

Corrigendum to “*Arithmetic Progressions on $y^2 = x^3 + k$* ”

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The proof of the upper bound $S_y(E) \leq 6$ in Theorem 2 is invalid. This establishes a gap in the proof, rather than a counterexample to the bound. The construction of a six-term progression survives, and the simultaneous-progression result in Theorem 3 is independent of the error. Throughout, progressions have nonzero common difference and $k \neq 0$.

1. The logical error. Bremner’s theorem [1] classifies certain polynomially parametrized solution curves of *arithmetic genus zero* on S_c : $X^3 + Y^3 + cZ^3 = c$. But it does not classify $S_c(\mathbb{Q})$. Indeed, Bremner explicitly states that these surfaces also have infinitely many polynomial parametric solutions whose image curves have positive arithmetic genus. Such an image can still have geometric genus zero if it is singular. Rational points on genus-one curves, and points outside the listed families, are likewise not excluded.

The unjustified step occurs on p. 5, immediately after equations (19)–(20), where the listed families are asserted to give *all* rational solutions, for $c = 4$ and $c = 12$. It occurs again on p. 6 after (24), for $c = 5$. Consequently both branches of the proposed exclusion of seven ordinates, with central ordinate nonzero or zero, are affected. Comparing parameters in a few families cannot rule out solutions outside those families.

2. A separate sign error. There is also a directly checkable error in family (ii) of Theorem 1. For the printed triple

$$(A, B, C) = \left(\frac{9}{c}t^4 - 3t, -\frac{9}{c}t^4, \frac{9}{c}t^3 - 1 \right)$$

one has $A^3 + B^3 + cC^3 = -c$, not c . Negating all three coordinates corrects this identity. This typographical repair does not repair the completeness argument in Theorem 2.

3. What can be retained. Put $r = 147q^3/8$ and $k = 197225q^6/64$, with $q \in \mathbb{Q}^\times$. Direct substitution verifies the six points

$$(-14q^2, \pm r), \quad (-7q^2/2, \pm 3r), \quad (35q^2/2, \pm 5r).$$

Their ordinates are $-5r, -3r, -r, r, 3r, 5r$, so the valid conclusion is $S_y(E) \geq 6$. Equality is not established by this construction. The parameter gives infinitely many distinct equations, but only one $\mathbb{Q}$ -isomorphism class: $(x, y) = (q^2X, q^3Y)$ removes q .

The four-point exclusion in Theorem 3 uses a separate permutation calculation, and the bound $S_{x,y}(E) \leq 3$ survives.

References

- [1] A. Bremner, *On diagonal cubic surfaces*, Manuscripta Math. **62** (1988), 21–32.