Safe dropping in incomplete decompositions

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Numerical simulations in science and engineering often lead to systems of sparse linear algebraic equations. Nowadays, efficient solvers of such systems are often hybrid techniques combining direct and iterative methods accompanied by preprocessing and postprocessing tools.

Our contribution targets the problem of safe dropping in incomplete factorizations for symmetric and positive definite systems. Such factorizations often represent a relaxed direct method used in combination with an iterative method as a preconditioner. Construction of the incomplete factorizations can be in some cases rather simple but in other cases it may heavily exploit algorithms, models, reorderings and implementations used in direct methods. This is the case of our particular strategy.

The safe dropping is developed for a particular technique that can explicitly monitor the factorization quality. This approach is the generalized Gram-Schmidt process where the accuracy of computed factors and the loss of orthogonality can be explicitly estimated even when performed incompletely. Recently developed theory for this decomposition in the floating-point arithmetic indicates that monitoring of the factorization quality can be based on the quantities that are directly available during the course of decomposition. Consequently, estimated level of the factor accuracy and loss of orthogonality can be used to develop an adaptive dropping strategy.

Our experiments confirm that combining this approach with pivoting can produce incomplete decompositions that are both sparse and powerful. While our results are still algorithm-specific, there is a relation between the factors obtained by the generalized Gram-Schmidt process and factors of the standard positive definite decomposition. We believe that the theory used here can help in better understanding of safe droppings also in the incomplete Cholesky decomposition.