Journal of Integer Sequences, Vol. 15 (2012),
Article 12.1.2

Some n-Color Compositions

Yu-hong Guo[1]
Department of Mathematics
Hexi University
Gansu, Zhangye, 734000
P. R. China
gyh7001@163.com

Abstract

An n-color odd composition is defined as an n-color composition with odd parts, and an n-color composition with parts $\neq 1$ is an n-color composition whose parts are > 1. In this paper, we get generating functions, explicit formulas and recurrence formulas for n-color odd compositions and n-color compositions with parts $\neq 1$.

1 Introduction

In the classical theory of partitions, compositions were first defined by MacMahon [1] as ordered partitions. For example, there are 5 partitions and 8 compositions of 4. The partitions are 4, 31, 22, 21^2, 1^4 and the compositions are 4, 31, 13, 22, 21^2, 121, $1^2 2$, 1^4.

Agarwal and Andrews [2] defined an n-color partition as a partition in which a part of size n can come in n different colors. They denoted different colors by subscripts: n_1, n_2, …, n_n. Analogous to MacMahon's ordinary compositions Agarwal [3] defined an n-color composition as an n-color ordered partition. Thus, for example, there are 21 n-color compositions of 4, viz.,

$$\begin{aligned}
&4_1, 4_2, 4_3, 4_4,\\
&3_1 1_1, 3_2 1_1, 3_3 1_1, 1_1 3_1, 1_1 3_2, 1_1 3_3,\\
&2_1 2_1, 2_1 2_2, 2_2 2_2, 2_2 2_1,\\
&2_1 1_1 1_1, 2_2 1_1 1_1, 1_1 2_1 1_1, 1_1 1_1 2_1, 1_1 2_2 1_1, 1_1 1_1 2_2,\\
&1_1 1_1 1_1 1_1.
\end{aligned}$$

[1] This work is supported by the Fund of the Education Department of Gansu Province (No. 200809-04) and the fund of Hexi University.

More properties of n-color compositions were found in [4, 5]. In 2006, G. Narang and Agarwal [6, 7] also defined an n-color self-inverse composition and gave some properties. In 2010, Guo [8] defined an n-color even self-inverse composition and proved some properties.

In this paper, we shall study some n-color compositions. We first give the following definitions.

Definition 1. An n-color odd composition is an n-color composition with odd parts.

Thus, for example, there are 7 n-color odd compositions of 4, viz.,

$$3_1 1_1, 3_2 1_1, 3_3 1_1,$$
$$1_1 3_1, 1_1 3_2, 1_1 3_3, 1_1 1_1 1_1 1_1.$$

Definition 2. An n-color composition with parts $\neq 1$ is an n-color composition whose parts are > 1.

For example, there are 17 n-color compositions with parts $\neq 1$ of 5, viz.,

$$5_1, 5_2, 5_3, 5_4, 5_5,$$
$$2_1 3_1, 2_1 3_2, 2_1 3_3, 2_2 3_1, 2_2 3_2, 2_2 3_3,$$
$$3_1 2_1, 3_2 2_1, 3_3 2_1, 3_1 2_2, 3_2 2_2, 3_3 2_2.$$

In section 2 we shall give generating functions, recurrence formulas and explicit formulas for n-color compositions above.

Agarwal [3] proved the following theorem.

Theorem 3. *([3]) Let $C(m, q)$ and $C(q)$ denote the enumerative generating functions for $C(m, \nu)$ and $C(\nu)$, respectively, where $C(m, \nu)$ is the number of n-color compositions of ν into m parts and $C(\nu)$ is the number of n-color compositions of ν. Then*

$$C(m, q) = \frac{q^m}{(1-q)^{2m}}, \tag{1}$$

$$C(q) = \frac{q}{1-3q+q^2}, \tag{2}$$

$$C(m, \nu) = \binom{\nu+m-1}{2m-1}, \tag{3}$$

$$C(\nu) = F_{2\nu}. \tag{4}$$

2 Main results

We denote the number of n-color odd compositions of ν by $C(o, \nu)$ and the number of n-color odd compositions of ν into m parts by $C(m, o, \nu)$, respectively. In this section, we first prove the following theorem.

Theorem 4. *Let $C(m,o,q)$ and $C(o,q)$ denote the enumerative generating functions for $C(m,o,\nu)$ and $C(o,\nu)$, respectively. Then*

$$C(m,o,q) = \frac{q^m(1+q^2)^m}{(1-q^2)^{2m}}, \tag{5}$$

$$C(o,q) = \frac{q+q^3}{1-q-2q^2-q^3+q^4}, \tag{6}$$

$$C(m,o,\nu) = \sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j}, \tag{7}$$

$$C(o,\nu) = \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j}. \tag{8}$$

where $(\nu-m)$ is even, and $(\nu-m)\ge 0$; $0\le i,j$ are integers.

Proof. Similar to the proof of Agarwal [3], we have

$$C(m,o,q) = \sum_{\nu=1}^{\infty} C(m,o,\nu)q^\nu = (q+3q^3+\cdots+)^m = \frac{q^m(1+q^2)^m}{(1-q^2)^{2m}}.$$

This proves (5).

$$C(o,q) = \sum_{m=1}^{\infty} C(m,o,q) = \sum_{m=1}^{\infty} \frac{q^m(1+q^2)^m}{(1-q^2)^{2m}} = \frac{q+q^3}{1-q-2q^2-q^3+q^4}.$$

We get (6).

On equating the coefficients of q^ν in (5), we have

$$C(m,o,\nu) = \sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j}.$$

Since ν is even if m is even, and ν is odd if m is odd, then $\nu-m$ is even. This proves (7).

Obviously $m\le\nu$, so (8) is also proven.

We complete the proof of this theorem. □

In this section, we also prove the following recurrence formula.

Theorem 5. *Let O_ν denote the number of n-color odd compositions of ν. Then*

$$O_1 = 1, O_2 = 1, O_3 = 4, O_4 = 7$$

and

$$O_\nu = O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3} - O_{\nu-4}, \ \textit{for}\ \nu > 4.$$

Proof. (Combinatorial) To prove that $O_\nu = O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3} - O_{\nu-4}$, we split the n-color compositions enumerated by $O_\nu + O_{\nu-4}$ into four classes:

(A) enumerated by O_ν with 1_1 on the right.

(B) enumerated by O_ν with 3_3 on the right.

(C) enumerated by O_ν with h_t on the right, $h > 1$, $1 \le t \le h-2$ (where, h is odd).

(D) enumerated by O_ν with h_t on the right, $h > 1, h-1 \le t \le h$ except 3_3 and those enumerated by $O_{\nu-4}$.

We transform the n-color odd compositions in class (A) by deleting 1_1 on the right. This produces n-color compositions enumerated by $O_{\nu-1}$. Conversely, for any n-color composition enumerated by $O_{\nu-1}$ we add 1_1 on the right to produce the elements of the class (A). In this way we prove that there are exactly $O_{\nu-1}$ elements in the class (A).

Similarly, we can produce $O_{\nu-3}$ n-color odd compositions in the class (B) by deleting 3_3 on the right.

Next, we transform the n-color odd compositions in class (C) by subtracting 2 from h, that is, replacing h_t by $(h-2)_t$. This transformation also establishes the fact that there are exactly $O_{\nu-2}$ elements in class (C). This correspondence being one to one.

Finally, we transform the elements in class (D) as follows: Subtract 2_2 from h_t on the right when $h > 3$, $h-1 \le t \le h$, that is, replace h_t by $(h-2)_{(t-2)}$; in this way we will get n-color odd compositions of $\nu - 2$ with part $h'_{t'}$ on the right, where, $h' > 1, t' \ge h' - 1$. After that we replace h_t by $(h-2)_{(t-1)}$ when $h = 3$, $t = 2$. This produces n-color odd compositions of $\nu - 2$ with part 1_1 on the right. To get the remaining n-color odd compositions from $O_{\nu-4}$, we add 2 to the right parts, that is, replace h_t by $(h+2)_t$ to get the n-color odd compositions of $(\nu-2)$ with part $h'_{t'}$ on the right, where, $h' > 1, 1 \le t' \le h' - 2$. We see that the number of n-color odd compositions in class (D) is also equal to $O_{\nu-2}$. Hence, $O_\nu + O_{\nu-4} = O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3}$. viz.,$O_\nu = O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3} - O_{\nu-4}$.

Thus, we complete the proof. □

We also give another proof of Theorem 5.

Proof. We have

$$\begin{aligned}
O_\nu &= \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&= \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+(i-1)-1}{2m-1}\binom{m}{j} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+(i-1)-1}{2m-2}\binom{m}{j} \\
&\quad \text{(by the binomial identity} \binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1}\text{)} \\
&= \sum_{m\le\nu-2}\sum_{i+j=\frac{(\nu-2)-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} + \binom{2\nu-2}{2\nu-1}\binom{\nu}{0} \\
&\quad + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} - \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-2}{2m-1}\binom{m}{j}
\end{aligned}$$

$$
\begin{aligned}
&= O_{\nu-2} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&\quad - \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+(i-2)-1}{2m-1}\binom{m}{j} - \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-3}{2m-2}\binom{m}{j} \\
&= O_{\nu-2} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&\quad - \sum_{m\le\nu-4}\sum_{i+j=\frac{(\nu-4)-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} - \binom{2\nu-2-1}{2\nu-1}\binom{\nu}{0} \\
&\quad - \binom{2(\nu-2)-2-1}{2(\nu-2)-1}\binom{\nu-2}{1} - \binom{2(\nu-2)-1-1}{2(\nu-2)-1}\binom{\nu-2}{0} \\
&\quad - \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-3}{2m-2}\binom{m}{j} \\
&= O_{\nu-2} - O_{\nu-4} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+(i-1)-1}{2m-1}\binom{m}{j} \\
&\quad + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-2}{2m-2}\binom{m}{j} - \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-3}{2m-2}\binom{m}{j} \\
&= 2O_{\nu-2} - O_{\nu-4} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2m+i-3}{2m-3}\binom{m}{j} \\
&= 2O_{\nu-2} - O_{\nu-4} + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2(m-1)+i-1}{2(m-1)-1}\binom{m-1}{j} \\
&\quad + \sum_{m\le\nu}\sum_{i+j=\frac{\nu-m}{2}} \binom{2(m-1)+i-1}{2(m-1)-1}\binom{m-1}{j-1} \\
&= 2O_{\nu-2} - O_{\nu-4} + \sum_{m\le\nu-1}\sum_{i+j=\frac{(\nu-1)-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&\quad + \sum_{m\le\nu-3}\sum_{i+j=\frac{(\nu-3)-m}{2}} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&= O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3} - O_{\nu-4}.
\end{aligned}
$$

So we have $O_\nu = O_{\nu-1} + 2O_{\nu-2} + O_{\nu-3} - O_{\nu-4}$. □

From recurrence formula above we have the following corollary easily.

Corollary 6. *If $\nu > 4$, then*

$$\sum_{m\le\nu-4}\Big(\sum_{i+j=\frac{\nu-m}{2}}\binom{2m+i-1}{2m-1}\binom{m}{j}-\sum_{i+j=\frac{\nu-1-m}{2}}\binom{2m+i-1}{2m-1}\binom{m}{j}$$
$$-2\sum_{i+j=\frac{\nu-2-m}{2}}\binom{2m+i-1}{2m-1}\binom{m}{j}-\sum_{i+j=\frac{\nu-3-m}{2}}\binom{2m+i-1}{2m-1}\binom{m}{j}$$
$$+\sum_{i+j=\frac{\nu-4-m}{2}}\binom{2m+i-1}{2m-1}\binom{m}{j}\Big)=0.$$

Next, we shall study n-color compositions with parts $\neq 1$. We denote the number of n-color compositions with parts $\neq 1$ of ν by $C_{\neq 1}(\nu)$ and the number of n-color compositions with parts $\neq 1$ of ν into m parts by $C_{\neq 1}(m,\nu)$, respectively. In this section, we present the following theorem.

Theorem 7. *Let $C_{\neq 1}(m,q)$ and $C_{\neq 1}(q)$ denote the enumerative generating functions for $C_{\neq 1}(m,\nu)$ and $C_{\neq 1}(\nu)$, respectively. Then*

$$C_{\neq 1}(m,q)=\frac{q^{2m}(2-q)^m}{(1-q)^{2m}}, \tag{9}$$

$$C_{\neq 1}(q)=\frac{2q^2-q^3}{1-2q-q^2+q^3}, \tag{10}$$

$$C_{\neq 1}(m,\nu)=\sum_{i+j=\nu-2m}(-1)^j 2^{m-j}\binom{2m+i-1}{2m-1}\binom{m}{j}, \tag{11}$$

$$C_{\neq 1}(\nu)=\sum_{m\le\frac{\nu}{2}}\sum_{i+j=\nu-2m}(-1)^j 2^{m-j}\binom{2m+i-1}{2m-1}\binom{m}{j}. \tag{12}$$

where $(\nu-2m)$ is an integer, and $(\nu-2m)\ge 0$; $0\le i,j$ are integers.

Proof. Similar to the proof of Agarwal [3], we have

$$C_{\neq 1}(m,q)=\sum_{\nu=1}^{\infty}C_{\neq 1}(m,\nu)q^\nu=(2q^2+3q^3+\cdots+)^m=\frac{q^{2m}(2-q)^m}{(1-q)^{2m}}.$$

This proves (9).

$$C_{\neq 1}(q)=\sum_{m=1}^{\infty}C_{\neq 1}(m,q)=\sum_{m=1}^{\infty}\frac{q^{2m}(2-q)^m}{(1-q)^{2m}}=\frac{2q^2-q^3}{1-2q-q^2+q^3}.$$

This proves (10).

On equating the coefficients of q^ν in (9), we have

$$C_{\neq 1}(m,\nu)=\sum_{i+j=\nu-2m}(-1)^j 2^{m-j}\binom{2m+i-1}{2m-1}\binom{m}{j}.$$

Since $\nu \geq 2m$, then $\nu - 2m \geq 0$, $i + j \geq 0$, and $0 \leq i, j$ are integers. This proves (11). Obviously $m \leq \frac{\nu}{2}$, therefore (12) is also proven.
We complete the proof of this theorem. □

In this section, we also prove the following recurrence formula.

Theorem 8. *Let $C_{\neq 1}(\nu)$ denote the number of n-color compositions with parts $\neq 1$ of ν. Then*

$$C_{\neq 1}(2) = 2, C_{\neq 1}(3) = 3, C_{\neq 1}(4) = 8,$$

and

$$C_{\neq 1}(\nu) = 2C_{\neq 1}(\nu - 1) + C_{\neq 1}(\nu - 2) - C_{\neq 1}(\nu - 3) \ \ \textit{for} \ \ \nu > 4.$$

Proof. (Combinatorial) To prove that $C_{\neq 1}(\nu) = 2C_{\neq 1}(\nu - 1) + C_{\neq 1}(\nu - 2) - C_{\neq 1}(\nu - 3)$, we split the n-color compositions enumerated by $C_{\neq 1}(\nu) + C_{\neq 1}(\nu - 3)$ into three classes:

(A) enumerated by $C_{\neq 1}(\nu)$ with 2_1 on the right.

(B) enumerated by $C_{\neq 1}(\nu)$ with h_t on the right, $h > 2$,$1 \leq t \leq h - 1$.

(C) enumerated by $C_{\neq 1}(\nu)$ with h_h on the right, $h \geq 2$ and those enumerated by $C_{\neq 1}(\nu - 3)$.

We transform the n-color compositions in class (A) by deleting 2_1 on the right. This produces n-color compositions enumerated by $C_{\neq 1}(\nu - 2)$. Conversely, for any n-color composition enumerated by $C_{\neq 1}(\nu - 2)$ we add 2_1 on the right to produce the elements of the class (A). In this way we prove that there are exactly $C_{\neq 1}(\nu - 2)$ elements in the class (A).

Next, we transform the n-color compositions in class (B) by subtracting 1 from h, that is, replacing h_t by $(h - 1)_t$; this transformation also establishes the fact that there are exactly $C_{\neq 1}(\nu - 1)$ elements in class (B). This correspondence being one to one.

Finally, we transform the elements in class (C) as follows: Subtract 1_1 from h_h on the right when $h > 2$, that is, replace h_h by $(h - 1)_{(h-1)}$; in this way we will get n-color compositions of $\nu - 1$ with part $h'_{h'} (h' > 1)$ on the right. We also replace h_h by $(h - 1)_{(h-1)}$ when $h = 2$. This produces n-color compositions of $\nu - 1$ with part 1_1 on the right. Now we delete 1_1 and add 1 to the preceding part of it. For example, $2_1 2_2 2_2 \longrightarrow 2_1 2_2 1_1 \longrightarrow 2_1 3_2$; $4_1 2_2 \longrightarrow 4_1 1_1 \longrightarrow 5_1$. Then we have n-color compositions of $\nu - 1$ with part h'_t on the right, where, $h' > 2, 1 \leq t \leq h' - 1$. To get the remaining n-color compositions from $C_{\neq 1}(\nu - 3)$, we set 2_1 on the right. This produces n-color compositions with parts $\neq 1$ of $\nu - 1$ with 2_1 on the right. We see that the number of n-color compositions in class (C) is also equal to $C_{\neq 1}(\nu - 1)$. Hence, $C_{\neq 1}(\nu) + C_{\neq 1}(\nu - 3) = 2C_{\neq 1}(\nu - 1) + C_{\neq 1}(\nu - 2)$. viz., $C_{\neq 1}(\nu) = 2C_{\neq 1}(\nu - 1) + C_{\neq 1}(\nu - 2) - C_{\neq 1}(\nu - 3)$.

Thus, we complete the proof. □

We also give another proof of Theorem 8.

Proof. We have

$$C_{\neq 1}(\nu) = \sum_{m \leq \frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m + i - 1}{2m - 1} \binom{m}{j}$$

$$
\begin{aligned}
&= \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+(i-1)-1}{2m-1}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-1}{2m-2}\binom{m}{j} \\
&\quad \text{(by the binomial identity} \binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1}\text{)} \\
&= \sum_{m\le\frac{\nu-1}{2}} \sum_{i+j=(\nu-1)-2m} (-1)^j 2^{m-j} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-3}\binom{m}{j} \\
&= C_{\neq 1}(\nu-1) + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-3}\binom{m-1}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2(m-1)+i-1}{2(m-1)-1}\binom{m-1}{j-1} \\
&= C_{\neq 1}(\nu-1) + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2}{2m-2}\binom{m-1}{j} \\
&\quad - \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m-1}{j} \\
&\quad + \sum_{m\le\frac{(\nu-3)}{2}} \sum_{i+j=(\nu-3)-2m} (-1)^{j+1} 2^{m-j} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&= C_{\neq 1}(\nu-1) - C_{\neq 1}(\nu-3) \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m-1}{j-1} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m-1}{j}
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-3}\binom{m-1}{j} \\
&= C_{\neq 1}(\nu-1) - C_{\neq 1}(\nu-3) \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-2}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2(m-1)+i-1}{2(m-1)-1}\binom{m-1}{j} \\
&= C_{\neq 1}(\nu-1) - C_{\neq 1}(\nu-3) \\
&\quad + \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2}{2m-1}\binom{m}{j} \\
&\quad - \sum_{m\le\frac{\nu}{2}} \sum_{i+j=\nu-2m} (-1)^j 2^{m-j} \binom{2m+i-2-1}{2m-1}\binom{m}{j} \\
&\quad + \sum_{m\le\frac{\nu-2}{2}} \sum_{i+j=(\nu-2)-2m} (-1)^j 2^{m+1-j} \binom{2m+i-1}{2m-1}\binom{m}{j} \\
&= C_{\neq 1}(\nu-1) - C_{\neq 1}(\nu-3) + C_{\neq 1}(\nu-1) - C_{\neq 1}(\nu-2) + 2C_{\neq 1}(\nu-2) \\
&= 2C_{\neq 1}(\nu-1) + C_{\neq 1}(\nu-2) - C_{\neq 1}(\nu-3).
\end{aligned}
$$

Thus we have $C_{\neq 1}(\nu) = 2C_{\neq 1}(\nu-1) + C_{\neq 1}(\nu-2) - C_{\neq 1}(\nu-3)$. □

3 Acknowledgement

The author would like to thank the referee for his/her suggestions and comments which have improved the quality of this paper.

References

[1] P. A. MacMahon, *Combinatory Analysis*, AMS Chelsea Publishing, 2001.

[2] A. K. Agarwal and G. E. Andrews, Rogers-Ramanujan identities for partitions with "n copies of n", *J. Combin. Theory Ser. A* **45** (1987), 40–49.

[3] A. K. Agarwal, n-colour compositions, *Indian J. Pure Appl. Math.* **31** (2000), 1421–1427.

[4] A. K. Agarwal, An analogue of Euler's identity and new combinatorial properties of n-colour compositions, *J. Comput. Appl. Math.* **160** (2003), 9–15.

[5] Yu-Hong Guo, Some identities between partitions and compositions, *Acta Math. Sinica (Chin. Ser.)* **50** (2007), 707–710.

[6] G. Narang and A. K. Agarwal, n-colour self-inverse compositions, *Proc. Indian Acad. Sci. Math. Sci.* **116** (2006), 257–266.

[7] G. Narang and A. K. Agarwal, Lattice paths and n-color compositions, *Discrete Math.* **308** (2008), 1732–1740.

[8] Yu-Hong Guo, n-colour even self-inverse compositions, *Proc. Indian Acad. Sci. Math. Sci.* **120** (2010), 27–33.

[9] G. E. Andrews, *The Theory of Partitions*, Encyclopedia of Mathematics and Its Applications, Cambridge University Press, 1998.

2010 *Mathematics Subject Classification*: Primary 05A17.
Keywords: n-color odd composition, n-color composition, generating function, explicit formula, recurrence formula.

Received June 12 2011 revised version received November 27 2011. Published in *Journal of Integer Sequences*, December 26 2011.

Return to Journal of Integer Sequences home page.