# Corrigendum to "A Note on Sumsets and Restricted Sumsets"

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We apologize for a mathematical error in Corollary 7 of our recently published article [1]. The bound in Corollary 7 is not optimal for $h_1 > 1$. The optimal (and corrected) bound is $|H\hat{}A| \geq \sum_{i=1}^{r}(h_i - h_{i-1})(k - h_i - 1) + h_1 + r$. The changes in the bound result in some changes in the proof of Corollary 7. Furthermore, this also results in some changes in the corresponding inverse result, which is Corollary 10. Below we present the complete and corrected proofs of Corollary 7 and Corollary 10. We also rectify a typo mistake in Corollary 11 by giving the correct statement.

**Corollary 7.** *Let $A$ be a set of $k$ nonnegative integers with $0 \in A$. Let $H = \{h_1, h_2, \ldots, h_r\}$ be a set of positive integers with $h_1 < h_2 < \cdots < h_r \leq k - 1$. Set $h_0 = 0$. Then*

$$|H\hat{}A| \geq \sum_{i=1}^{r}(h_i - h_{i-1})(k - h_i - 1) + h_1 + r. \tag{1}$$

*This lower bound is optimal.*

*Proof.* Let $A = \{0, a_1, a_2, \ldots, a_{k-1}\}$, where $0 < a_1 < a_2 < \cdots < a_{k-1}$. Set $A' = A \setminus \{0\}$. For $i = 1, 2, \ldots, h_1$, let

$$s_i = \sum_{j=1,\, j \neq h_1 - i + 1}^{h_1} a_j.$$

Then it is easy to see that $\{0\} \cup H\hat{}A' \subseteq H\hat{}A$ if $h_1 = 1$ and $\{s_1, s_2, \ldots, s_{h_1}\} \cup H\hat{}A' \subseteq H\hat{}A$ if $h_1 > 1$, where $s_1 < s_2 < \cdots < s_{h_1} < \min(H\hat{}A')$. So, by Theorem 6, we get

$$|H\hat{}A| \geq |H\hat{}A'| + h_1 \geq \sum_{i=1}^{r}(h_i - h_{i-1})(k - h_i - 1) + h_1 + r. \tag{2}$$

Furthermore, the optimality of the lower bound in (1) can be verified by taking $A = [0, k-1]$ and $H = [1, r]$, where $k, r$ are positive integers with $r \leq k - 1$. □

**Corollary 10.** *Let $A$ be a set of $k \geq 7$ nonnegative integers with $0 \in A$. Let $H = \{h_1, h_2, \ldots, h_r\}$ be a set of $r \geq 2$ positive integers with $h_1 < h_2 < \cdots < h_r \leq k-2$. Set $h_0 = 0$. If*

$$|H\hat{}A| = \sum_{i=1}^{r} (h_i - h_{i-1})(k - h_i - 1) + h_1 + r,$$

*then $H = h_1 + [0, r-1]$ and $A = \min(A \setminus \{0\}) \cdot [0, k-1]$.*

*Proof.* Let $A = \{0, a_1, a_2, \ldots, a_{k-1}\}$, where $0 < a_1 < a_2 < \cdots < a_{k-1}$. Set $A' = A \setminus \{0\}$. The equality $|H\hat{}A| = \sum_{i=1}^{r}(h_i - h_{i-1})(k - h_i - 1) + h_1 + r$ together with (2) implies $|H\hat{}A'| = \sum_{i=1}^{r}(h_i - h_{i-1})(k - 1 - h_i) + r$. By applying Theorem 9 on $H$ and $A'$, we obtain $H = h_1 + [0, r-1]$ and $A' = \min(A') \cdot [1, k-1]$. Hence, $H = h_1 + [0, r-1]$ and $A = \min(A') \cdot [0, k-1]$. This completes the proof of the corollary. □

**Corollary 11.** *[2, Theorem 2.2, Corollary 2.3] Let $A$ be a set of $k \geq 7$ nonnegative integers and $H = [0, r]$ with $2 \leq r \leq k-1$. If $0 \notin A$ and $|H\hat{}A| = rk - \frac{r(r-1)}{2} + 1$, then $A = d \cdot [1, k]$ for some positive integer $d$.*

*If $0 \in A$, $r \leq k-2$, and $|H\hat{}A| = rk - \frac{r(r+1)}{2} + 1$, then $A = d \cdot [0, k-1]$ for some positive integer $d$.*

# References


[1] J. Bhanja, A note on sumsets and restricted sumsets, *J. Integer Seq.* **24** (2021), Article 21.4.2.

[2] J. Bhanja and R. K. Pandey, Inverse problems for certain subsequence sums in integers, *Discrete Math.* **343** (2020), 112148.