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On The Distribution of Consecutive Generalized r-Free Integers in Piatetski-Shapiro Sequences

Angkana Sripayap and Teerapat Srichan[1]
Department of Mathematics
Faculty of Science
Kasetsart University
Bangkok 10900
Thailand
fscianr@ku.ac.th
fscitrp@ku.ac.th

Pinthira Tangsupphathawat
Department of Mathematics
Faculty of Science and Technology
Phranakhon Rajabhat University
Bangkok 10220
Thailand
t.pinthira@hotmail.com

Abstract

We use Rieger's technique to generalize a previous result on the distribution of consecutive r-free integers in Piatetski-Shapiro sequences.

1 Introduction

Let r be a fixed integer ≥ 2. A positive integer n is called r-free whenever it is not divisible by the r-th power of a prime. By convention, 2-free and 3-free integers are called square-free and cube-free, respectively. The Piatetski-Shapiro sequence of parameter c is defined by

$$\mathbb{N}^c = \{\lfloor n^c \rfloor\}_{n\in\mathbb{N}} \qquad (c > 1, c \notin \mathbb{N}),$$

[1]Corresponding author.

where $\lfloor z \rfloor$ is the integer part of $z \in \mathbb{R}$. The Piatetski-Shapiro sequence was introduced by Piatetski-Shapiro [9] to study prime numbers in a sequence of the form $\lfloor f(n) \rfloor$, where $f(n)$ is a polynomial. The study of the distribution of square-free and cube-free integers in Piatetski-Shapiro sequences has a long and rich history; see [2, 3, 4, 10, 12, 16]. The distribution of consecutive square-free and cube-free integers in Piatetski-Shapiro sequences is also a topic of interest. In 2018 Dimitrov [5] proved that for every fixed $1 < c < 7/6$, there exist infinitely many consecutive square-free integers of the form $\lfloor n^c \rfloor$, $\lfloor n^c \rfloor + 1$ by showing that

$$\sum_{\substack{x/2 < n \le x \\ \lfloor n^c \rfloor,\, \lfloor n^c \rfloor + 1 \text{ are square-free}}} 1 = \frac{1}{2} \prod_p \left(1 - \frac{2}{p^2}\right) x + O\left(x^{\frac{6c+1}{8}+\varepsilon}\right), \qquad \text{for } 1 < c < \frac{7}{6}. \tag{1}$$

Very recently, Tangsupphathawat, Srichan, and Laohakosol [15] used Rieger's technique in [10] to improve the range of c and the error term in Dimitrov's work in (1). They showed that for $1 < c < 3/2$, and sufficiently small $\varepsilon > 0$,

$$\sum_{\substack{n \le x \\ \lfloor n^c \rfloor,\, \lfloor n^c \rfloor + 1 \text{ are square-free}}} 1 = \prod_p \left(1 - \frac{2}{p^2}\right) x + O\left(x^{\frac{2c+1}{4}+\varepsilon}\right) \qquad (x \to \infty). \tag{2}$$

In the case of cube-free numbers, Zhang and Li [16] proved that, for any ε less than 10^{-10}, one has

$$\sum_{\substack{n \le x \\ \lfloor n^c \rfloor \text{ is cube-free}}} 1 = \left(\frac{1}{\zeta(3)} + o(x^{\varepsilon})\right) x \qquad \text{for } 1 < c < \frac{11}{6}.$$

In 2018, Dimitrov [6] used the method of Zhang and Li to prove that

$$\sum_{\substack{n \le x \\ \lfloor n^c \rfloor, \lfloor n^c \rfloor + 1 \text{ are cube-free}}} 1 = \prod_p \left(1 - \frac{2}{p^3}\right) x + O(x^{1-\delta^2/2}), \qquad \text{for } 1 < c < \frac{31}{17}, \tag{3}$$

where $0 < \delta < \min\{\frac{31-17c}{9c-9}, 10^{-10}\}$ is a sufficiently small constant.

From these articles, it is interesting to study similar problems of counting integers in Piatetski-Shapiro sequences which belong to larger classes such as the (k,r)-integers, defined below. In 1966, Subbarao and Harris [13] generalized the notion of r-free integers as follows: let k and r be fixed positive integers with $1 < r < k$. A positive integer n is called a (k,r)-integer if n is of the form $n = a^k b$, where $a, b \in \mathbb{N}$ and b is r-free. They noticed that in the limiting case when $k \to \infty$, a (k,r)-integer becomes an r-free integer.

Recently, the second author [11] studied the distribution of (k,r)-integer, considered as generalized r-free integers, in Piatetski-Shapiro sequences. He proved that for all pairs of exponents (κ_1, λ_1) and (κ_2, λ_2) satisfying

$$r\lambda_1 - \kappa_1 < 1,\ r\lambda_2 - \kappa_2 > 1,\ k\lambda_1 - \kappa_1 > 1,\ \frac{r(\lambda_2 - \lambda_1) - (\kappa_2 - \kappa_1)}{\lambda_2(1+\kappa_1) - \lambda_1(1+\kappa_2)} > 1,$$

we have

$$\sum_{\substack{n\le N\\ \lfloor n^c\rfloor \text{ is a } (k,r)\text{-integer}}} 1 = \frac{\zeta(k)}{\zeta(r)}N + O(N^{(c/r)+\delta(\kappa_1,\lambda_1,\kappa_2,\lambda_2)}\log N),$$

for $1 < c < \frac{r(\lambda_2-\lambda_1)-(\kappa_2-\kappa_1)}{\lambda_2(1+\kappa_1)-\lambda_1(1+\kappa_2)}$, where $\delta(\kappa_1,\lambda_1,\kappa_2,\lambda_2) = \frac{(\lambda_2\kappa_1-\lambda_1\kappa_2)+r^{-1}(\kappa_2-\kappa_1)}{\lambda_2(1+\kappa_1)-\lambda_1(1+\kappa_2)}$.

In this work, we use use Rieger's technique [10] to generalize the previous result on the distribution of consecutive r-free integers in Piatetski-Shapiro sequences, by proving the following result:

Theorem 1. *Let k, r be integers with $1 < r < k$. For a large $N \in \mathbb{N}$, let*

$$T_c(N;k,r) := \sum_{\substack{n\le N\\ \lfloor n^c\rfloor,\, \lfloor n^c\rfloor+1 \text{ are } (k,r)\text{-integers}}} 1$$

denote the number of positive integers $n \le N$ such that $\lfloor n^c\rfloor$ and $\lfloor n^c\rfloor + 1$ are (k,r)-integers. For $1 < c < \frac{3}{2}$, we have, as $N \to \infty$,

$$T_c(N;k,2) = N\sum_{m=1}^{\infty}\frac{\tau(m)\lambda_{k,2}(m)}{m} + O(N^{c/2+1/4}\log N), \tag{4}$$

and for $1 < c < 2$, we have, as $N \to \infty$,

$$T_c(N;k,r) = N\sum_{m=1}^{\infty}\frac{\tau(m)\lambda_{k,r}(m)}{m} + O(N^{c/3+1/3}\log N), \qquad \text{if} \quad r \ge 3,$$

where $\lambda_{k,r}(n)$ is a multiplicative function defined by

$$\lambda_{k,r}(p^a) = \begin{cases} 1, & \text{if } a \equiv 0 \pmod{k};\\ -1, & \text{if } a \equiv r \pmod{k};\\ 0, & \text{otherwise,}\end{cases} \tag{5}$$

and $\tau(n)$ denotes the number of divisors of n.

As mentioned above, in the limiting case when $k \to \infty$, a $(k,2)$-integer becomes a square-free integer, which leads to the following corollary.

Corollary 2. *For $1 < c < \frac{3}{2}$, we have, as $N \to \infty$,*

$$T_c(N;\infty,2) = N\prod_p\left(1-\frac{2}{p^2}\right) + O(N^{c/2+1/4}\log N), \tag{6}$$

where the big O-term is independent of k.

Proof. For a fixed large N, every positive integer a, and a positive r-free integer b, the inequality $a^{\lfloor N \rfloor} b \le N$ holds only when $a = 1$. This indicates that in the interval $[1, N]$, every (N, r)-integer is an r-free integer. Thus, as $N \to \infty$, the function $T_c(N; \infty, 2)$ counts the number of positive integers $n \le N$ such that $\lfloor n^c \rfloor$ and $\lfloor n^c \rfloor + 1$ are square-free integers. In view of (5), we have

$$\sum_{m=1}^{\infty} \frac{\lambda_{N,r}(m)}{m^s} = \frac{\zeta(Ns)}{\zeta(rs)}, \quad s > \frac{1}{r}.$$

When $N \to \infty$, since $\zeta(Ns) = 1$, we have,

$$\sum_{m=1}^{\infty} \frac{\lambda_{N,2}(m)}{m^s} = \frac{1}{\zeta(2s)}.$$

The main term of (6) follows from (4) of Theorem 1 and we get

$$\sum_{m=1}^{\infty} \frac{\tau(m)\lambda_{N,2}(m)}{m} = \sum_{\substack{d,\, t \\ \gcd \gcd(d,t)=1}} \frac{\mu(d)\mu(t)}{d^2 t^2} = \prod_{p} \Big(1 - \frac{2}{p^2}\Big).$$

Since the big O-term of (4) is independent to k, Corollary 2 follows. □

Similarly, since an $(\infty, 3)$-integer becomes a cube-free integer, we obtain an improved result for (3) in the following corollary.

Corollary 3. *For $1 < c < 2$, we have, as $N \to \infty$,*

$$T_c(N; \infty, 3) = N \prod_{p} \Big(1 - \frac{2}{p^3}\Big) + O(N^{c/3+1/3} \log N).$$

2 Lemmas

We collect now some lemmas needed later.

Lemma 4. *[14, Lemma 2.6] Let*

$$q_{k,r}(n) = \begin{cases} 1, & \text{if } n \text{ is a } (k,r)\text{-integer;} \\ 0, & \text{if } n \text{ is not a } (k,r)\text{-integer,} \end{cases}$$

denote the characteristic function of the set of (k, r)-integers. Then

$$q_{k,r}(n) = \sum_{a^k b^r c = n} \mu(b).$$

Let

$$d(r,k,n) := \sum_{n_1^r n_2^k = n} 1$$

denote the number of ways of writing an integer n in the form $n = n_1^r n_2^k$, and put

$$D(r,k,x) = \sum_{n \le x} d(r,k,n).$$

In the proof of our main result, we need the following estimate for the function $D(r,k,x)$ whose proof can be found in [8, Section 14.3].

Lemma 5. *For a sufficiently large $x \in \mathbb{R}$, we have*

$$D(r,k,x) \ll x^{1/r}.$$

Lemma 6. *For $x \ge 1$, we have*

$$\sum_{m \le x} \lambda_{k,r}(m) \ll x^{1/r}.$$

Proof. In view of (5), Lemmas 4 and 5, we have

$$\sum_{m \le x} \lambda_{k,r}(m) = \sum_{a^k b^r \le x} \mu(b) \ll \sum_{a^k b^r \le x} 1 \ll x^{1/r}.$$

□

The proof of Theorem 1 makes use of the following estimate, due originally to Rieger [10], for the number of integers n up to x such that $\lfloor n^c \rfloor$ belongs to an arithmetic progression.

Lemma 7. *([10]) For $1 < c < 2$, let x be a positive real number, and let q and a be two integers such that $0 \le a < q \le x^c$. Then*

$$\sum_{\substack{n \le x \\ \lfloor n^c \rfloor \equiv a \, (\mathrm{mod}\, q)}} 1 = \frac{x}{q} + \begin{cases} O\left(\frac{x^{(c+4)/7}}{q^{1/7}}\right), & \textit{for } q < x^{c-5/4}; \\ O\left(\frac{x^{(c+1)/3}}{q^{1/3}}\right), & \textit{for } x^{c-5/4} \le q < x^{c-1/2}; \\ O\left(\frac{x^c}{q}\right), & \textit{for } x^{c-1/2} \le q < x^c. \end{cases}$$

The proofs of the next two lemmas make use of ideas similar to those in Lemma 2.1 of [1].

Lemma 8. *Let $A_c(x;k,r)$ and $B_c(x;k,r)$ denote the number of 6-tuples $(d_1, t_1, d_2, t_2, u, v)$ satisfying the conditions*

$$d_2^k t_2^r v - d_1^k t_1^r u = 1, \qquad d_1^k t_1^r u \le x^c. \tag{7}$$

I) If $x^c < d_1^k t_1^r d_2^k t_2^r \le x^{4c/3}$*, then*

$$A_c(x;k,r) \ll x^{4c/3r}\log x.$$

II) If $x^{4c/3} < d_1^k t_1^r d_2^k t_2^r \le x^{2c}$*, then*

$$B_c(x;k,r) \ll x^{2c/3}\log x.$$

Proof. I) For a fixed choice of d_1, t_1, d_2 and t_2 satisfying (7), we have $d_1^k t_1^r u \equiv -1 \pmod{d_2^k t_2^r}$, which fixes the value of u modulo $d_2^k t_2^r$. In view of (7), the total number of possibilities for u is $O(1 + x^c/d_1^k t_1^r d_2^k t_2^r)$. By (7), the value of v is fixed for a given choice of d_1, t_1, d_2, t_2, u . Then, by Lemma 5, we have

$$\begin{aligned}
A_c(x;k,r) &\ll \sum_{x^c < d_1^k t_1^r d_2^k t_2^r \le x^{4c/3}} \Big(1 + \frac{x^c}{d_1^k t_1^r d_2^k t_2^r}\Big) \\
&\ll \sum_{x^c < m \le x^{4c/3}} \tau(m) d(k,r;m)\Big(1 + \frac{x^c}{m}\Big) \\
&\ll \sum_{x^c < m \le x^{4c/3}} d(k,r;m)\Big(\log m + \frac{x^c \log m}{m}\Big) \\
&\ll x^{4c/3r}\log x + x^{c/r}\log x \ll x^{4c/3r}\log x.
\end{aligned}$$

II) From (7), we have $uv d_1^k t_1^r d_2^k t_2^r \le x^c(x^c+1)$, whence $uv \le x^c(x^c+1)x^{-4c/3}$ for every 6-tuple counted by $B_c(x)$. From a divisor argument, the total number of choices for u, v is therefore bounded by $O(x^{2c/3}\log x)$. For ever such choice of u, v, the number of solutions in d_1, t_1, d_2, t_2 of the equation $d_2^k t_2^r v - d_1^k t_1^r u = 1$ is $O(\log x)$; see [7]. □

3 Proof of Theorem 1

From $q_{k,r}(n) = \sum_{d|n} \lambda_{k,r}(d)$, we have

$$\begin{aligned}
T_c(N;k,r) &= \sum_{n \le N} q_{k,r}(\lfloor n^c \rfloor) q_{k,r}(\lfloor n^c \rfloor + 1) \\
&= \sum_{n \le N} \Big(\sum_{d | \lfloor n^c \rfloor} \lambda_{k,r}(d)\Big)\Big(\sum_{t | \lfloor n^c \rfloor + 1} \lambda_{k,r}(t)\Big) \\
&= \Big(\sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^c}} + \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt > N^c}}\Big) \lambda_{k,r}(d)\lambda_{k,r}(t) \sum_{\substack{n \le N \\ \lfloor n^c \rfloor \equiv 0 \,(\mathrm{mod}\, d) \\ \lfloor n^c \rfloor + 1 \equiv 0 \,(\mathrm{mod}\, t)}} 1.
\end{aligned}$$

In view of Lemma 8, we have

$$\begin{aligned}
T_c(N;k,r) = &\sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^c}} \lambda_{k,r}(d)\lambda_{k,r}(t) \sum_{\substack{n \le N \\ \lfloor n^c \rfloor \equiv 0 \,(\mathrm{mod}\, d) \\ \lfloor n^c \rfloor + 1 \equiv 0 \,(\mathrm{mod}\, t)}} 1 + \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt > N^c}} \lambda_{k,r}(d)\lambda_{k,r}(t) \sum_{\substack{n \le N \\ \lfloor n^c \rfloor \equiv 0 \,(\mathrm{mod}\, d) \\ \lfloor n^c \rfloor + 1 \equiv 0 \,(\mathrm{mod}\, t)}} 1 \\
=: &\ \Sigma_1 + O(N^{2c/3}\log N).
\end{aligned}$$

By the Chinese remainder theorem, there is a positive integer α, unique modulo dt, satisfying the congruence system $\alpha \equiv 0 \pmod{d}$ and $\alpha + 1 \equiv 0 \pmod{t}$. Thus,

$$\Sigma_1 = \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^c}} \lambda_{k,r}(d)\lambda_{k,r}(t) \sum_{\substack{n \le N \\ \lfloor n^c \rfloor \equiv \alpha \, (\mathrm{mod}\ dt)}} 1.$$

In view of Lemma 7, we have

$$\Sigma_1 = N \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^c}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} + O\Big(N^{(c+4)/7}\Big| \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^{c-5/4}}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{(dt)^{1/7}}\Big|\Big)$$
$$+ O\Big(N^{(c+1)/3}\Big| \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ N^{c-5/4} < dt \le N^{c-1/2}}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{(dt)^{1/3}}\Big|\Big) + O\Big(N^c\Big| \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ N^{c-1/2} < dt \le N^c}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt}\Big|\Big).$$

In view of Lemma 6, we note that,

$$\sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt \le N^{c-5/4}}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{(dt)^{1/7}} = \sum_{m \le N^{c-5/4}} \frac{\tau(m)\lambda_{k,r}(m)}{m^{1/7}}$$
$$\ll \sum_{m \le N^{c-5/4}} \frac{\lambda_{k,r}(m)}{m^{1/7-\varepsilon}} \ll \begin{cases} (N^{c-5/4})^{\varepsilon+1/r-1/7}, & \text{if } r < 7; \\ \log N, & \text{if } r \ge 7, \end{cases}$$

$$\sum_{\substack{d,t \\ \gcd(d,t)=1 \\ N^{c-5/4} < dt \le N^{c-1/2}}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{(dt)^{1/3}} = \sum_{N^{c-5/4} < m \le N^{c-1/2}} \frac{\tau(m)\lambda_{k,r}(m)}{m^{1/3}}$$
$$\ll \sum_{N^{c-5/4} < m \le N^{c-1/2}} \frac{\lambda_{k,r}(m)}{m^{1/3-\varepsilon}} \ll \begin{cases} (N^{c-1/2})^{\varepsilon+1/r-1/3}, & \text{if } r = 2; \\ \log N, & \text{if } r \ge 3, \end{cases}$$

and

$$\sum_{\substack{d,t \\ \gcd(d,t)=1 \\ N^{c-1/2} < dt \le N^c}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} = \sum_{N^{c-1/2} < m \le N^c} \frac{\tau(m)\lambda_{k,r}(m)}{m}$$
$$\ll \sum_{N^{c-1/2} < m \le N^c} \frac{\lambda_{k,r}(m)}{m^{1-\varepsilon}} \ll (N^{c-1/2})^{\varepsilon+1/r-1}.$$

Thus,

$$\Sigma_1 = N \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt\le N^c}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} + \begin{cases} O(N^{c/2+1/4})\log N, & \text{if } r=2; \\ O(N^{c/3+1/3})\log N, & \text{if } r\ge 3. \end{cases}$$

Note that

$$\begin{aligned} \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt\le N^c}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} &= \Bigg(\sum_{\substack{d,t \\ \gcd(d,t)=1}} + \sum_{\substack{d,t \\ \gcd(d,t)=1 \\ dt> N^c}} \Bigg)\Bigg(\frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt}\Bigg) \\ &= \sum_{\substack{d,t \\ \gcd(d,t)=1}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} + O\Bigg(\sum_{m>N^c} \frac{\tau(m)\lambda_{k,r}(m)}{m}\Bigg) \\ &= \sum_{\substack{d,t \\ \gcd(d,t)=1}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} + O\Big(N^{c/r-c}\log N\Big). \end{aligned} \tag{8}$$

Since $\frac{c}{r} - c < 0$, the sum on the right hand side of (8) converges when $r > 1$. Thus,

$$\Sigma_1 = N \sum_{\substack{d,t \\ \gcd(d,t)=1}} \frac{\lambda_{k,r}(d)\lambda_{k,r}(t)}{dt} + \begin{cases} O(N^{c/2+1/4})\log N, & \text{if } r=2; \\ O(N^{c/3+1/3})\log N, & \text{if } r\ge 3, \end{cases}$$

which completes the proof of Theorem 1.

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