{1}{4} - \delta}.$$

The equation

$$\sqrt{\frac{1}{4} + \delta} - \sqrt{\frac{1}{4} - \delta} = \frac{1}{2}$$

has a unique solution in $(0, 1/4)$, namely, $\sqrt{3}/8$, and the assertion follows. $\square$

6 Greedily defined sequences

In the following theorem we show how sequence A128135 emerges as a subsequence of a sequence defined by a greedy integer recurrence. Such greedily defined integer sequences have been studied by Venkatachala [10], Avdispahić and Zejnulahi [1], and Shallit [5]. While the result we obtain does not directly settle a conjecture from the OEIS, we observed it while working on a conjecture stated in A248982. We shall elaborate on it at the end of this section.

Theorem 9. *Let $a_1 = 1$ and, for $n \ge 2$, let a_n be the least positive integer such that the average of $a_1, \ldots, a_{n-1}$ is a power of 2. Then*

$$a_n = \begin{cases} (n+1)2^{\frac{n}{2}-1}, & \text{if } n \text{ is even;} \\ 2^{\frac{n-1}{2}}, & \text{otherwise.} \end{cases}$$

Proof. Set $s_n = \sum_{i=1}^{n} a_i$. We claim that $s_n = n2^{\lfloor \frac{n}{2} \rfloor}$ (this sequence is A132344). To see that, we proceed by induction on n. The base case, namely $n = 1$, obviously holds. Now assume that the assertion holds for every $1 \le i \le n-1$, where $n \ge 2$. Let ℓ be a nonnegative integer such that $s_n = n2^{\ell}$. We have

$$a_n = s_n - s_{n-1} = n2^{\ell} - (n-1)2^{\lfloor \frac{n-1}{2} \rfloor}. \tag{6}$$

Clearly, if $\ell \ge \lfloor (n-1)/2 \rfloor$, then $a_n > 0$. We claim that the converse also holds. Indeed, suppose that $a_n > 0$ but $\ell < \lfloor (n-1)/2 \rfloor$. Then

$$n2^{\lfloor \frac{n-1}{2} \rfloor - 1} - (n-1)2^{\lfloor \frac{n-1}{2} \rfloor} > 0 \iff \frac{n}{n-1} > 2.$$

But since $n \ge 2$, we have $n/(n-1) \le 2$.

Now assume that n is even. If $\ell = \lfloor (n-1)/2 \rfloor$, then, by (6) and the induction hypothesis, $a_n = 2^{\lfloor \frac{n-1}{2} \rfloor} = 2^{\frac{n-2}{2}} = a_{n-1}$, in violation of the distinctness condition. Trying the next best candidate $\ell = \lfloor (n-1)/2 \rfloor + 1$, we have, by (6),

$$a_n = n2^{\lfloor \frac{n-1}{2} \rfloor + 1} - (n-1)2^{\lfloor \frac{n-1}{2} \rfloor} = (n+1)2^{\lfloor \frac{n-1}{2} \rfloor},$$

which is obviously adequate. Thus, $s_n = n2^{\lfloor \frac{n}{2} \rfloor}$.

Now consider a_{n+1} and let ℓ be a nonnegative integer such that $s_{n+1} = (n+1)2^{\ell}$. We have

$$a_{n+1} = s_{n+1} - s_n = (n+1)2^{\ell} - n2^{\lfloor \frac{n}{2} \rfloor}.$$

Here, $\ell = \lfloor n/2 \rfloor$ is possible, leading to $a_{n+1} = 2^{\lfloor \frac{n}{2} \rfloor}$ and $s_{n+1} = (n+1)2^{\lfloor \frac{n}{2} \rfloor}$, concluding the proof of the induction step. □

As mentioned earlier, we observed the statement of the previous theorem while working on a conjecture stated in A248982, which is defined to be the sequence of distinct least positive numbers such that the average of the first n terms is a Fibonacci number. Let $(a_n)_{n \ge 1}$ be this sequence. Refining the conjecture stated in A248982 regarding a closed-form formula for $(a_n)_{n \ge 1}$, it seems that, for $n \ge 10$, we have

$$a_n = \begin{cases} nF\left(\frac{n}{2} + 3\right) - (n-1)F\left(\frac{n}{2} + 2\right), & \text{if } n \text{ is even;} \\ F\left(\frac{n+1}{2} + 2\right), & \text{otherwise.} \end{cases}$$

A proof of this likely proceeds along similar lines as the previous theorem. Nevertheless, we were not able to show that the two sets

$$\left\{ nF\left(\frac{n}{2} + 3\right) - (n-1)F\left(\frac{n}{2} + 2\right) \ : \ n \ge 1 \text{ is even} \right\},$$
$$\left\{ F\left(\frac{n+1}{2} + 2\right) \ : \ n \ge 1 \text{ is odd} \right\},$$

are disjoint, or, equivalently, that for every $n \in \mathbb{N}$, the number $F(n+2) + 2nF(n+1)$ is not a Fibonacci number. We conjecture that this is so. Notice that a similar sequence is the Les Marvin sequence A007502$(n) = F(n) + (n-1)F(n-1)$.

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